



**PROBLEM sp-02-Q.3.1:**(Circle exactly one answer<sup>1</sup> for each system,  $S_i$ )

$S_1$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_2$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_3$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_4$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_5$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_6$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_7$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_8$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9

**PROBLEM sp-02-Q.3.2:**(Circle exactly one answer for each system,  $S_i$ )

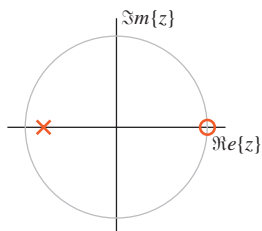
$S_1$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_2$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_3$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_4$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_5$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_6$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_7$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_8$ :	(A)	(B)	(C)	(D)	(E)	(F)	None

**PROBLEM sp-02-Q.3.4:**(Circle exactly one answer for each part)

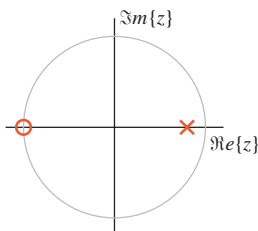
(a)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(b)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(c)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(d)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(e)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(f)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(g)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(h)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

<sup>1</sup>If more than one answer is circled, the response will be considered wrong and will receive no credit.

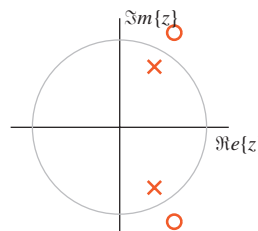
**PROBLEM sp-02-Q.3.1:**



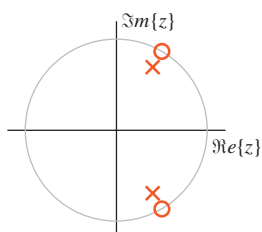
**Pole-Zero Plot #1**



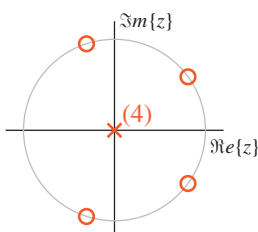
**Pole-Zero Plot #2**



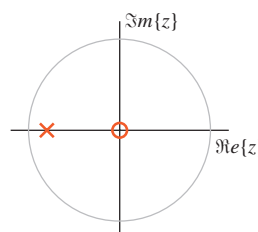
**Pole-Zero Plot #3**



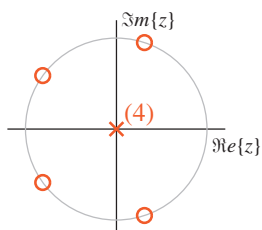
**Pole-Zero Plot #4**



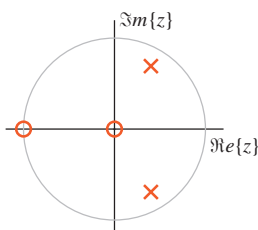
**Pole-Zero Plot #5**



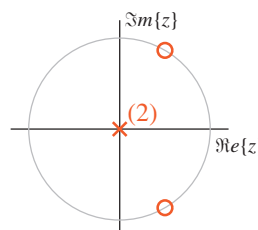
**Pole-Zero Plot #6**



**Pole-Zero Plot #7**



**Pole-Zero Plot #8**



**Pole-Zero Plot #9**

For each of systems below<sup>2</sup> determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. **Mark your answers on the answer sheet provided.**

*Note:* the unit circle is shown for reference.

$$\mathcal{S}_1 : y[n] = 0.8y[n - 1] - 0.64y[n - 2] + 6.4x[n] - 8x[n - 1] + 10x[n - 2]$$

$$\mathcal{S}_2 : H(z) = \frac{10}{3}(1 - z^{-1} + z^{-2})$$

$$\mathcal{S}_3 : y[n] = 2x[n] - 2x[n - 1] + 2x[n - 2] - 2x[n - 3] + 2x[n - 4]$$

$$\mathcal{S}_4 : H(z) = \frac{1.8(1 + z^{-1})}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$\mathcal{S}_5 : y[n] = 0.8y[n - 1] + x[n] + x[n - 1]$$

