GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #3

DATE: 5-April-02 COURSE: ECE 2025

NAME: ______________________________ GT #: gt______________
LAST, First

Recitation Section: **CIRCLE THE DAY & TIME** when your Recitation Section meets:

L02:Tues-9:30am (Bordelon) L04:Tues-12:00pm (Yezzi) L05:Thurs-1:30pm (Williams)

L06:Tues-1:30pm (Bordelon) L07:Thur-3:00pm (Williams) L08:Tues-3:00pm (Smith

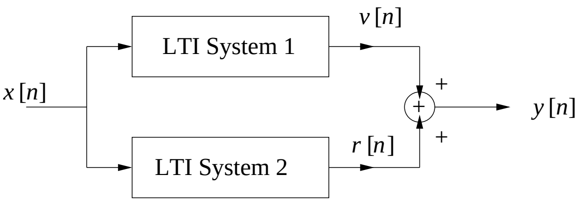
L11:Mon-3:00pm (Glytsis) L14:Mon-4:00pm (McClellan) RPK: (Abler) Vald: (Fares)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2} \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem 1: (20%)

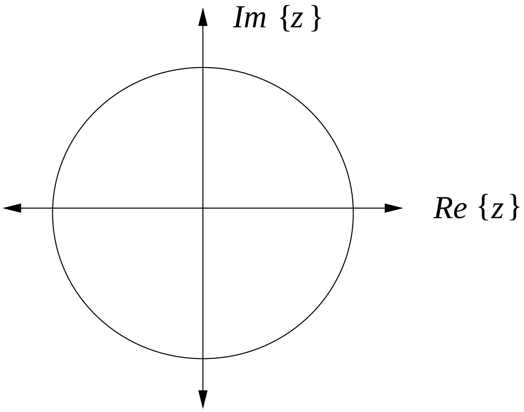
Consider the parallel form LTI system depicted below.



System 1 is defined by the difference equation $v[n] = x[n] - x[n-5]$.

System 2 is defined by the system function $H_2(z) = z^{-1} + \frac{1}{4}z^{-2} + z^{-5}$.

(a) Determine the system function $H_1(z)$ associated with System 1 and plot the zeros of $H_1(z)$.



$H_1(z) =$

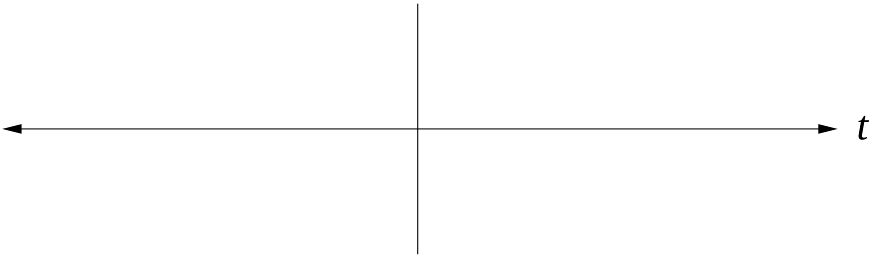
(b) Determine the impulse response of the overall parallel form system. That is, find $h[n]$ such that $y[n] = x[n] * h[n]$.

$h[n] =$

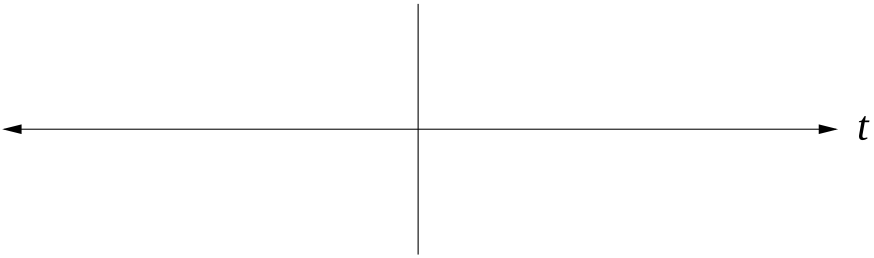
Problem 2: (20%)

Let $x(t) = u(t) - u(t-4)$.

(a) Sketch $\frac{d}{dt}x(t)$. Carefully label your plot.



(b) Sketch $x(2-t)$. Carefully label your plot.



