Statically Indeterminate Systems 2

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BMED 3410: Introduction to Biomechanics
September 21, 2022
Lecture 9

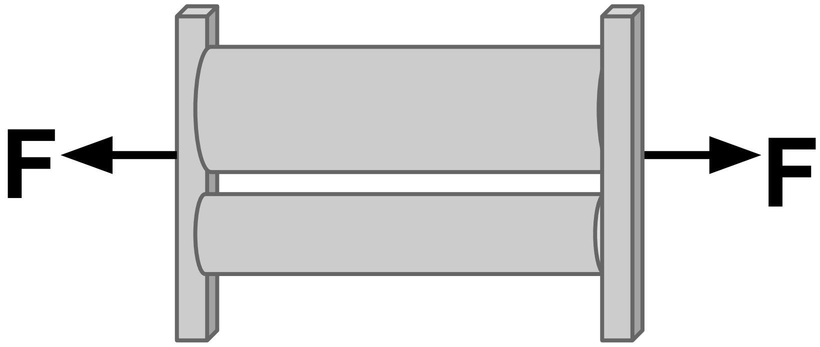
Outline

1. Exam #1 in class a week from today
2. Practice problems available at
    a. https://drive.google.com/drive/u/0/folders/1aHujb5E6tYOGjiMUnr3LUVtruFIbn21H
3. Static Indeterminacy
    a. Example Problems
    b. Linear systems
    c. Design problem

Statically Indeterminate Example

Physical Model

- Same material, different cross-sectional areas
  - large and small rubber bands with evenly spaced marks
- What do you expect to happen?
- What does this imply?
- What assumption can we make?



Note: If we knew the moments and the geometry, we could use statics to find the forces applied to each of the rubber bands.

The Deformations are Related

$$\delta_{total\_1} = \delta_{total\_2}$$

Thermal Expansion

image from: http://www.tmkpackers.co.nz/uploads/products/TMK_Flames_Lamp_Oil.JPG

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Linear Coefficient of Thermal Expansion