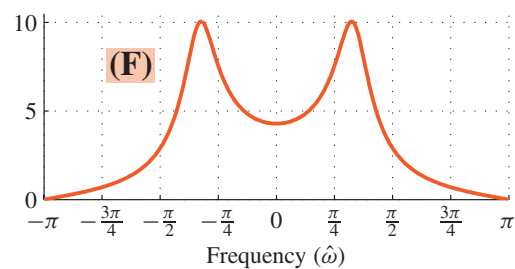
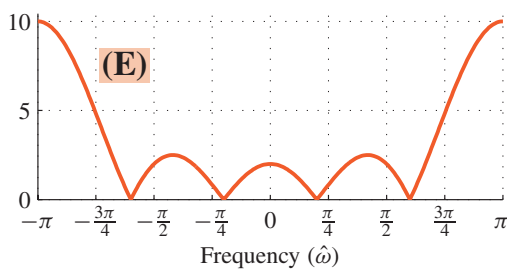
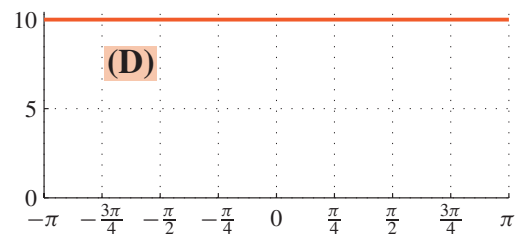
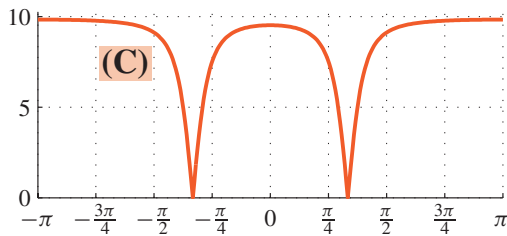
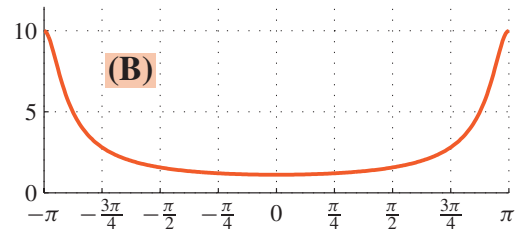
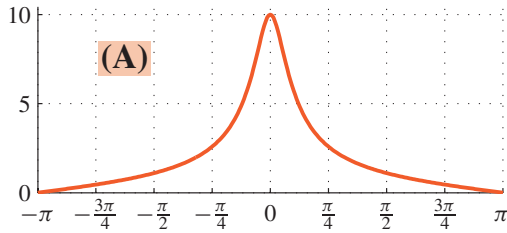
$$\mathcal{S}_6 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$\mathcal{S}_7 : y[n] = -0.8y[n - 1] + 2x[n]$$

$$\mathcal{S}_8 : H(z) = 2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

<sup>2</sup>These same systems are also used in the next problem.

**PROBLEM sp-02-Q.3.2:**



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. **Mark your answers on the answer sheet provided.**

*Note:* the frequency axis is  $\hat{\omega}$ .

$$S_1 : y[n] = 0.8y[n-1] - 0.64y[n-2] + 6.4x[n] - 8x[n-1] + 10x[n-2]$$

$$S_2 : H(z) = \frac{10}{3}(1 - z^{-1} + z^{-2})$$

$$S_3 : y[n] = 2x[n] - 2x[n-1] + 2x[n-2] - 2x[n-3] + 2x[n-4]$$

$$S_4 : H(z) = \frac{1.8(1 + z^{-1})}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_5 : y[n] = 0.8y[n-1] + x[n] + x[n-1]$$

$$S_6 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_7 : y[n] = -0.8y[n-1] + 2x[n]$$

$$S_8 : H(z) = 2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

**PROBLEM sp-02-Q.3.3:**

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

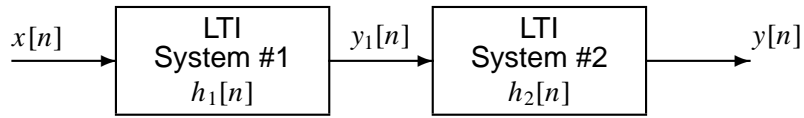


Figure 1: Cascade connection of two discrete-time LTI systems.

