GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 6-April-01 COURSE: ECE 2025

NAME: ______________________ STUDENT #: ______________
LAST, FIRST

Recitation Section: Circle the date & time when your Recitation Section meets (not Lab):

Mon-3p (L11:McClellan)	M-4:30p (L13:Frazier)			
Tues-9:30a (L01:Casinovi)	T-Noon (L03:Casinovi)	T-1:30p (L05:Bordelon)	T-3p (L07:Bordelon)	T-4:30p (L09:Casinovi)
Thur-9:30a (L02:Bordelon)	Th-Noon (L04:Bordelon)	Th-1:30p (L06:Smith)	Th-3p (L08:Smith)	Th-4:30p (L09:Casinovi)
Th-6p (L10:Casinovi)				

- Write your name on the front page ONLY.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning CLEARLY to receive any partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

Problem spr-01-Q.3.1:

In each of the following cases, simplify the expression **as much as possible** using the properties of the continuous-time unit impulse signal. In part (d) find the requested $h(t)$. Provide some **explanation** or intermediate steps for each answer.

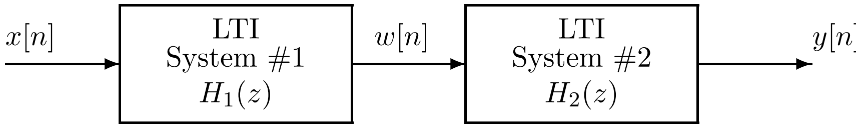
(a) $\sin[5\pi(t-2)]u(t-2)\delta(t-3) =$

(b) $\displaystyle\int_{-\infty}^{t+3} \delta(\tau)d\tau =$

(c) $\dfrac{d}{dt}\{\sin[5\pi(t-2)]u(t-2)\} =$

(d) $\{\sin[5\pi(t-2)]u(t-2)\} * h(t) = 4\sin[5\pi(t-5)]u(t-5)$ (find $h(t)$ that satisfies this equation)

Problem spr-01-Q.3.2:
A cascade of two FIR discrete-time systems is depicted by the following block diagram:



The first system is is defined by the following z-transform system function:

$$H_1(z) = (1 + z^{-2}).$$

The second system is defined by the z-transform system function

$$H_2(z) = (1 - re^{j\hat{\omega}_0}z^{-1})(1 - re^{-j\hat{\omega}_0}z^{-1}) = 1 - 2r\cos(\hat{\omega}_0)z^{-1} + r^2z^{-2}.$$

(a) If the input to the first system is

$$x[n] = 4\cos\left(\frac{\pi}{4}n\right) \qquad -\infty < n < \infty,$$

determine the output, $w[n]$, of the **first** system. *Express your answer for $w[n]$ as a single cosine.*

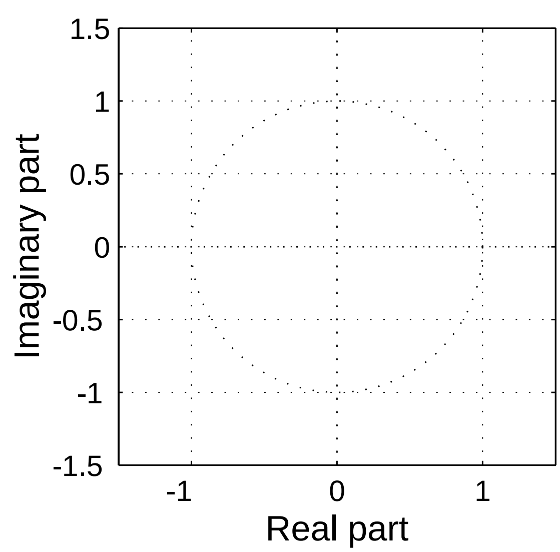
$w[n] =$

(b) For the input of part (a), how should r and $\hat{\omega}_0$ in the system function of the **second** system, $H_2(z)$, be chosen so that $y[n] = 0$ for $-\infty < n < \infty$?

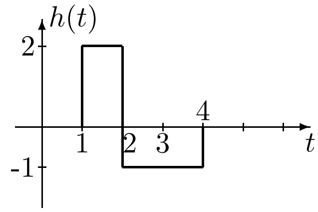
$r =$

$\hat{\omega}_0 =$

(c) For r and $\hat{\omega}_0$ found in part (b) and the given system function $H_1(z)$, determine *all* the zeros of the **overall** system function $H(z)$ and plot them in the z-plane. *If you were unable to find values for r and $\hat{\omega}_0$ in part (b), use values of $r = .5$ and $\hat{\omega}_0 = \pi/2$ for this part.*

