GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 1-Mar-02 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ________________

LAST, FIRST

Recitation Section: **Circle the day & time** when your recitation section meets:

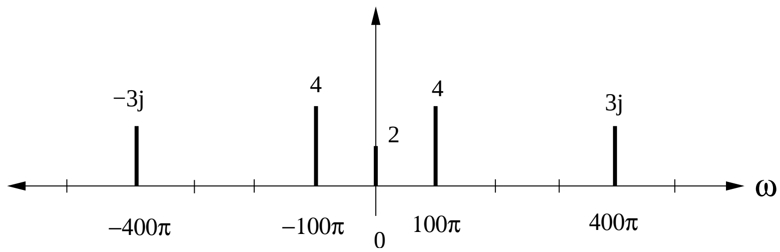
L02:Tues-9:30am (Bordelon)	L04:Tues-12:00pm (Yezzi)	L05:Thurs-1:30pm (Williams)
L06:Tues-1:30pm (Bordelon)	L07:Thurs-3:00pm (Williams)	L08:Tues-3:00pm (Smith)
L11:Mon-3:00pm (Glytsis)	L14:Mon-4:00pm (McClellan)	RPK: (Abler) Vald: (Fares)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem SPRING-02-Q.1.1:

Shown in the figure is a spectrum plot for the periodic signal $x(t)$.



(a) Determine the period T_0 of $x(t)$.

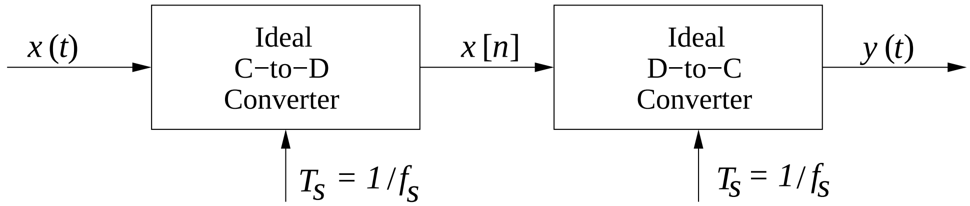
$T_0 =$

(b) A periodic signal of this type can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}.$$

If the Fourier series coefficients of $x(t)$ are denoted by a_k, $k = 0, \pm 1, \pm 2, \pm 3, \ldots$, determine which coefficients have non-zero value. List these Fourier series coefficients and their values in the box below.

Problem SPRING-02-Q.1.2:



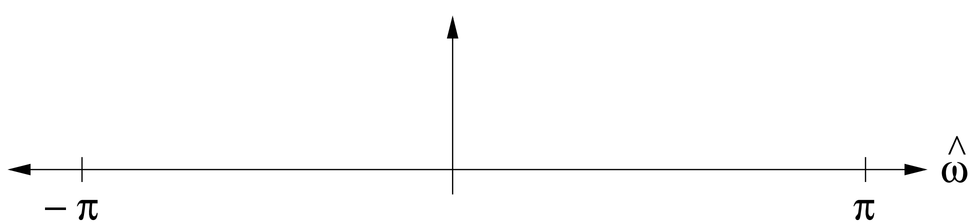
Shown in the figure above is an ideal C-to-D converter that samples $x(t)$ with a sampling period T_s to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$.

Let $x(t) = 6\cos(2\pi(8000)t) + 8\sin(2\pi(6000)t)$

(a) What is the <u>minimum</u> sampling rate such that $y(t) = x(t)$?

$f_s =$

(b) Sketch the digital spectrum of $x[n]$ when $f_s = 20000$ samples/sec. Carefully label the amplitudes and frequencies in your sketch.



(c) If we under-sample $x(t)$, aliases of the spectral components can appear in the baseband of the digital spectrum. What is the <u>maximum</u> sampling rate $f_{s\,\max}$ such that the spectrum of $x[n]$ will have a non-zero DC component?

$f_{s\,\max} =$

Determine the amplitude value for the DC component of $x[n]$ at this sampling rate.