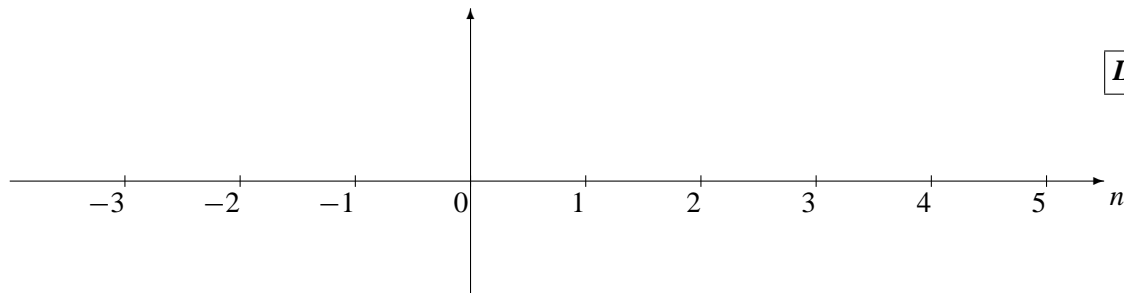
- (a) Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{9 - 10z^{-1}}{1 - 0.9z^{-1}}$$

and System #2 is described by the impulse response

$$h_2[n] = 10\delta[n - 3] - 19\delta[n - 4] + 9\delta[n - 5]$$

Determine the impulse response sequence,  $h_1[n]$ , of the first system. Give your answer as a *plot*.



- (b) Determine the output,  $y[n]$ , of the overall cascade system when the input,  $x[n]$ , is a *unit-step* signal.  
*Hint:* The output,  $y[n]$ , will be finite-length.

**PROBLEM sp-02-Q.3.4:**

For each of the following expressions, select the correct match from the second list below.

*Write your answers on the answer sheet provided.* (The operator \* denotes convolution.)

(a)  $\int_{-\infty}^0 \delta(t-3)dt$

(b)  $u(3)$

(c)  $e^{-t}u(t) * \delta(t-3)$

(d)  $u(t-1) * u(t-2)$

(e)  $e^{-t}u(t)\delta(t-3)$

(f)  $\delta(t-1) * \delta(t-2)$

(g)  $e^{-t}u(t) * u(t-3)$

(h)  $\frac{d}{dt} \{e^{-t}u(t-3)\}$

---

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1]  $-e^{-t}u(t-3) + e^{-3}\delta(t-3)$

[2]  $(t-3)u(t-3)$

[3]  $(1 - e^{-t+3})u(t-3)$

[4]  $e^{-(t-3)}u(t-3)$

[5]  $e^{-3}\delta(t-3)$

[6] 0

[7]  $\delta(t-3)$

[8] 1

[9]  $e^{-3}$

[10]  $-e^{-t}u(t-3)$

[11]  $u(t-3)$

**GEORGIA INSTITUTE OF TECHNOLOGY**  
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING  
**QUIZ #3**

DATE: 11-Apr-03

COURSE: ECE-2025

NAME: Answer Key  
LAST, FIRST

GT #: Version-1

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Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L01:Tues-9:30am (McLaughlin)		L02:Thur-9:30am (Barry)
	L03:Tues-Noon (McLaughlin)		L04:Thur-Noon (Barry)
	L05:Tues-1:30pm (Li)		
L11:M-3pm (McClellan)	L07:Tues-3pm (Li)	L12:W-3pm (Hayes)	L08:Thur-3pm (Williams)
	L09:Tues-4:30pm (Zhou)	L14:W-4:30pm (Hayes)	
	L10:Tues-6pm (Zhou)		RPK:Thur-Late (Tugcu)

- 
- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
  - Closed book, but a calculator is permitted.
  - One page ( $8\frac{1}{2}'' \times 11''$ ) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
  - Justify your reasoning clearly to receive partial credit.  
Explanations are also required to receive full credit for any answer.
  - You must write your answer **on the answer sheet** or in the space provided on the exam paper itself.  
Only these answers will be graded. Circle your answers, or write them in the boxes provided.  
If space is needed for scratch work, use the backs of previous pages.

