BMED 3400 Axial Loading Chart

version 1.1

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January 26, 2016

variable	*units*	*relationship to other variables*	*name*	*description*
L	meters, m		length of the object	length of axially loaded non-rigid object; model tends to work better for long narrow objects
x	meters, m	$0 \le x \le L$	location of a cross-section	location of cross section along length of object prior to loading; cross section is normal to axis of object
$\omega(x)$	$\frac{N}{m}$	$\int_{-\epsilon}^{L+\epsilon} \omega(x)\,dx = 0$ due to static equilibrium	applied distributed load	applied load points in $+x$ direction $\Rightarrow \omega(x) < 0$; applied load points in $-x$ direction $\Rightarrow \omega(x) > 0$
$P(x)$	newtons, N	$P(x) = \int_{-\epsilon}^{x} \omega(\alpha)\,d\alpha = -\int_{x}^{L+\epsilon} \omega(\alpha)\,d\alpha$	internal axial load	total internal force normal to cross section at x; $P > 0$ if cross-sectional sliver of length dx is in tension
$A(x)$	m^2	$A(x) = \int_{shape(x)} 1\,dA$	cross-sectional area	area of cross section at x
$\sigma(x)$	pascals, $\frac{N}{m^2}$	$\sigma(x) = \frac{P(x)}{A(x)}$, $\sigma = \frac{dF}{dA}$, $P(x) = \int_{A(x)} \sigma(x)\,dA$	normal stress	average normal stress across the cross section at x; uniform stress model; $\sigma > 0$ when in tension
$E(x)$	pascals, $\frac{N}{m^2}$	$E(x) = \frac{\sigma(x)}{\varepsilon(x)}$	Young's modulus	model of the material; slope of linear approximation to stress-strain curve at 0,0; stiffness of the material
$\varepsilon(x)$	unitless, %, $\frac{m}{m}$	$\varepsilon(x) = \frac{\sigma(x)}{E(x)}$, $\varepsilon(x) = \frac{P(x)}{E(x)A(x)}$, $\varepsilon = \frac{d\delta}{dx}$	normal strain	% change in length of cross-sectional sliver of length dx located at x; $\varepsilon > 0$ represents length increase
$\delta(x)$	meters, m	$\delta(x) = \int_{-\epsilon}^{x} \varepsilon(\alpha)\,d\alpha$, $\delta_{total} = \delta(L)$, $\delta(0) = 0$	deformation	distance cross section at x displaces due to load; $\delta = 0 \Rightarrow$ no change; $\delta > 0 \Rightarrow$ displaced in positive x direction