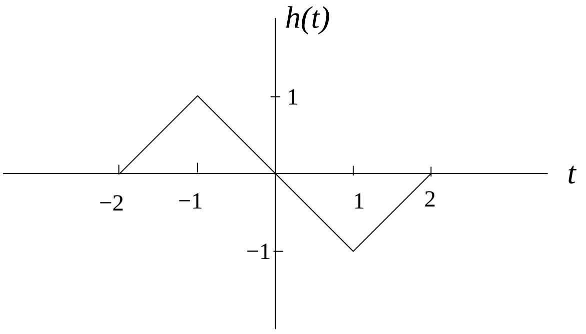
(c) If $x(t) * x(t-3) * \delta(t-T) = x(t-5) * x(t-6)$, determine the numerical value of T.

$T =$

Problem 3: (20%)

Consider the LTI System whose output is $y(t) = x(t) * h(t)$, where $x(t) = u(t)$

and $h(t)$ is given by



(a) Determine $y(-1)$, the value of $y(t)$ for $t = -1$.

$y(-1) =$

(b) You should be able to see that $y(t) = 0$ in two regions: $T_1 \leq t \leq T_2$ and $T_3 \leq t \leq T_4$. Determine T_1, T_2, T_3, and T_4. **Explain carefully to receive full credit.**

$T_1 =$ __________, $T_2 =$ __________, $T_3 =$ __________, $T_4 =$ __________

(c) Is this system causal? yes or no (Circle one)

Problem 4: (20%)

Let $h(t) = \delta(t + 10) + 2\delta(t) + \delta(t - 10)$.

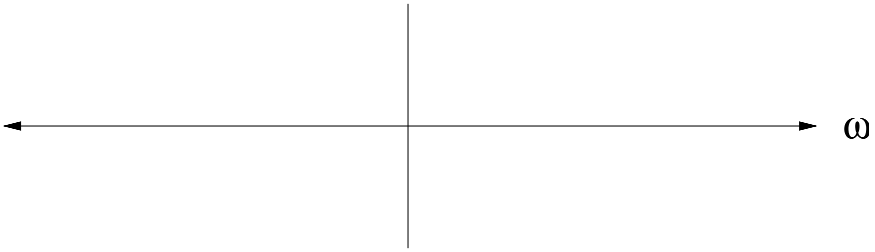
(a) Find $H(j\omega)$.

$H(j\omega) =$

(b) Let $y(t) = h(t - 10)$. Find the phase of $Y(j\omega)$, the Fourier transform of $y(t)$.

$\angle Y(j\omega) =$

(c) If $x(t) = \dfrac{\sin 100\pi t}{\pi t} - \dfrac{\sin 50\pi t}{\pi t}$, plot $X(j\omega)$. Carefully label your plot.



Problem 5: (20%)

Assume that $x(t)$ is the periodic function given by

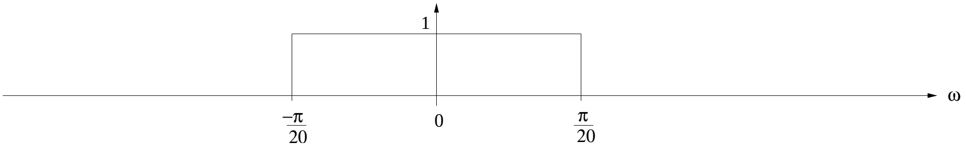
$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 50k) = \sum_{k=-\infty}^{\infty} \frac{1}{50} e^{j\omega_0 kt}.$$

(a) Determine the value of the fundamental frequency ω_0.

$\omega_0 =$

(b) Suppose that $x(t)$ is the input to an LTI system with the frequency response illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| \leq \dfrac{\pi}{20} \\ 0 & |\omega| > \dfrac{\pi}{20} \end{cases}$$