<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	25	
2	25	
3	25	
4	25	

**PROBLEM sp-02-Q.3.1:**(Circle exactly one answer<sup>1</sup> for each system,  $S_i$ )

$S_1$ :	#1	#2	<b>#3</b>	#4	#5	#6	#7	#8	#9
$S_2$ :	#1	#2	#3	#4	#5	#6	#7	#8	<b>#9</b>
$S_3$ :	#1	#2	#3	#4	<b>#5</b>	#6	#7	#8	#9
$S_4$ :	#1	#2	#3	#4	#5	#6	#7	<b>#8</b>	#9
$S_5$ :	#1	<b>#2</b>	#3	#4	#5	#6	#7	#8	#9
$S_6$ :	#1	#2	#3	<b>#4</b>	#5	#6	#7	#8	#9
$S_7$ :	#1	#2	#3	#4	#5	<b>#6</b>	#7	#8	#9
$S_8$ :	#1	#2	#3	#4	#5	#6	<b>#7</b>	#8	#9

**PROBLEM sp-02-Q.3.2:**(Circle exactly one answer for each system,  $S_i$ )

$S_1$ :	(A)	(B)	(C)	<b>(D)</b>	(E)	(F)	None
$S_2$ :	(A)	(B)	(C)	(D)	(E)	(F)	<b>None</b>
$S_3$ :	(A)	(B)	(C)	(D)	<b>(E)</b>	(F)	None
$S_4$ :	(A)	(B)	(C)	(D)	(E)	<b>(F)</b>	None
$S_5$ :	<b>(A)</b>	(B)	(C)	(D)	(E)	(F)	None
$S_6$ :	(A)	(B)	<b>(C)</b>	(D)	(E)	(F)	None
$S_7$ :	(A)	<b>(B)</b>	(C)	(D)	(E)	(F)	None
$S_8$ :	(A)	(B)	(C)	(D)	(E)	(F)	<b>None</b>

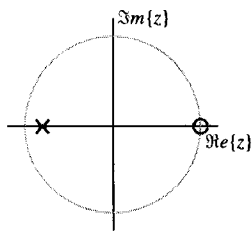
**PROBLEM sp-02-Q.3.4:**(Circle exactly one answer for each part)

(a)	[1]	[2]	[3]	[4]	[5]	<b>[6]</b>	[7]	[8]	[9]	[10]	[11]
(b)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	<b>[8]</b>	[9]	[10]	[11]
(c)	[1]	[2]	[3]	<b>[4]</b>	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(d)	[1]	<b>[2]</b>	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(e)	[1]	[2]	[3]	[4]	<b>[5]</b>	[6]	[7]	[8]	[9]	[10]	[11]
(f)	[1]	[2]	[3]	[4]	[5]	[6]	<b>[7]</b>	[8]	[9]	[10]	[11]
(g)	[1]	[2]	<b>[3]</b>	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(h)	<b>[1]</b>	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

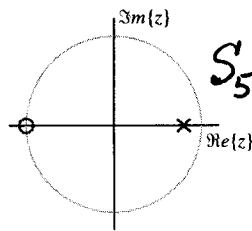
<sup>1</sup>If more than one answer is circled, the response will be considered wrong and will receive no credit.