DC Value =

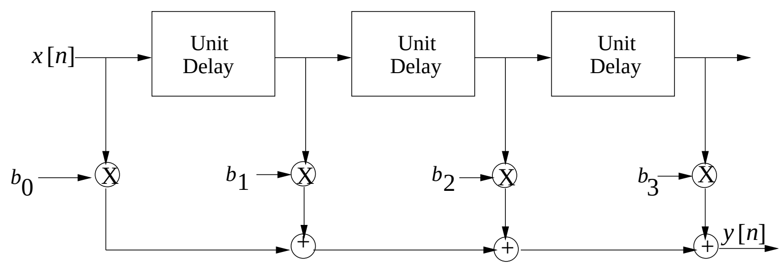
Problem SPRING-02-Q.1.3:

The following problem considers three different discrete-time systems. In each case, the input is $x[n]$ and the output is $y[n]$.

(a) If an LTI system has impulse response $h[n] = \frac{3}{4}\delta[n] - \frac{1}{2}\delta[n-1] + 2\delta[n-2]$, determine the difference equation that relates $x[n]$ and $y[n]$.

$y[n] =$

(b) If an LTI system is described by the block diagram below



where $b_0 = 1$, $b_1 = 0$, $b_2 = \frac{1}{2}$, $b_3 = \frac{1}{2}$, determine its impulse response $h[n]$.

$h[n] =$

(c) If a system is defined by the relation

$$y[n] = x[n^2] + (x[n-1])^2,$$

indicate which of the statements below is true or false by circling the appropriate T or F.

i. The system is linear. T or F

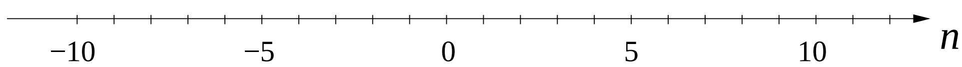
ii. The system is time-invariant. T or F

iii. The system is causal. T or F

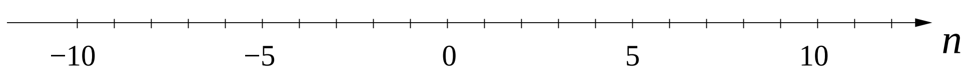
Problem SPRING-02-Q.1.4:

Let $x[n] = u[n] - u[n-7]$ and $h[n] = \begin{cases} (\frac{1}{2})^n & 0 \le n \le 3 \\ 0 & \text{otherwise.} \end{cases}$

(a) Plot $x[n]$.

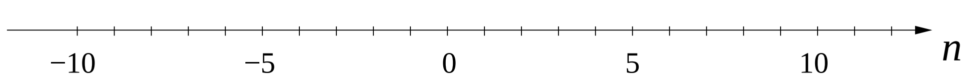


Plot $h[n]$.



Label the amplitudes for each sample.

(b) If we now assume $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ and $y[n] = x[n] * h[n]$, where $h[n]$ is as defined above, plot $y[n]$ on the axis below.



Problem SPRING-02-Q.1.5:

Let $h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$ be the impulse response of an LTI system and let

$$x[n] = 2e^{j(\pi/2)n}, \qquad -\infty < n < \infty$$

be the input to that system.

(a) Determine the frequency response $\mathcal{H}(\hat{\omega})$ of $h[n]$.[1]

$\mathcal{H}(\hat{\omega}) =$

(b) If $y[n] = h[n] * x[n]$, the output is a complex exponential of the form $Ae^{j(\omega_o n+\phi)}$, where A is a real positive number. Determine A, ϕ and ω_o.

$A =$

$\phi =$

$\omega_o =$

[1]We have also used the notation $H(e^{j\hat{\omega}})$ for the frequency response; i.e. $\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}})$.

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 1-Mar-02 COURSE: ECE 2025

NAME: Answer Key (LAST, FIRST) STUDENT #: A

Recitation Section: **Circle the day & time** when your recitation section meets:

L02:Tues-9:30am (Bordelon) L04:Tues-12:00pm (Yezzi) L05:Thurs-1:30pm (Williams)

L06:Tues-1:30pm (Bordelon) L07:Thurs-3:00pm (Williams) L08:Tues-3:00pm (Smith)