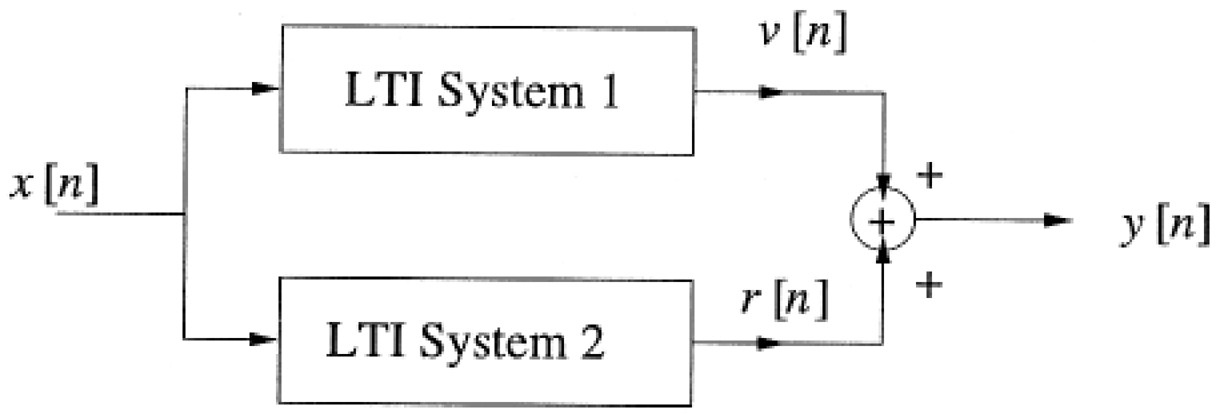


Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. Your answer should be expressed in terms of only real quantities. (Hint: Plot the spectrum of $x(t)$ on the plot of the frequency response.)

$y(t) =$

Problem 1: (20%)

Consider the parallel form LTI system depicted below.

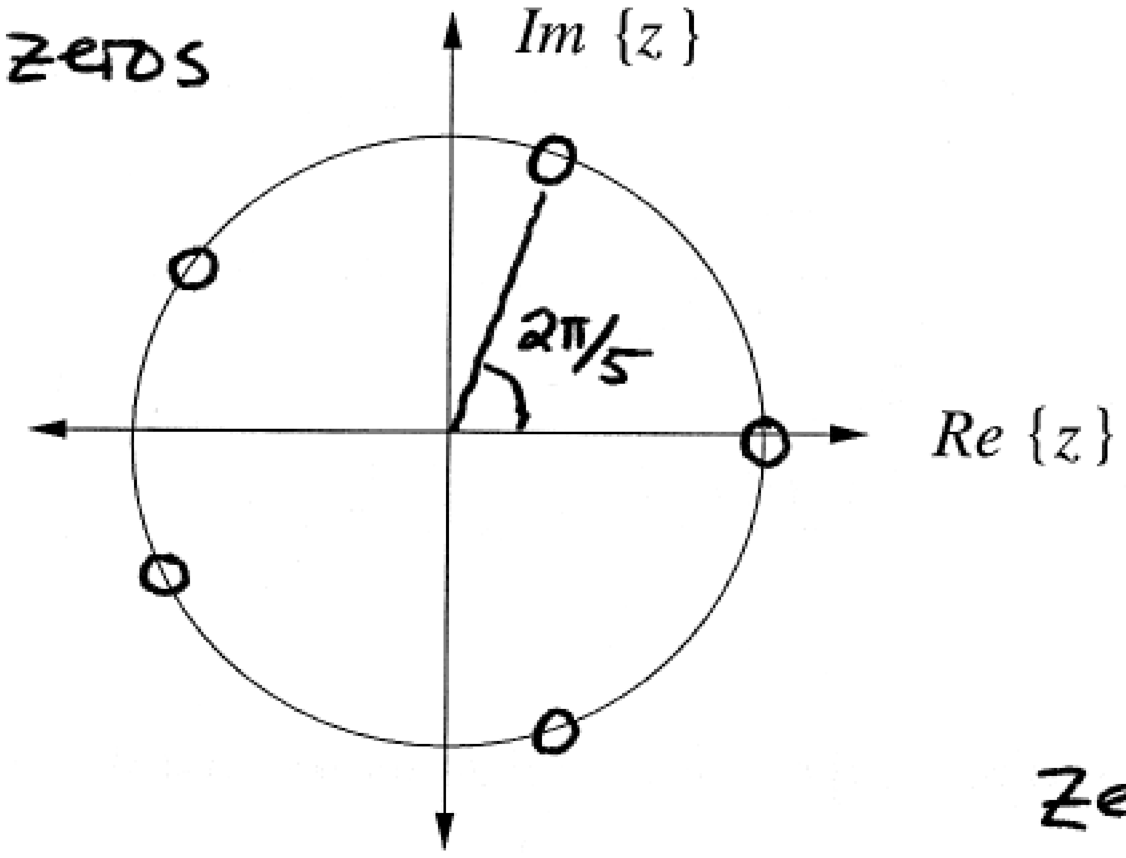


System 1 is defined by the difference equation $v[n] = x[n] - x[n-5]$.