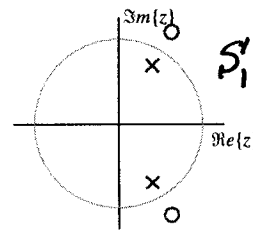
**PROBLEM sp-02-Q.3.1:**



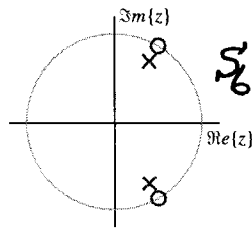
**Pole-Zero Plot #1**



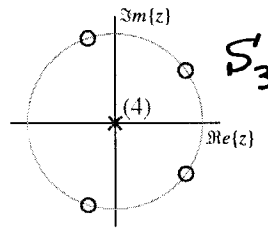
**Pole-Zero Plot #2**



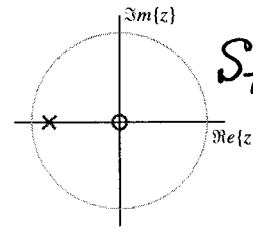
**Pole-Zero Plot #3**



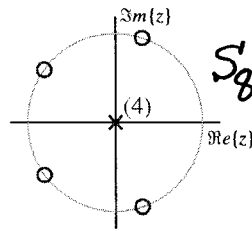
**Pole-Zero Plot #4**



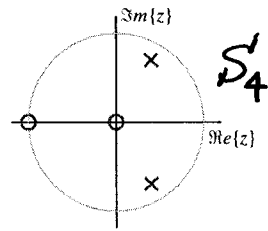
**Pole-Zero Plot #5**



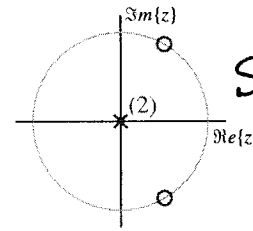
**Pole-Zero Plot #6**



**Pole-Zero Plot #7**



**Pole-Zero Plot #8**



**Pole-Zero Plot #9**

For each of systems below<sup>2</sup> determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. **Mark your answers on the answer sheet provided.**

Note: the unit circle is shown for reference.

✓ All-pass, zeros outside u.c.

$S_1 : y[n] = 0.8y[n-1] - 0.64y[n-2] + 6.4x[n] - 8x[n-1] + 10x[n-2]$

$S_2 : H(z) = \frac{10}{3}(1 - z^{-1} + z^{-2})$  FIR 2 zeros at  $e^{\pm j\pi/3}$

$S_3 : y[n] = 2x[n] - 2x[n-1] + 2x[n-2] - 2x[n-3] + 2x[n-4]$  FIR, 4 zeros

$S_4 : H(z) = \frac{1.8(1 + z^{-1})}{1 - 0.8z^{-1} + 0.64z^{-2}}$  — zero at  $z = -1$   
2 poles