$$\varepsilon_{therm}(x) = \alpha(x) \Delta T(x)$$

$$\delta_{therm}(x) = \int_{-\epsilon}^{x} \varepsilon_{therm}(\beta)\, d\beta$$

image from: http://www.tmkpackers.co.nz/uploads/products/TMK_Flames_Lamp_Oil.JPG

Statically Indeterminate Example

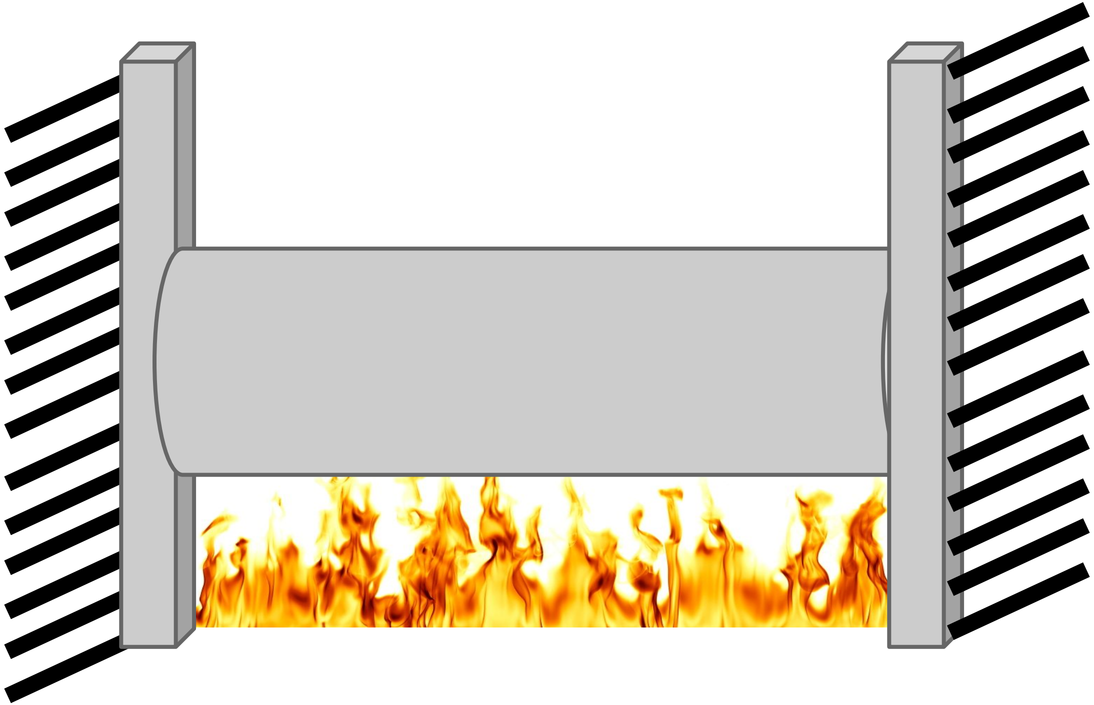


image from: http://www.tmkpackers.co.nz/uploads/products/TMK_Flames_Lamp_Oil.JPG

We Know the Total Deformation

$$\delta_{total} = 0$$

We Know the Total Deformation

$$\delta_{therm} + \delta_{force} = 0$$

What does our model predict?

- If you double the axial load applied to an object, what will happen to the normal stress?
- If you add an axial load to the load already being applied to an object and the object remains in static equilibrium, what will happen to the normal stress?
- What will happen to the total deformation of the object in these situations?
- Do these properties have a name?

What does our model predict?

- If you double the axial load applied to an object, what will happen to the normal stress? **The stress doubles.**
- If you add an axial load to the load already being applied to an object and the object remains in static equilibrium, what will happen to the normal stress? **The stresses add.**
- What will happen to the total deformation of the object in these situations? **same operations**
- Do these properties have a name? **superposition principle**

The Superposition Principle

A function $F(x)$ that satisfies the superposition principle is called a linear function. Superposition can be defined by two simpler properties: additivity

$$F(x_1 + x_2) = F(x_1) + F(x_2)$$

and homogeneity

$$F(ax) = aF(x)$$

for scalar a.

copied from http://en.wikipedia.org/wiki/Superposition_principle

Linear System

A general deterministic system can be described by an operator, H, that maps an input, $x(t)$, as a function of t to an output, $y(t)$, a type of black box description. A system is linear if and only if it satisfies the superposition principle, or equivalently both the additivity and homogeneity properties. So, given two arbitrary inputs

$$x_1(t)$$
$$x_2(t)$$

as well as their respective zero-state outputs

$$y_1(t) = H\{x_1(t)\}$$
$$y_2(t) = H\{x_2(t)\}$$

then a linear system must satisfy

$$\alpha y_1(t) + \beta y_2(t) = H\{\alpha x_1(t) + \beta x_2(t)\}$$

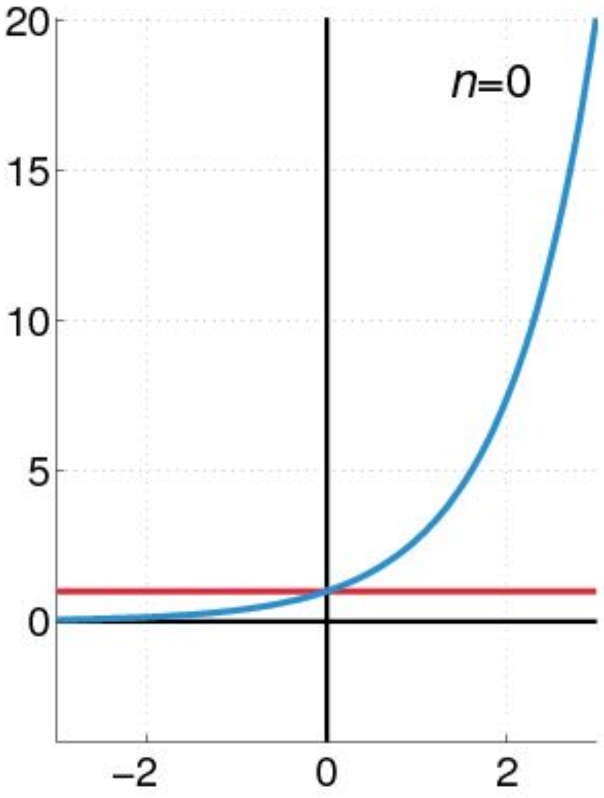
for any scalar values α and β, for any input signals $x_1(t)$ and $x_2(t)$, and for all time t.

Copied from https://en.wikipedia.org/wiki/Linear_system and modified

Linear Systems

"Classifying systems as linear and nonlinear is like classifying the Universe as bananas and non-bananas" - unknown.