System 2 is defined by the system function $H_2(z) = z^{-1} + \frac{1}{4}z^{-2} + z^{-5}$.

(a) Determine the system function $H_1(z)$ associated with System 1 and plot the zeros of $H_1(z)$.

There are 5 zeros



$H_1(z) = 1 - z^{-5}$

Zeros are roots of

$1 - z^{-5}$

$1 - z^{-5} = 0$

$z^5 - 1 = 0$

$z^5 = 1 = e^{j2\pi k}$, $k=0,1,2,3,4$

$z = e^{j2\pi k/5}$

(b) Determine the impulse response of the overall parallel form system. That is, find $h[n]$ such that $y[n] = x[n] * h[n]$.

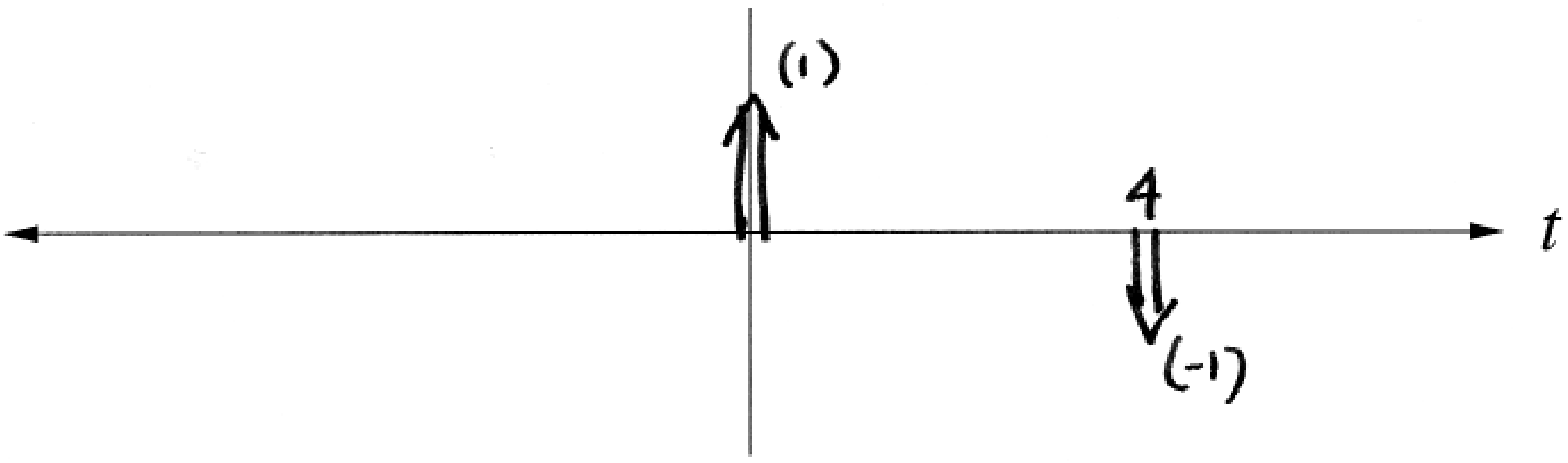
$h[n] = \delta[n] + \delta[n-1] + \frac{1}{4}\delta[n-2]$

$$H(z) = H_1(z) + H_2(z)$$
$$= (1 - z^{-5}) + (z^{-1} + \tfrac{1}{4}z^{-2} + z^{-5})$$
$$= 1 + z^{-1} + \tfrac{1}{4}z^{-2}$$

Problem 2: (20%)