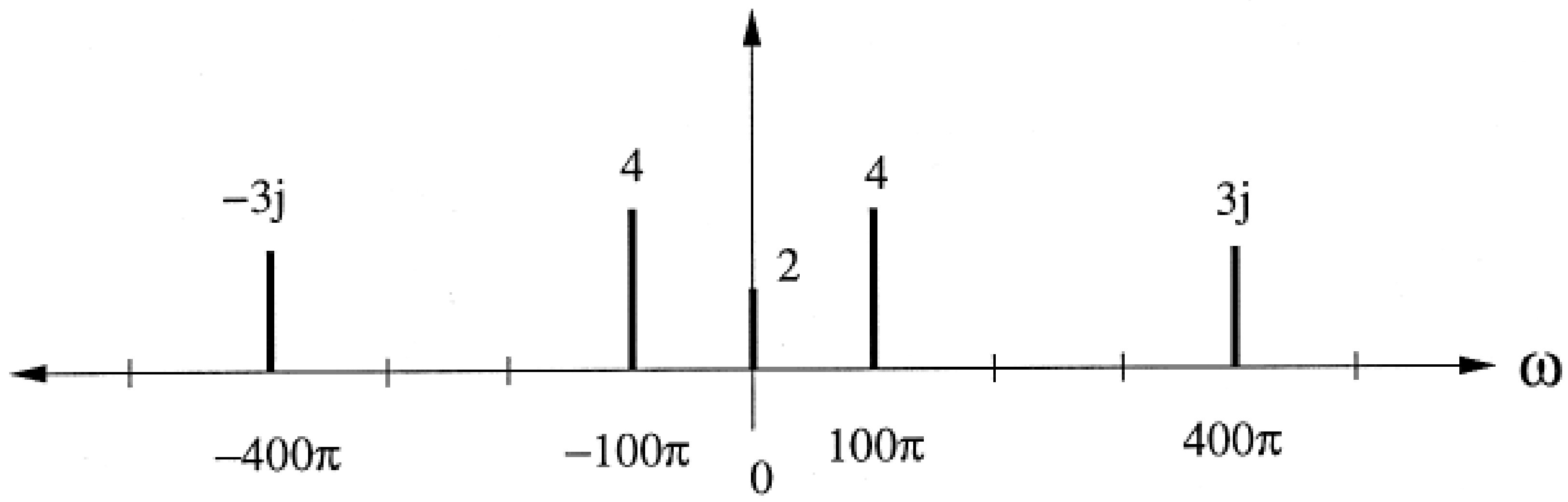
L11:Mon-3:00pm (Glytsis) L14:Mon-4:00pm (McClellan) RPK: (Abler) Vald: (Fares)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem SPRING-02-Q.1.1:

Shown in the figure is a spectrum plot for the periodic signal $x(t)$.



(a) Determine the period T_0 of $x(t)$.

$\omega_0 = \gcd(100\pi, 400\pi) = 100\pi$ rad/s

$f_0 = 50$ Hz ← fundamental freq.

$T_0 = 1/f_0 = 1/50$ sec

$T_0 =$ 1/50 sec.

(b) A periodic signal of this type can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}.$$

If the Fourier series coefficients of $x(t)$ are denoted by a_k, $k = 0, \pm1, \pm2, \pm3, \ldots$, determine which coefficients have non-zero value. List these Fourier series coefficients and their values in the box below.