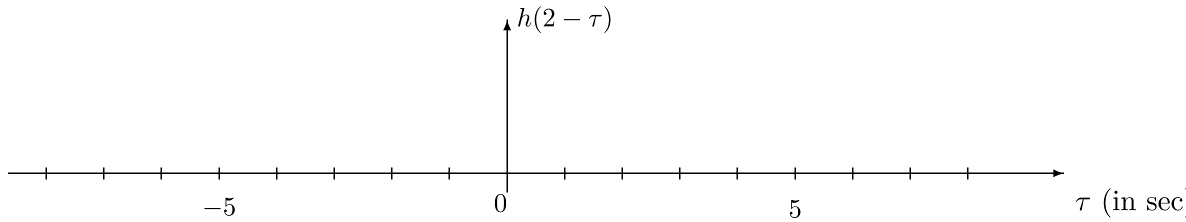


Problem spr-01-Q.3.3:
A linear time-invariant system has impulse response:



(a) Is the LTI system stable? Give a reason to support your answer.

(b) Plot $h(t-\tau)$ versus τ, for $t=2$. Label your plot carefully.



(c) If the input is $x(t)=u(t)$, use the convolution integral to find $y(2)$; i.e., $y(t)$ when $t=2$.

(d) It can be seen that, for the input $x(t)=u(t)$ and the given impulse response, the output is $\boxed{y(t)=0 \text{ for } t<T_1 \text{ and for } t>T_2}$. Find T_1 and T_2. $\boxed{\textbf{Explain}}$ your answers. *You may "flip and shift" either $x(t)$ or $h(t)$, whichever leads to the easiest solution.*

Problem spr-01-Q.3.4:

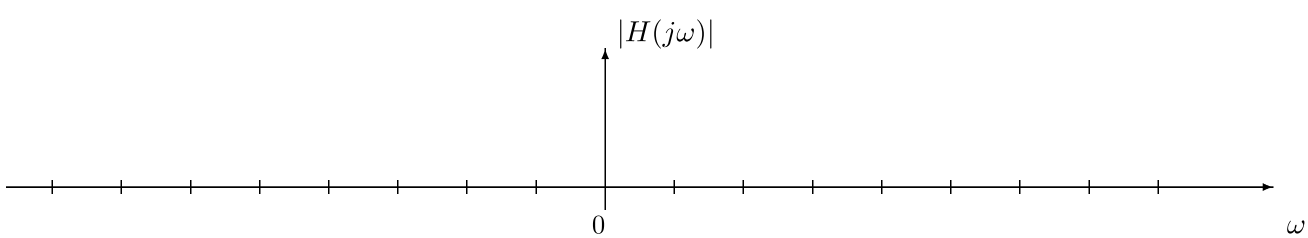
In each of the following cases, determine the Fourier transform, or inverse Fourier transform. Give your answer as a $\boxed{\text{simple}}$ formula or a plot. $\boxed{\textbf{Explain}}$ each answer by stating which property and transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = 20e^{-7(t-1)}u(t-1)$.

(b) Find $s(t)$ when $S(j\omega) = e^{-j\omega/2}[j\pi\delta(\omega + 10\pi) - j\pi\delta(\omega - 10\pi)]$.

(c) Find $H(j\omega)$ when $h(t) = \dfrac{\sin(6\pi t)}{\pi t} - \dfrac{\sin(4\pi t)}{\pi t}$.

(d) Plot $|H(j\omega)|$ found in part (c) on the graph below.



Problem spr-01-Q.3.1:
In each of the following cases, simplify the expression **as much as possible** using the properties of the continuous-time unit impulse signal. In part (d) find the requested $h(t)$. Provide some **explanation** or intermediate steps for each answer.

(a) $\sin[5\pi(t-2)]u(t-2)\delta(t-3) =$

Since $f(t)\delta(t-3) = f(3)\delta(t-3)$, we get

$$\sin[5\pi(3-2)]\,u(3-2)\,\delta(t-3)$$

$$= \sin[5\pi]\,u(1)\,\delta(t-3) = (0)(1)\delta(t-3) = 0$$

(b) $\int_{-\infty}^{t+3} \delta(\tau)d\tau = \quad u(t)\Big|_{-\infty}^{t+3} = u(t+3) - \cancel{u(-\infty)}^{\to 0} = u(t+3)$

(c) $\frac{d}{dt}\{\sin[5\pi(t-2)]u(t-2)\} =$

$$= \underbrace{\sin[5\pi(t-2)]\delta(t-2)}_{\text{at } t=2} + 5\pi\cos[5\pi(t-2)]u(t-2)$$

$$= \cancel{\sin[5\pi(0)]}^{\to 0}\,\delta(t-2) + 5\pi\cos[5\pi(t-2)]u(t-2)$$

$$= 5\pi\cos[5\pi(t-2)]u(t-2)$$

(d) $\{\sin[5\pi(t-2)]u(t-2)\} * h(t) = 4\sin[5\pi(t-5)]u(t-5)$ (find $h(t)$ that satisfies this equation)