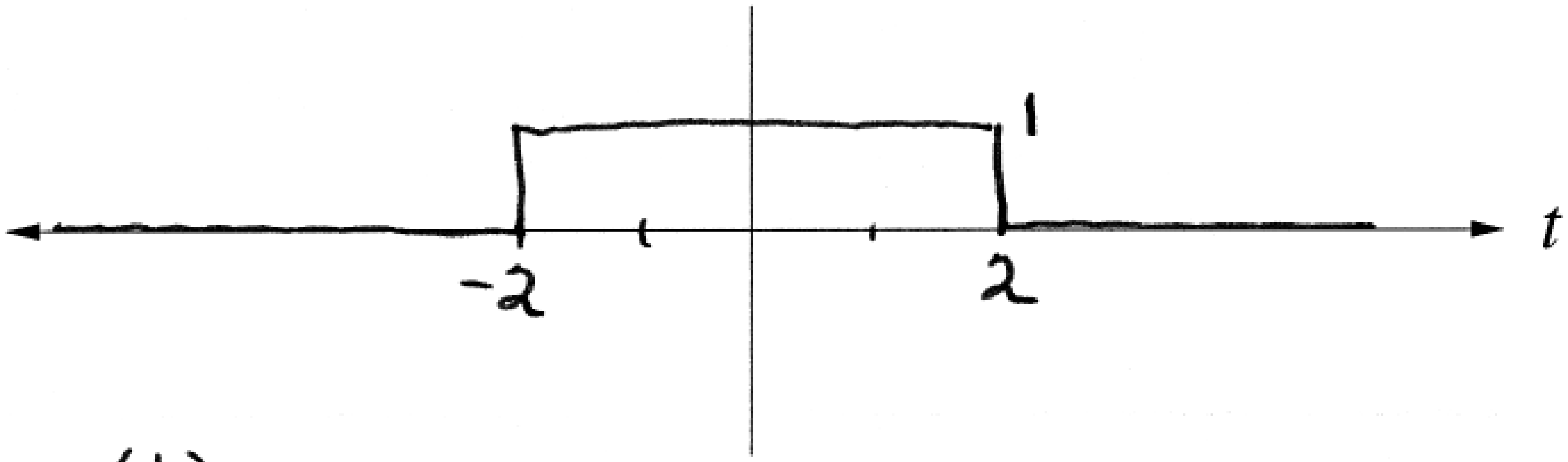
Let $x(t) = u(t) - u(t-4)$.

(a) Sketch $\frac{d}{dt}x(t)$. Carefully label your plot.

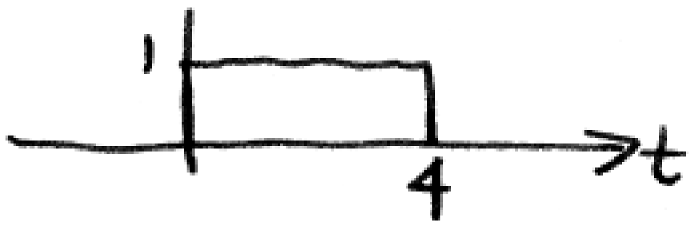


$$\frac{d}{dt}\{u(t)-u(t-4)\} = \delta(t) - \delta(t-4)$$

(b) Sketch $x(2-t)$. Carefully label your plot.



Sketch $x(t)$



Then flip and slide by 2.

(c) If $x(t) * x(t-3) * \delta(t-T) = x(t-5) * x(t-6)$, determine the numerical value of T.

$T =$ 8 secs

Let $x(t) * x(t)$ be called $s(t)$.

