BMED 3400: Notes on Trapeze Stand Design Problem

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Version 0.9, January 19, 2016

1 Goal

Design a new cable for the standing trapeze that will be unlikely to fail when used appropriately.

2 Design Parameters

Select a material, steel or nylon, and the radius, r, for the cylindrical part.

3 Stress Predicted by Our Model of Simple Axial Loading

Our model for simple axial loading predicts that the stress, σ, throughout the part will be uniform with

$$\sigma = \frac{P}{A}, \tag{1}$$

where P defines the applied load and A is the cross-sectional area of the cylinder, which defines its geometry.

4 Optimizing the Design of Our Part

We want to find the minimum geometry, A_{min}, we can use for the part without it failing, given the maximum load, P_{max}, and the maximum stress the material can handle, σ_{max} (i.e., yield strength of the material). We want to minimize the geometry to reduce the size and weight of the part to reduce the costs to produce and ship it. So, based on our model

$$\sigma_{max} = \frac{P_{max}}{A_{min}} \tag{2}$$

and

$$A_{min} = \frac{P_{max}}{\sigma_{max}}. \tag{3}$$

4.1 Estimate Parameters for Our Part, P_{max} and σ_{max}

First, we find P_{max}, which is the maximum load the part must handle without plastic deformation. We specified the maximum mass of the person using the device to be, $m_{max} = 200kg$. We then used a 50% safety margin (`https://en.wikipedia.org/wiki/Factor_of_safety`) to find

$$F = m * g = m_{max} * 10\frac{m}{s^2} = 2 \times 10^3 N \tag{4}$$

and

$$P_{max} = F * 1.5 = 3 \times 10^3 N. \tag{5}$$

We then approximated σ_{max} based on information on the internet as

$$\sigma_{max_{steel}} = 200MPa = 200 \times 10^6 Pa = 2 \times 10^8 Pa \tag{6}$$

and

$$\sigma_{max_{nylon}} = 40MPa = 40 \times 10^6 Pa = 4 \times 10^7 Pa. \tag{7}$$

4.2 Estimate the Minimum Cross-sectional Area, A_{min}

Consequently,

$$A_{min_{steel}} = \frac{P_{max}}{\sigma_{max_{steel}}} = \frac{3 \times 10^3}{2 \times 10^8} = 1.5 \times 10^{-5} m^2 \tag{8}$$

and

$$A_{min_{nylon}} = \frac{P_{max}}{\sigma_{max_{nylon}}} = \frac{3 \times 10^3}{4 \times 10^7} = 0.75 \times 10^{-4} m^2. \tag{9}$$

4.3 Estimate the Minimum Volume, V_{min}

Since the volume for a cylinder is $V = A * L$, this implies that the minimum volume for the part would be $V_{min_{steel}} = A_{min_{steel}} * L$ and $V_{min_{nylon}} = A_{min_{nylon}} * L$. So,

$$\frac{V_{min_{nylon}}}{V_{min_{steel}}} = \frac{A_{min_{nylon}}}{A_{min_{steel}}} = \frac{0.75 \times 10^{-4} m^2}{1.5 \times 10^{-5} m^2} = \frac{3}{4} * \frac{2}{3} * 10 = 5. \tag{10}$$

4.4 Interpret Our Results

Our results suggest that **the nylon part would need to have 5 times the volume of the steel part**. This means it would require 5 times the amount of material to manufacture it.

Likewise, since $A = \pi r^2$,

$$r_{min_{steel}} = \sqrt{\frac{1}{\pi} A_{min_{steel}}} \approx \sqrt{\frac{1}{3}\frac{3}{2} \times 10^{-5} m^2} = \sqrt{\frac{1}{2} \times 10^{-5} m^2} = 2.2mm \quad (11)$$

$$r_{min_{nylon}} = \sqrt{\frac{1}{\pi} A_{min_{nylon}}} \approx \sqrt{\frac{1}{3}\frac{3}{4} \times 10^{-4} m^2} = \sqrt{\frac{1}{4} \times 10^{-4} m^2} = 5mm, \quad (12)$$

giving diameters that approximately differ by a factor of 2 with a $1cm$ diameter for the nylon cylinder and approximately a $\frac{1}{2}cm$ diameter for the steel cylinder.

Interestingly, for at least one vendor (`https://www.1st-chainsupply.com/technical/sschainspecs.pdf`), the $\frac{1}{2}cm$ diameter we found is close to the 0.2" diameter ($0.508cm$) of the steel tubes used to produce their $\frac{3}{16}$" stainless steel chains rated for $830lb \approx 376kg$. Since a chain link consists of two cylinders in parallel, this would correspond with twice the cross-sectional area in our model and the ability to support twice our $200kg$ load. Notably, $400kg$ is close to $376kg$, which indicates that our modeling has resulted in reasonable predictions. However, among other uncertainties, we do not know what safety margin this supplier uses with their specifications, if any.