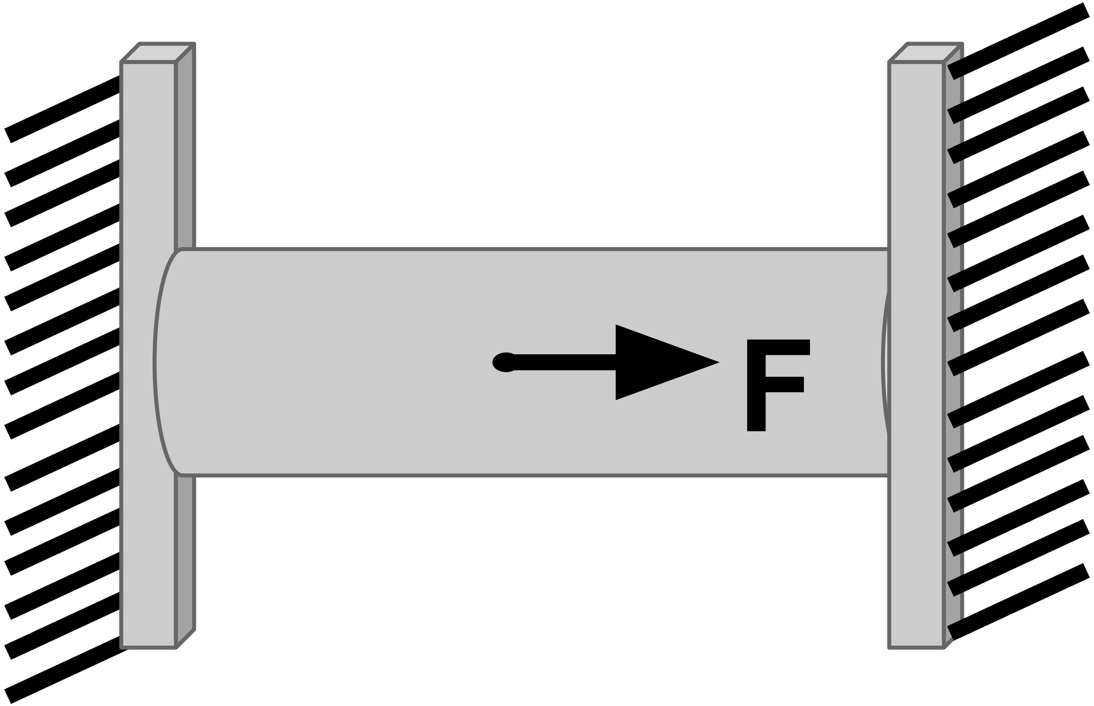
Taylor Series



"The exponential function e^x (in blue), and the sum of the first n+1 terms of its Taylor series at 0 (in red)."

gif and caption from Taylor Series Wikipedia page on 2/4/2016

Statically Indeterminate Example



Example Design Problem

You are designing a new compression bioreactor that will use a single actuator to simultaneously apply distinct compressive stresses to the cylindrical samples by inserting a compliant cylinder beneath each sample. You will first model this system in order to select the stiffness of the underlying cylinders.

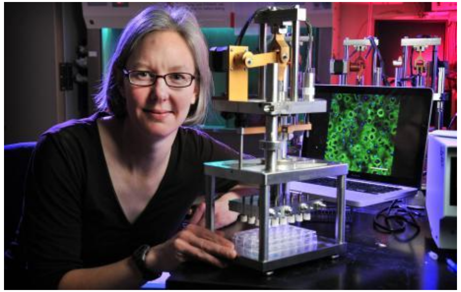


Image of Prof. Stephanie J. Bryant at CU Boulder from http://www.colorado.edu/engineering/features-cue-home/mechanically-conditioned-tissues-offer-hope-better-healing
Lab page: https://www.colorado.edu/bryantgroup/
Faculty page: https://www.colorado.edu/certificate/iqbiology/stephanie-bryant

Problem Solving Heuristic

1. **Understand the question**
   a. **Read it carefully.**
   b. **What is it asking?**
   c. **What would an answer look like?**
2. Draw an overall diagram of the system
3. Identify the component of interest
4. Isolate the component
   a. draw a free-body diagram
5. Model the component
   a. select a model
   b. apply the model
6. Solve for the quantities of interest
7. Interpret and check results (iterate as necessary …)

Understand the Problem

- Search for “compression bioreactor”
  - http://goo.gl/W5YCJl
- Mechanical stimulation
  - influences cell behavior (e.g., migration, proliferation, differentiation)
  - samples can include tissue or cell-laden hydrogels
  - often a uniaxial cyclic load
- Example system
  - Popp J.R., Roberts J.J., Gallagher D.V., Anseth K.S., Bryant S.J., Quinn T.P. An instrumented bioreactor for mechanical stimulation and real-time, nondestructive evaluation of engineered cartilage tissue. Journal of Medical Devices: 6(2): 021006 (7 pages) (2012).

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