Then $x(t-5) * x(t-6)$ is $s(t-11)$ because convolution is time-invariant

$x(t) * x(t-3) = s(t-3)$

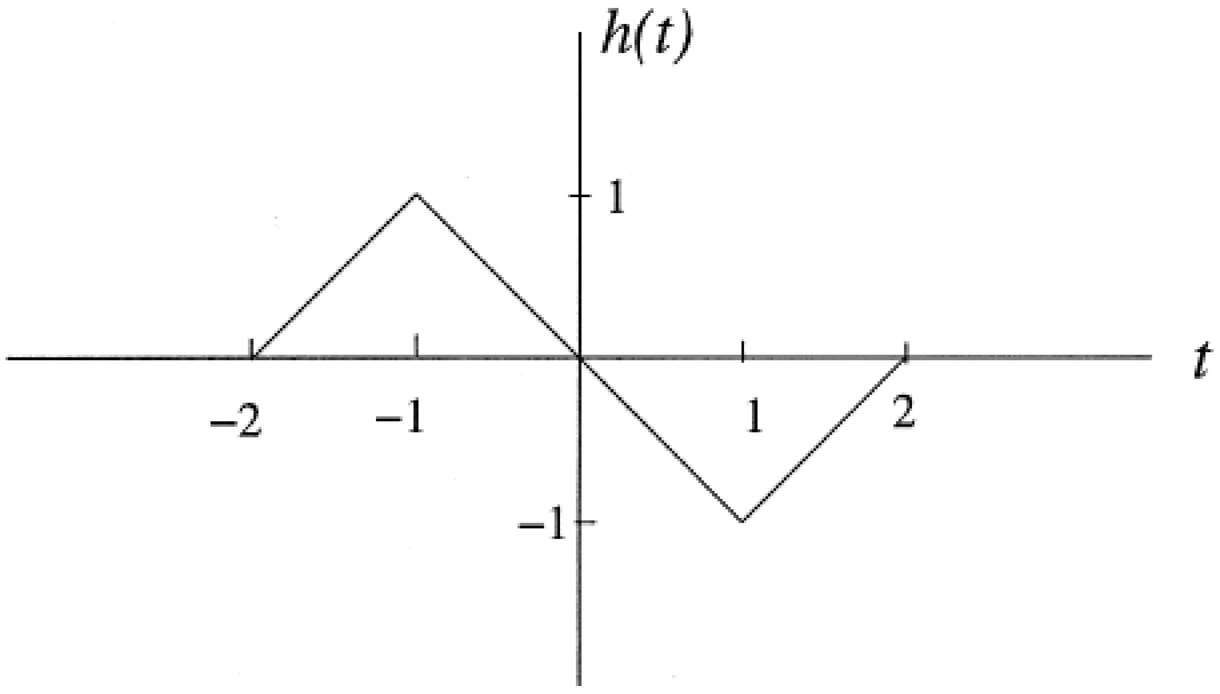
Thus $s(t-3) \underbrace{* \, \delta(t-T)}_{\text{time-shifting}} = s(t-11)$

need $T = 8$

Problem 3: (20%)

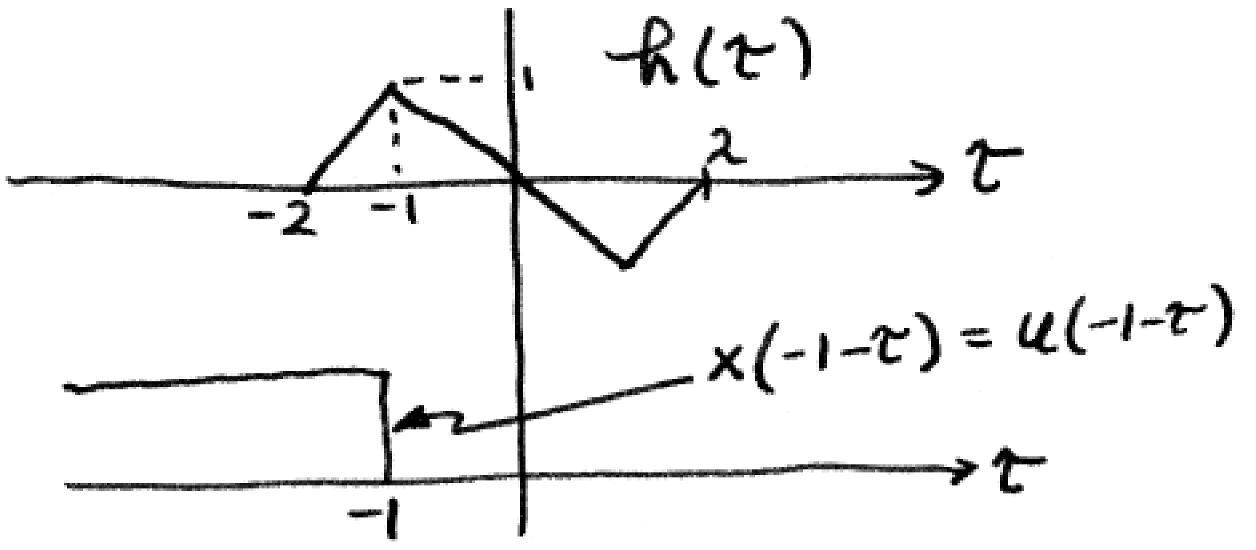
Consider the LTI System whose output is $y(t) = x(t) * h(t)$, where $x(t) = u(t)$

and $h(t)$ is given by

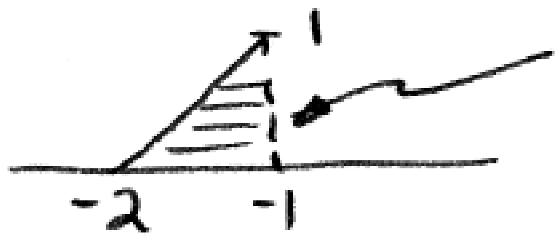


(a) Determine $y(-1)$, the value of $y(t)$ for $t = -1$.