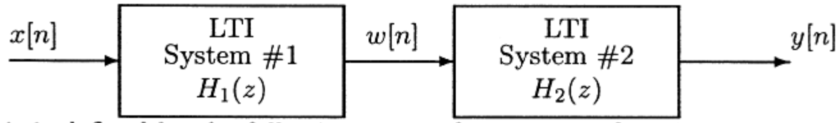
h(t) must do two things: Multiply by 4 and shift right by 3 secs.

$$\Rightarrow \quad h(t) = 4\delta(t-3)$$

$$x(t) * 4\delta(t-3) = 4x(t-3)$$

Problem spr-01-Q.3.2:

A cascade of two FIR discrete-time systems is depicted by the following block diagram:



The first system is is defined by the following z-transform system function:

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(a) If the input to the first system is

$$x[n] = 4\cos\left(\frac{\pi}{4}n\right) \qquad -\infty < n < \infty,$$

determine the output, $w[n]$, of the **first** system. *Express your answer for $w[n]$ as a single cosine.* Use $H_1(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/4$

$$H_1(e^{j\hat{\omega}}) = 1 + e^{-j2\hat{\omega}}$$

$$H_1(e^{j\pi/4}) = 1 + e^{-j2\pi/4} = 1 + e^{-j\pi/2} = 1 - j = \sqrt{2}\,e^{-j\pi/4}$$

$$\boxed{w[n] = 4\sqrt{2}\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{4}\right)}$$

(b) For the input of part (a), how should r and $\hat{\omega}_0$ in the system function of the **second** system, $H_2(z)$, be chosen so that $y[n] = 0$ for $-\infty < n < \infty$?

we need to null out $\hat{\omega} = \pm\pi/4$. Thus we need zeros on the unit circle. The zeros of $H_2(z)$ are: $re^{\pm j\hat{\omega}_0}$

$\boxed{r = 1}$ $\boxed{\hat{\omega}_0 = \pi/4}$

(c) For r and $\hat{\omega}_0$ found in part (b) and the given system function $H_1(z)$, determine *all* the zeros of the **overall** system function $H(z)$ and plot them in the z-plane. *If you were unable to find values for r and $\hat{\omega}_0$ in part (b), use values of $r = .5$ and $\hat{\omega}_0 = \pi/2$ for this part.*

$H(z) = H_1(z)H_2(z)$, so we need the zeros of both $H_1(z)$ and $H_2(z)$.

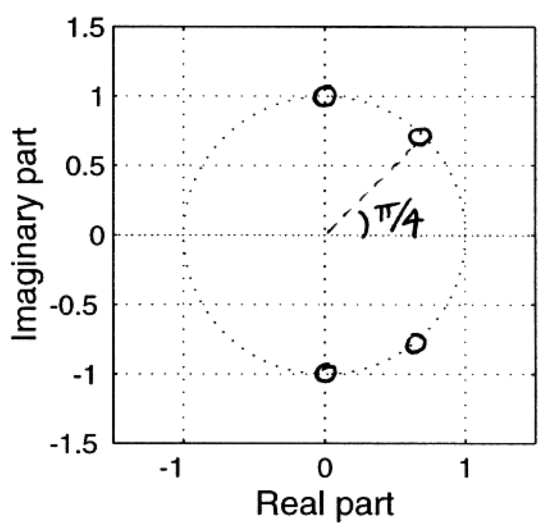
for $H_1(z)$, the zeros are

$1 + z^{-2} = 0$

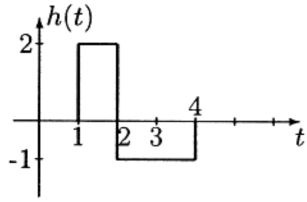
$z^2 = -1 \Rightarrow z = \pm j$

for $H_2(z)$ the zeros are

$re^{\pm j\hat{\omega}_0} = 1e^{\pm j\pi/4}$



Problem spr-01-Q.3.3:
A linear time-invariant system has impulse response:

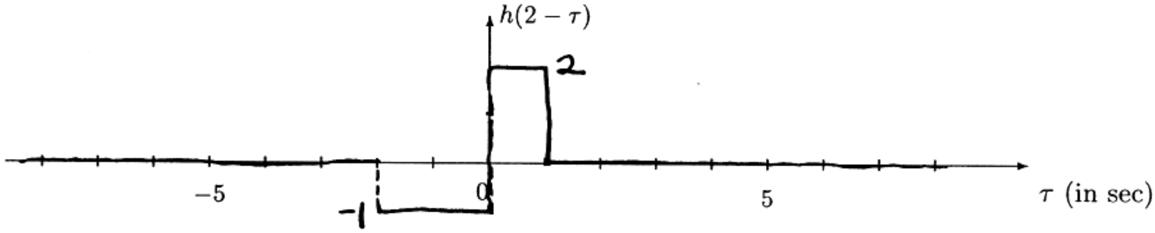


(a) Is the LTI system stable? Give a reason to support your answer.