There are non-zero coeffs for $k = \pm1, \pm4, 0$

$a_0 = 2$

$a_1 = 4$

$a_{-1} = 4$

$a_4 = 3j = 3e^{j\pi/2}$

$a_{-4} = -3j = 3e^{-j\pi/2}$

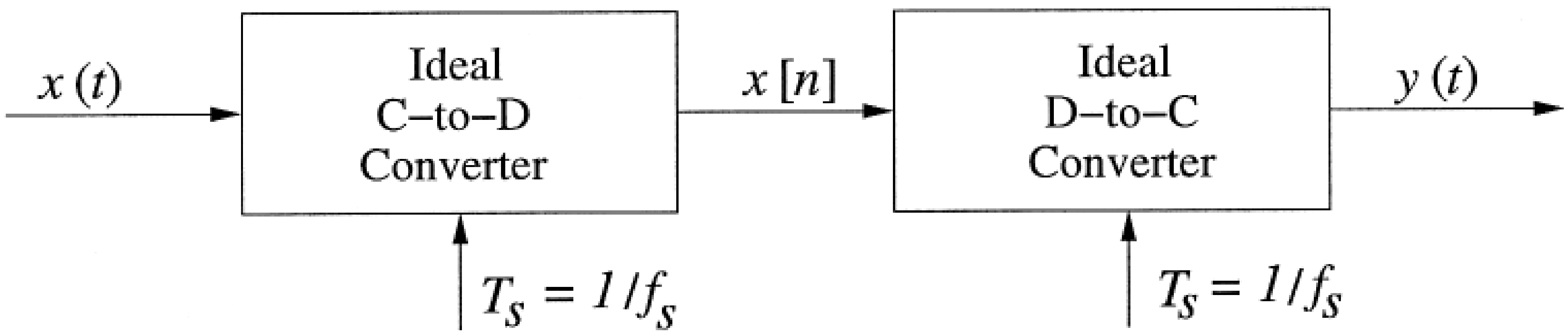
$a_2 = 0$

$a_{-2} = 0$

$a_3 = 0$

$a_{-3} = 0$

Problem SPRING-02-Q.1.2:



Shown in the figure above is an ideal C-to-D converter that samples $x(t)$ with a sampling period T_s to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$.

Let $x(t) = 6\cos(2\pi(8000)t) + 8\sin(2\pi(6000)t)$ ← $8\cos(2\pi(6000)t - \pi/2)$

(a) What is the <u>minimum</u> sampling rate such that $y(t) = x(t)$?

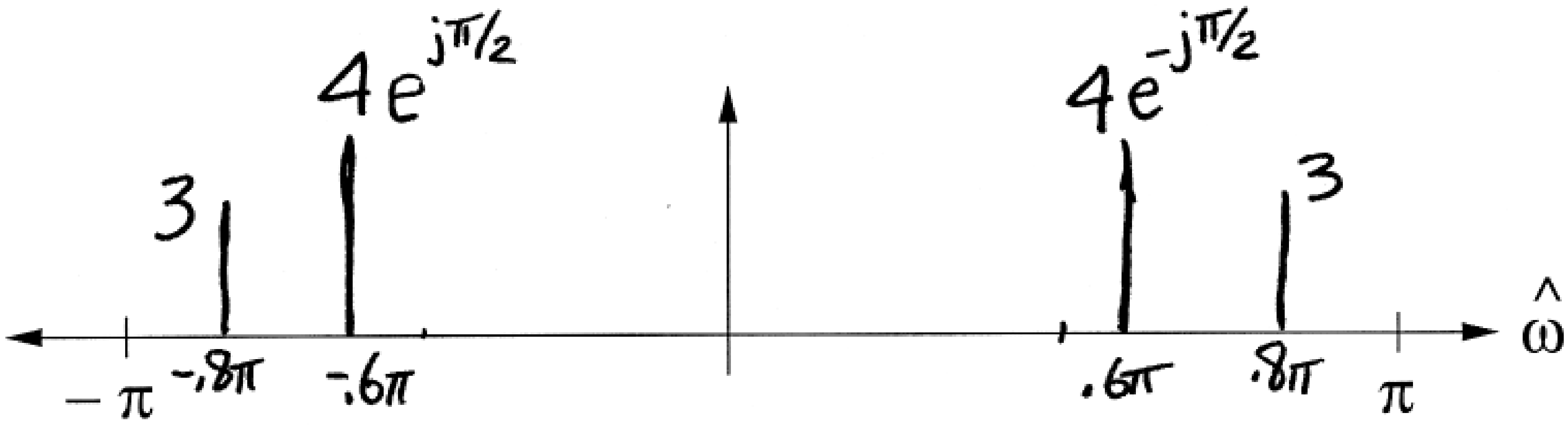
$f_s =$ 16,000 Hz

Sampling Thm ⇒ $f_s \geq 2 f_{MAX}$
$f_{MAX} = 8000$ Hz

(b) Sketch the digital spectrum of $x[n]$ when $f_s = 20{,}000$ samples/sec. Carefully label the amplitudes and frequencies in your sketch.

$\hat{\omega}_1 = 2\pi \frac{8000}{20000} = 0.8\pi$

$\hat{\omega}_2 = 2\pi \frac{6000}{20000} = 0.6\pi$



(c) If we under-sample $x(t)$, aliases of the spectral components can appear in the baseband of the digital spectrum. What is the <u>maximum</u> sampling rate $f_{s\,\max}$ such that the spectrum of $x[n]$ will have a non-zero DC component?

Need $\hat{\omega} = 2\pi$. If $f_s = 8000$, $\hat{\omega} = 2\pi \frac{8000}{8000}$

$f_{s\,\max} =$ 8000 Hz

Determine the amplitude value for the DC component of $x[n]$ at this sampling rate.