$y(-1) =$ 1/2



The overlap is from $\tau = -2$ to $\tau = -1$, so the integral is $\int_{-2}^{-1} h(\tau)\,d\tau$ = Area of triangle



(b) You should be able to see that $y(t) = 0$ in two regions: $T_1 \leq t \leq T_2$ and $T_3 \leq t \leq T_4$. Determine T_1, T_2, T_3, and T_4. **Explain carefully to receive full credit.**

$T_1 =$ $-\infty$, $T_2 =$ -2, $T_3 =$ 2, $T_4 =$ ∞

There is no overlap until the flipped $u(t-\tau)$ slides over $x(\tau)$ at $\tau = -2$. Thus $T_2 = -2$.
When $u(t-\tau)$ completely overlaps $x(\tau)$ the result for $y(t)$ is the area of $h(\tau)$. The areas of the 2 triangles cancel, so $y(t) = 0$ for $t \geq 2$

(c) Is this system causal? yes or [✓ no] (Circle one)

NOT Causal because $h(t) \neq 0$ for $t < 0$.

Problem 4: (20%)

Let $h(t) = \delta(t+10) + 2\delta(t) + \delta(t-10)$.

(a) Find $H(j\omega)$.

$$\boxed{H(j\omega) = e^{+j10w} + 2 + e^{-j10w}}$$

Simplify:

$$H(jw) = 2 + 2\cos(10w)$$

(purely real)

(b) Let $y(t) = h(t-10)$. Find the phase of $Y(j\omega)$, the Fourier transform of $y(t)$.