$S_5 : y[n] = 0.8y[n-1] + x[n] + x[n-1]$  pole at 0.8, zero at -1

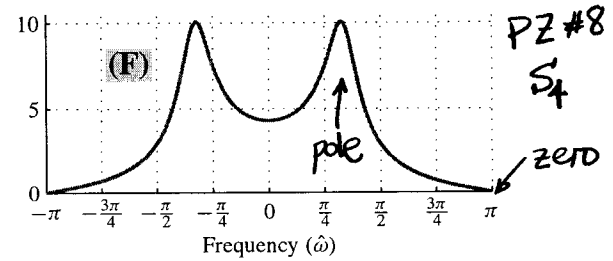
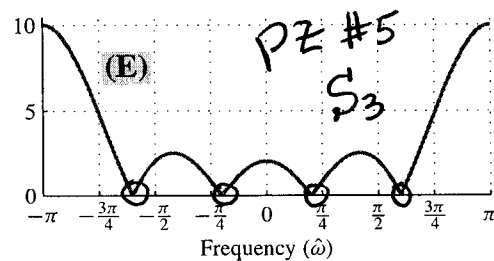
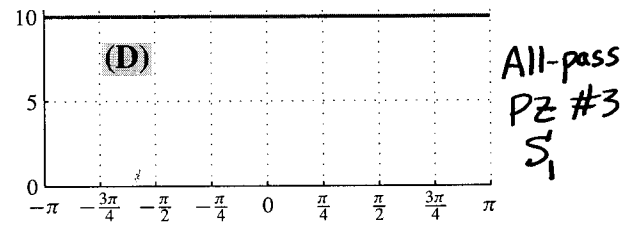
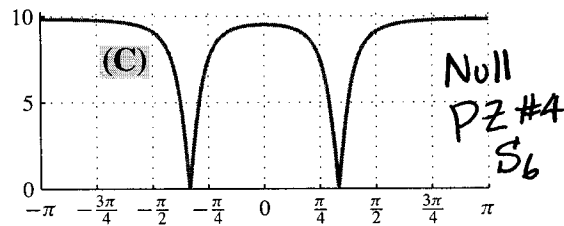
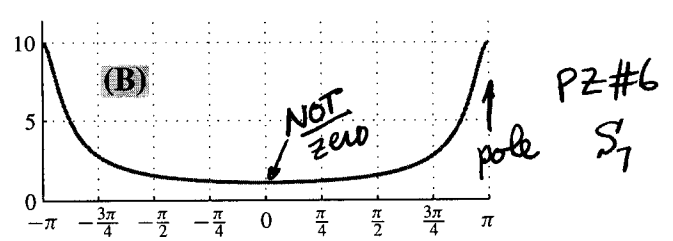
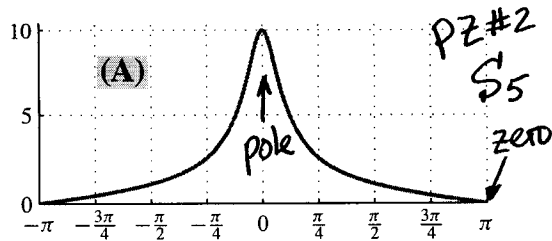
$S_6 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$  Nulling: zeros @  $e^{\pm j\pi/3}$  on u.c.

$S_7 : y[n] = -0.8y[n-1] + 2x[n]$  pole at -0.8, zero at  $z = 0$

$S_8 : H(z) = 2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$  FIR, 4 zeros

<sup>2</sup>These same systems are also used in the next problem.

**PROBLEM sp-02-Q.3.2:**



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. **Mark your answers on the answer sheet provided.**

Note: the frequency axis is  $\hat{\omega}$ .

$$S_1: y[n] = 0.8y[n-1] - 0.64y[n-2] + 6.4x[n] - 8x[n-1] + 10x[n-2]$$

$$S_2: H(z) = \frac{10}{3}(1 - z^{-1} + z^{-2})$$

$$S_3: y[n] = 2x[n] - 2x[n-1] + 2x[n-2] - 2x[n-3] + 2x[n-4]$$

$$S_4: H(z) = \frac{1.8(1 + z^{-1})}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_5: y[n] = 0.8y[n-1] + x[n] + x[n-1]$$

$$S_6: H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_7: y[n] = -0.8y[n-1] + 2x[n]$$

$$S_8: H(z) = 2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

**PROBLEM sp-02-Q.3.3:**

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

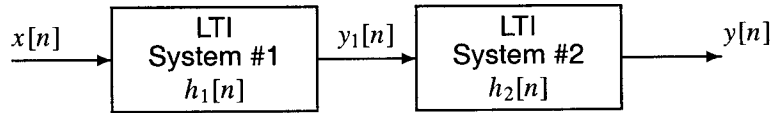


Figure 1: Cascade connection of two discrete-time LTI systems.

