BMED 3400: Notes on the Surgical Robot Design Problem

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1 Goal

Given a physical sample of a material, evaluate the feasibility of using the material to produce a cable for a surgical robot that connects a motor to the surgical robot's gripper.

2 Model of Simple Axial Loading

Our model for simple axial loading predicts that the elongation of the cable, δ, will be

$$\delta = \frac{PL}{AE}, \tag{1}$$

where P is the tensile load applied to the cable, L is the length of the cable, A is the cross-sectional area of the cable, and E is Young's modulus for the material sample under consideration.

3 Estimate the Cable's Elongation to Decide on Feasibility

If the elongation of the cable, δ, is too large, the cable will be poorly suited to transmit the force applied by the robot's motor on one end to the robot's gripper on the other end.

3.1 Estimate the Cable's Geometry: L and A

Based on the apparent geometry of the Da Vinci Surgical System from Intuitive Surgical, we can make the following approximations for the cable's geometry:

$$L = \frac{1}{2}m$$
$$r = 1mm = 1 \times 10^{-3}m$$
$$A = \pi r^2 \approx 3 * (1 \times 10^{-3}m)^2 = 3 \times 10^{-6}m^2.$$

3.2 Estimate the Load on the Cable: P

We can then estimate the load, P, applied to the cable based on a scientific paper, [1], that suggested that $10N$ of grip force is enough for many surgical procedures. Based on a free body diagram (FBD) of the gripper and the moment equation for static equilibrium, we find that

$$P = F_{cable} = \frac{d_{tissue}}{d_{cable}} F_{tissue} \tag{2}$$

where d_{tissue} is the moment arm for the force applied by the tissue, $F_{tissue} = 10N$ is the force applied to the gripper's finger, and d_{cable} is the moment arm for the force applied by the cable to the gripper's finger.

The moments occur with respect to the pin joint around which the gripper's finger rotates. We can estimate them as $d_{tissue} = 2cm = 2 \times 10^{-2}m$ and $d_{cable} = \frac{1}{2}cm = \frac{1}{2} \times 10^{-2}m$ based on the apparent geometry of a representative gripper finger. This results in

$$P = \frac{d_{tissue}}{d_{cable}} F_{tissue} = \frac{2 \times 10^{-2}m}{\frac{1}{2} \times 10^{-2}m} 10N = 40N \tag{3}$$

3.3 Characterize the Material Sample: E

There are two clear ways that we can use our model of simple axial loading to estimate Young's modulus, E, for our material sample.

3.3.1 Directly Estimate E

The first is to directly estimate E with

$$\delta_{sample} = \frac{P_{sample} L_{sample}}{A_{sample} E}, \tag{4}$$

which results in

$$E = \frac{P_{sample} L_{sample}}{A_{sample} \delta_{sample}}. \tag{5}$$

3.3.2 Estimate Stress-strain Curve and Then Estimate E

The other clear option is to first estimate a stress-strain curve from the material and then estimate E from this stress-strain curve. For this approach, we estimate the stress, $\sigma_{sample} = \frac{P_{sample}}{A_{sample}}$, and strain, $\epsilon_{sample} = \frac{\delta_{sample}}{L_{sample}}$, for a variety of loads, P_{sample}, applied to the material sample. We can then find the slope of the resulting stress-strain curve at the origin to estimate E. By estimating the stress and strain for many loads, one can gain a better understanding of the material's behavior, including how linear it is.

3.3.3 Example of Directly Estimating E

Using a ruler, I first measured the initial length of a wide rubber band material sample, L_{sample}, without applying a load. This resulted in a length of $L_{sample} = 9.5cm = 9.5 \times 10^{-2}m$. I then applied a tensile load of $P_{sample} = 5N$ with a spring scale to the sample and measured the resulting length, $L_{sample_{loaded}} = 15cm$.

I also measured the dimensions of the cross section of the rubber band when not applying a load to find that a single rectangular cross section had width $w = 6mm = 6 \times 10^{-3}m$ and height $h = 1mm = 1 \times 10^{-3}m$, which results in a cross-sectional area for a single rectangle of $A_{rectangle} = wh = 6 \times 10^{-3}m * 1 \times 10^{-3}m = 6 \times 10^{-6}m^2$. Since I applied the load to the rubber band without cutting it, the cross-sectional area had two rectangles and a total cross-sectional area of $A_{sample} = 2A_{rectangle} = 12 \times 10^{-6}m^2$.

These measurements result in the following estimate for E. We first find

$$\delta_{sample} = L_{sample_{loaded}} - L_{sample} = 15cm - 9.5cm = 5.5cm = 5.5 \times 10^{-2}m \tag{6}$$

Then, we use our model of simple axial loading to find

$$\begin{aligned} E &= \frac{P_{sample} L_{sample}}{A_{sample} \delta_{sample}} = \frac{5N * 9.5 \times 10^{-2}m}{12 \times 10^{-6}m^2 * 5.5 \times 10^{-2}m} \\ &= \frac{47.5 \times 10^{-2}Nm}{66 \times 10^{-8}m^3} = 0.72 \times 10^6 \frac{N}{m^2} = 0.72MPa \end{aligned}$$

3.4 Put it All Together to Estimate Elongation: δ

$$\delta = \frac{PL}{AE} = \frac{40N\frac{1}{2}m}{3 \times 10^{-6}m^2}\frac{1}{E} = \frac{20Nm}{3 \times 10^{-6}m^2}\frac{1}{E} = \frac{\frac{2}{3} \times 10^7 \frac{N}{m}}{E} \tag{7}$$

$$\delta = \frac{\frac{2}{3} \times 10^7 \frac{N}{m}}{E} = \frac{\frac{2}{3} \times 10^7 \frac{N}{m}}{0.72 \times 10^6 \frac{N}{m^2}} = 0.93 \times 10m = 9.3m \tag{8}$$

4 Interpret Our Results

It is infeasible to use the material to produce the cable. Our model suggests that an elongation of almost 10 meters would be required to produce grip forces for surgery. This is completely unreasonable, especially since the surgical robot is at the scale of a meter. Due to the low Young's modulus for the material the cable would be too compliant. A stiffer material should be considered.

References

[1] J. Westwood *et al.*, "Quantifying surgeon grasping mechanics in laparoscopy using the blue dragon system," *Medicine Meets Virtual Reality 12: Building a Better You: the Next Tools for Medical Education, Diagnosis, and Care*, vol. 98, p. 34, 2004.