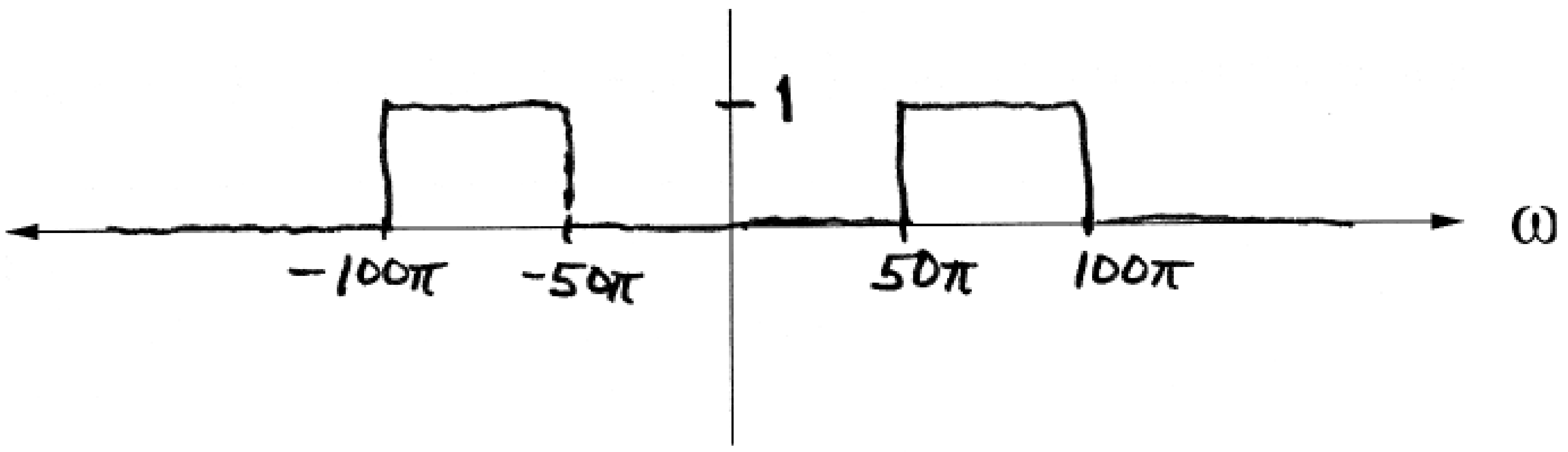
$$\boxed{\angle Y(j\omega) = -10w}$$

$$Y(jw) = e^{-j10w} H(jw)$$

$$= e^{-j10w} \underbrace{(2 + 2\cos(10w))}_{\text{Magnitude}}$$

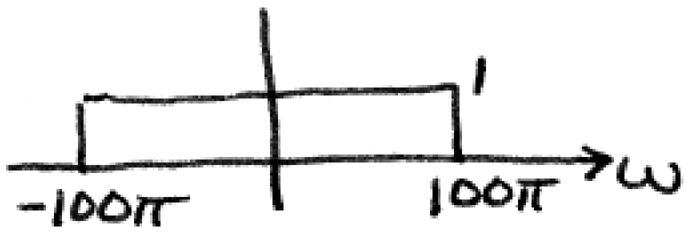
phase

(c) If $x(t) = \dfrac{\sin 100\pi t}{\pi t} - \dfrac{\sin 50\pi t}{\pi t}$, plot $X(j\omega)$. Carefully label your plot.

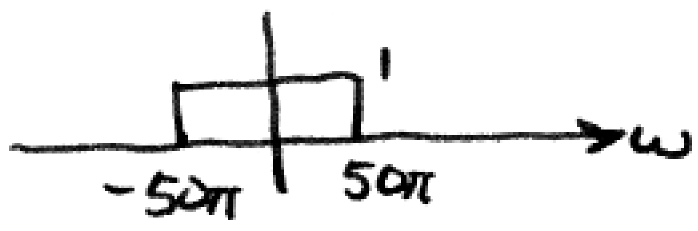


Each "sinc" transforms to a rectangle, so you must subtract the two rectangles

$$\frac{\sin 100\pi t}{\pi t} \xrightarrow{F.T.}$$



$$\frac{\sin 50\pi t}{\pi t} \xrightarrow{F.T.}$$



Problem 5: (20%)

Assume that $x(t)$ is the periodic function given by

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-50k) = \sum_{k=-\infty}^{\infty} \frac{1}{50} e^{j\omega_0 kt}.$$

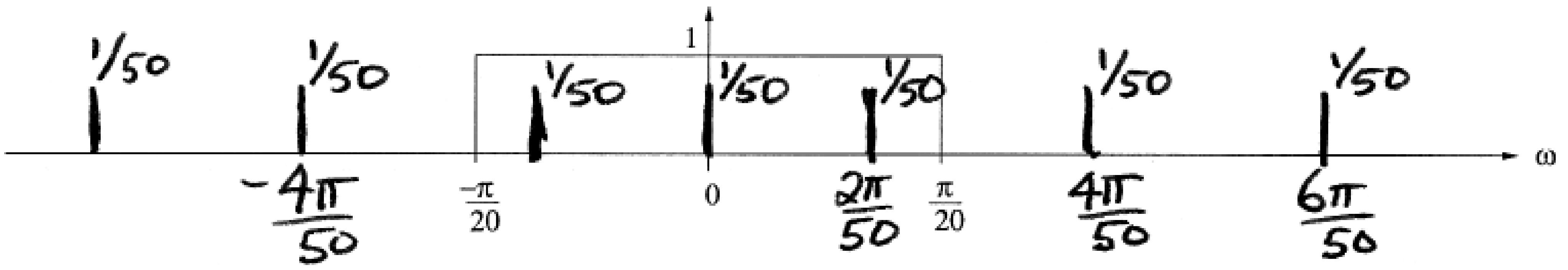
(a) Determine the value of the fundamental frequency ω_0.

$$\boxed{\omega_0 = 2\pi/T = 2\pi/50 \text{ rad/s}}$$

period = 50 sec.

(b) Suppose that $x(t)$ is the input to an LTI system with the frequency response illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| \leq \dfrac{\pi}{20} \\ 0 & |\omega| > \dfrac{\pi}{20} \end{cases}$$



Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. Your answer should be expressed in terms of only real quantities. (Hint: Plot the spectrum of $x(t)$ on the plot of the frequency response.)

$$\boxed{y(t) = \frac{1}{50} + \frac{2}{50}\cos\left(\frac{2\pi}{50}t\right)}$$

Only 3 spectrum lines are passed by the LPF

$$y(t) = \frac{1}{50} + \frac{1}{50} e^{-j\frac{2\pi}{50}t} + \frac{1}{50} e^{j\frac{2\pi}{50}t}$$