DC Value = 6

At $f_s = 8000$ Hz, the DC is
$6\cos\left(2\pi \frac{8000 n}{8000}\right) = 6\cos(2\pi n) = 6$

Problem SPRING-02-Q.1.3:

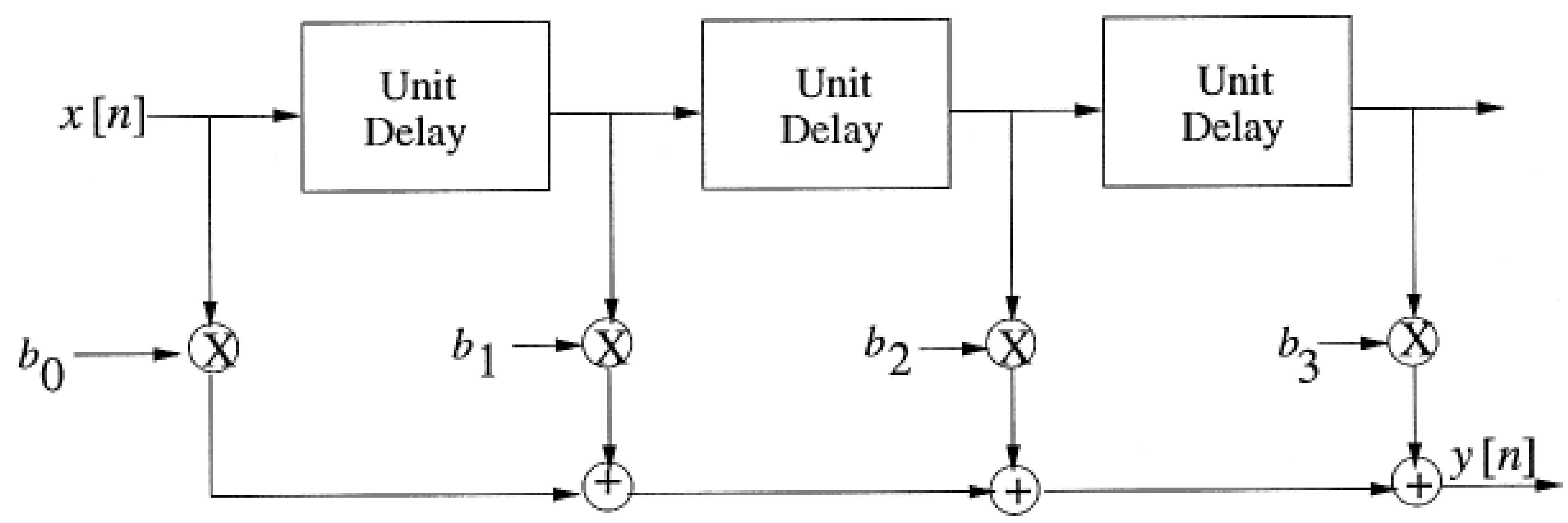
The following problem considers three different discrete-time systems. In each case, the input is $x[n]$ and the output is $y[n]$.

(a) If an LTI system has impulse response $h[n] = \frac{3}{4}\delta[n] - \frac{1}{2}\delta[n-1] + 2\delta[n-2]$, determine the difference equation that relates $x[n]$ and $y[n]$.

$y[n] = \frac{3}{4}x[n] - \frac{1}{2}x[n-1] + 2x[n-2]$

$b_k = \{\frac{3}{4}, -\frac{1}{2}, 2\}$ from $h[n]$

(b) If an LTI system is described by the block diagram below



where $b_0 = 1$, $b_1 = 0$, $b_2 = \frac{1}{2}$, $b_3 = \frac{1}{2}$, determine its impulse response $h[n]$. $= \sum_{k=0}^{M} b_k \delta[n-k]$

$h[n] = \delta[n] + \frac{1}{2}\delta[n-2] + \frac{1}{2}\delta[n-3]$

(c) If a system is defined by the relation

$$y[n] = x[n^2] + (x[n-1])^2,$$

indicate which of the statements below is true or false by circling the appropriate T or F.

i. The system is linear. T or (F)

ii. The system is time-invariant. T or (F)

iii. The system is causal. T or (F)