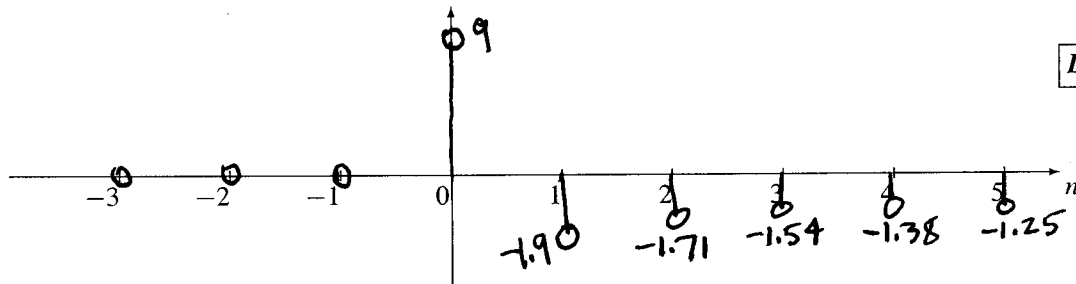
- (a) Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{9 - 10z^{-1}}{1 - 0.9z^{-1}}$$

and System #2 is described by the impulse response

$$h_2[n] = 10\delta[n - 3] - 19\delta[n - 4] + 9\delta[n - 5]$$

Determine the impulse response sequence,  $h_1[n]$ , of the first system. Give your answer as a *plot*.



$$H_1(z) = \frac{9}{1 - 0.9z^{-1}} - \frac{10z^{-1}}{1 - 0.9z^{-1}} \Rightarrow h_1[n] = 9(0.9)^n u[n] - 10(0.9)^{n-1} u[n-1]$$

$$h_1[0] = 9 \quad h_1[2] = 9(0.9)^2 - 10(0.9) = 7.29 - 9 = -1.71$$

$$h_1[1] = 9(0.9) - 10 = -1.9 \quad h_1[3] = 9(0.9)^3 - 10(0.9)^2 = -1.539$$

- (b) Determine the output,  $y[n]$ , of the overall cascade system when the input,  $x[n]$ , is a *unit-step* signal.

*Hint:* The output,  $y[n]$ , will be finite-length.

$$Y(z) = U(z) \frac{9 - 10z^{-1}}{1 - 0.9z^{-1}} \cdot (10z^{-3} - 19z^{-4} + 9z^{-5})$$

$$\frac{1}{1 - z^{-1}} \quad Y(z) = \frac{(9 - 10z^{-1})(10 - 19z^{-1} + 9z^{-2})z^{-3}}{1 - 0.9z^{-1}}$$

$$= (9 - 10z^{-1})10z^{-2} = 90z^{-2} - 100z^{-3}$$

$$y[n] = 90\delta[n-2] - 100\delta[n-3]$$

**PROBLEM sp-02-Q.3.4:**

For each of the following expressions, select the correct match from the second list below.

Write your answers on the answer sheet provided. (The operator \* denotes convolution.)

(a)  $\int_{-\infty}^0 \delta(t-3) dt = 0$  

(b)  $u(3) = 1$

(c)  $e^{-t}u(t) * \delta(t-3) = e^{-(t-3)}u(t-3)$

(d)  $u(t-1) * u(t-2) = (t-3)u(t-3)$  because  $u(t) * u(t) = tu(t)$

(e)  $e^{-t}u(t)\delta(t-3) = e^{-3}u(3)\delta(t-3) = e^{-3}\delta(t-3)$

(f)  $\delta(t-1) * \delta(t-2) = \delta(t-3)$

(g)  $e^{-t}u(t) * u(t-3) = (1 - e^{-(t-3)})u(t-3)$

(h)  $\frac{d}{dt} \{e^{-t}u(t-3)\} = -e^{-t}u(t-3) + e^{-t}\delta(t-3)$   
↖  $e^{-3}$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1]  $-e^{-t}u(t-3) + e^{-3}\delta(t-3)$  (h)

[2]  $(t-3)u(t-3)$  (d)

[3]  $(1 - e^{-t+3})u(t-3)$  (g)

[4]  $e^{-(t-3)}u(t-3)$  (c)

[5]  $e^{-3}\delta(t-3)$  (e)

[6]  $0$  (a)

[7]  $\delta(t-3)$  (f)

[8]  $1$  (b)

[9]  $e^{-3}$

[10]  $-e^{-t}u(t-3)$

[11]  $u(t-3)$