Yes, the system is stable because

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_1^2 2\,dt + \int_2^4 |-1|\,dt = 2 + 2 = 4 < \infty$$

(b) Plot $h(t-\tau)$ versus τ, for $t = 2$. Label your plot carefully. Flip & shift by 2



(c) If the input is $x(t) = u(t)$, use the convolution integral to find $y(2)$; i.e., $y(t)$ when $t = 2$.

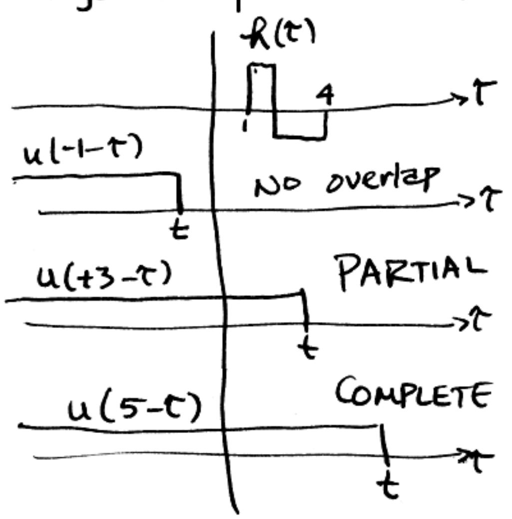
$$y(2) = \int_{-\infty}^{\infty} x(\tau) h(2-\tau)\,d\tau = \int_{-\infty}^{\infty} u(\tau) h(2-\tau)\,d\tau = \int_0^{\infty} h(2-\tau)\,d\tau$$

$$= \int_0^1 2\,d\tau = 2$$

use the picture above to get $h(2-\tau)$

(d) It can be seen that, for the input $x(t) = u(t)$ and the given impulse response, the output is $\boxed{y(t) = 0 \text{ for } t < T_1 \text{ and for } t > T_2}$. Find T_1 and T_2. **Explain** your answers. *You may "flip and shift" either $x(t)$ or $h(t)$, whichever leads to the easiest solution.*

If we flip & slide $x(t) = u(t)$ we get the pictures below:



When $t < 1$, there is No overlap, so $y(t) = 0$

When $t \geq 4$, there is Complete overlap, but the integral of $h(\tau)$ is zero.

$\boxed{T_1 = 1}$ $\boxed{T_2 = 4}$

Problem spr-01-Q.3.4:
In each of the following cases, determine the Fourier transform, or inverse Fourier transform. Give your answer as a $\boxed{\text{simple}}$ formula or a plot. $\boxed{\textbf{Explain}}$ each answer by stating which property and transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = 20e^{-7(t-1)}u(t-1)$.

shift by 1 $\Rightarrow e^{-j\omega}$

$$X(j\omega) = 20e^{-j\omega}\frac{1}{7+j\omega}$$

0.5

(b) Find $s(t)$ when $S(j\omega) = e^{-j\omega/2}[j\pi\delta(\omega+10\pi) - j\pi\delta(\omega-10\pi)]$.

time-shift by ½

$$s(t) = 0.5j\,e^{-j10\pi(t-\frac{1}{2})} - 0.5j\,e^{+j10\pi(t-\frac{1}{2})}$$
$$= 0.5j\,e^{j5\pi}e^{-j10\pi t} - 0.5j\,e^{-j5\pi}e^{j10\pi t}$$
$$= -0.5j\,e^{-j10\pi t} + 0.5j\,e^{j10\pi t}$$

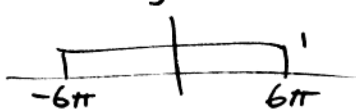
$$0.5e^{j(10\pi t+\pi/2)} + 0.5e^{-j(10\pi t+\pi/2)}$$
$$= \cos(10\pi t + \pi/2)$$
or $-\sin(10\pi t)$

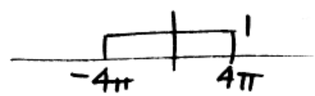
(c) Find $H(j\omega)$ when $h(t) = \dfrac{\sin(6\pi t)}{\pi t} - \dfrac{\sin(4\pi t)}{\pi t}$.

$$H(j\omega) = [u(\omega+6\pi) - u(\omega-6\pi)] - [u(\omega+4\pi) - u(\omega-4\pi)].$$

rectangle from -6π to $+6\pi$

rectangle: -4π to $+4\pi$





(d) Plot $|H(j\omega)|$ found in part (c) on the graph below.