If $x[n] = \delta[n]$, $y[n] = \delta[n] + \delta[n-1]$

$x[n] = 2\delta[n] \longrightarrow y[n] = 2\delta[n] + 4\delta[n-1]$

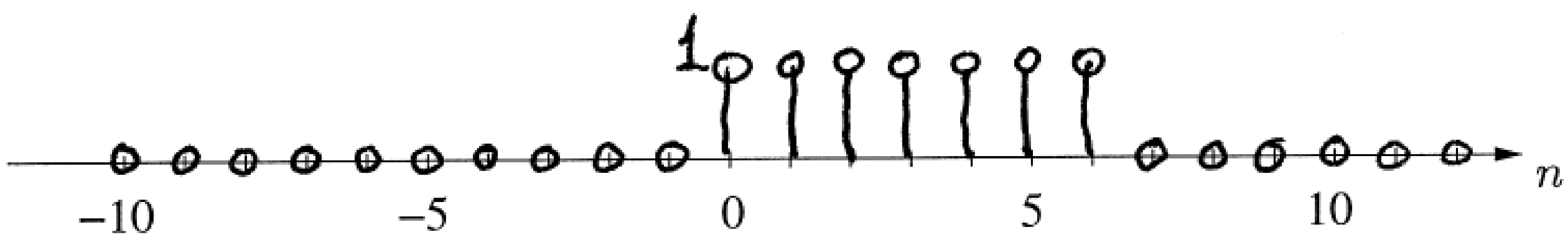
$x[n] = \delta[n+1] \longrightarrow y[n] = 0 + \delta[n]$

$x[n] = \delta[n-1] \longrightarrow y[n] = \delta[n+1] + \delta[n-1] + \delta[n-2]$

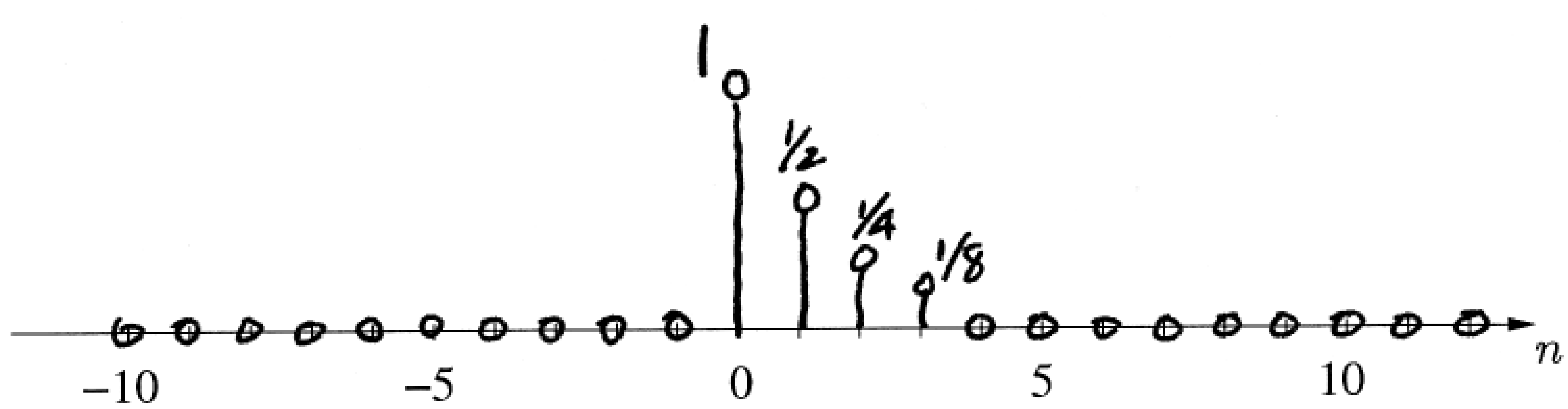
Problem SPRING-02-Q.1.4:

Let $x[n] = u[n] - u[n-7]$ and $h[n] = \begin{cases} (\frac{1}{2})^n & 0 \leq n \leq 3 \\ 0 & \text{otherwise.} \end{cases}$

(a) Plot $x[n]$.



Plot $h[n]$.



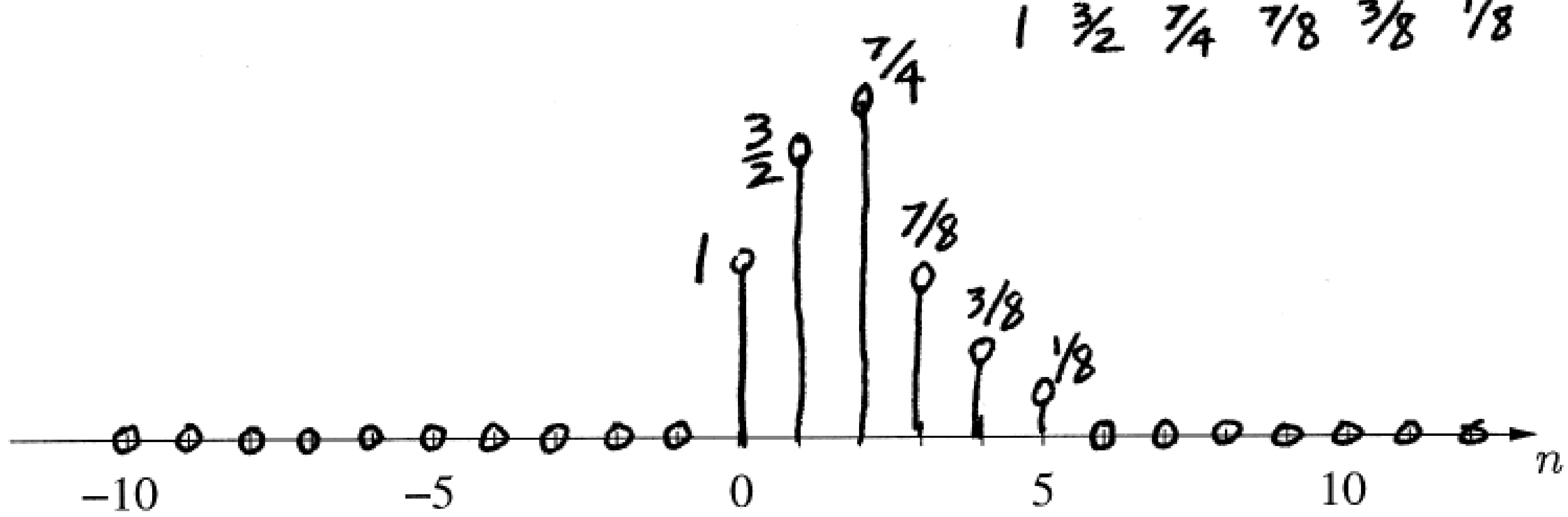
Label the amplitudes for each sample.

(b) If we now assume $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ and $y[n] = x[n] * h[n]$, where $h[n]$ is as defined above, plot $y[n]$ on the axis below.

1 ½ ¼ ⅛
1 1 1

1 ½ ¼ ⅛
1 ½ ¼ ⅛
1 ½ ¼ ⅛

1 3/2 7/4 7/8 3/8 1/8



Problem SPRING-02-Q.1.5:

Let $h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$ be the impulse response of an LTI system and let

$$x[n] = 2e^{j\pi/2n}, \qquad -\infty < n < \infty$$

be the input to that system.

(a) Determine the frequency response $\mathcal{H}(\hat{\omega})$ of $h[n]$.[1]

$\mathcal{H}(\hat{\omega}) =$ $(2+2\cos\hat{\omega})e^{-j\hat{\omega}}$

$\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) = e^{-j\hat{\omega}}(2+2\cos\hat{\omega})$

(b) If $y[n] = h[n] * x[n]$, the output is a complex exponential of the form $Ae^{j(\omega_o n+\phi)}$, where A is a real positive number. Determine A, ϕ and ω_o.

$A =$ 4

$\phi =$ $-\pi/2$

$\omega_o =$ $\pi/2$

Use $\mathcal{H}(\hat{\omega})$ at $\hat{\omega} = \frac{\pi}{2}$

$\mathcal{H}(\frac{\pi}{2}) = (2+2\cos\frac{\pi}{2})e^{-j\pi/2}$

$= 2e^{-j\pi/2}$

$y[n] = \mathcal{H}(\frac{\pi}{2}) \cdot 2e^{j\frac{\pi}{2}n}$

$= 2e^{-j\pi/2} \cdot 2e^{j\frac{\pi}{2}n}$

$= 4e^{-j\pi/2}e^{j\frac{\pi}{2}n}$

[1] We have also used the notation $H(e^{j\hat{\omega}})$ for the frequency response; i.e. $\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}})$.