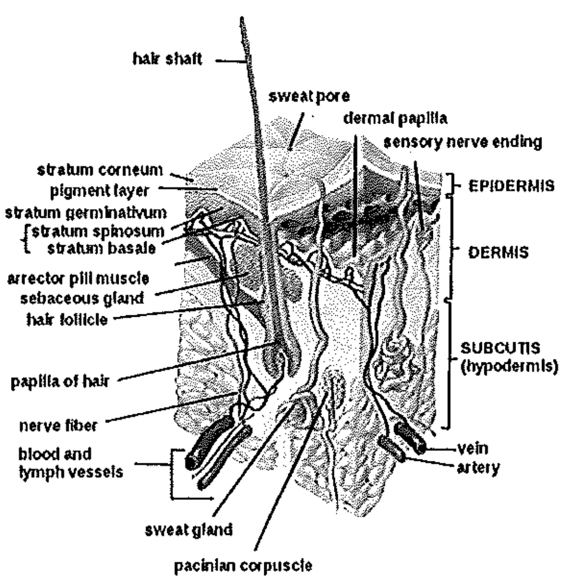
Problem 3 (33 points)

Standard hypodermic needles have long been used in medicine to deliver substances to the human body. One function of hypodermic needles is to enable substances to penetrate the 20 micrometer outer layer of skin named the stratum corneum (see figure below from Wikipedia). However, the use of hypodermic needles is considered invasive and undesirable by many patients, in part because of the pain associated with their use.

Microneedles are a new method of drug delivery that makes use of many small needles that penetrate the stratum corneum in order to deliver substances with less pain due to their small size. As shown in the figure below from an article in the Journal of Biomechanics by Georgia Tech researchers*, a microneedle can be substantially smaller than even small cross-section (large gauge) hypodermic needles.

One of the challenges of microneedle design is for the microneedle to penetrate human skin (i.e., the stratum corneum) without fracturing due to the axial load. In this exam problem, you must model a microneedle and analyze your model's properties with respect to key design parameters.

For this problem, model the microneedle as a solid truncated cone with a circular top (small sharp tip) of radius r_t, a circular bottom of radius r_b, and height of h. Assume that the radius of the cone from the top to the bottom is $r(x)=\frac{r_b-r_t}{h}x+r_t$ where $x=0$ at the top of the truncated cone and $x=h$ at the bottom of the truncated cone. The truncated cone will be made of titanium with $E=100\,GPa$ and will fracture with compressive stress greater than $1\,GPa$. Researchers have found that pushing this microneedle into human skin with force $F_{penetrate}\geq\frac{1}{2}\pi*10^9*r_t^2+\frac{1}{2}\pi*10^{-3}\,Newtons$ penetrates the skin and that lower forces do not penetrate the skin. Researchers have also found that microneedles only fracture before they penetrate the skin.



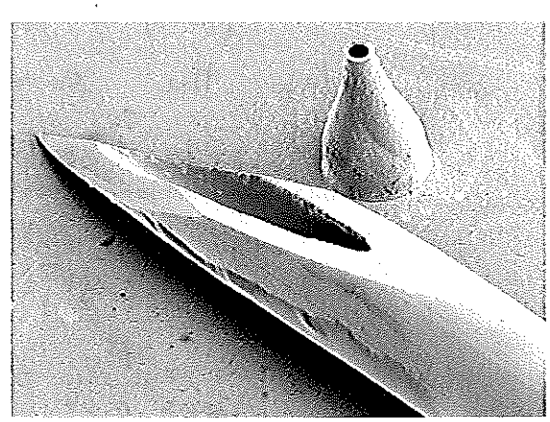
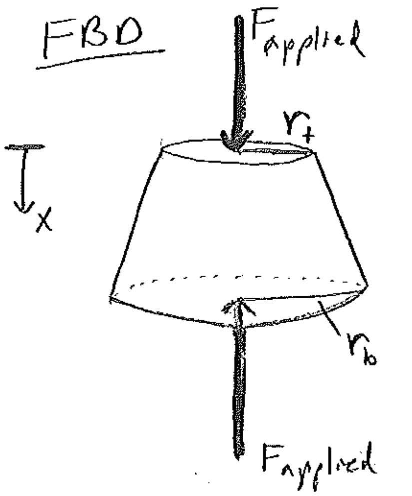


Fig. 1. Scanning electron micrograph of a 500-µm tall microneedle next to the tip of a 27 gauge hypodermic needle.

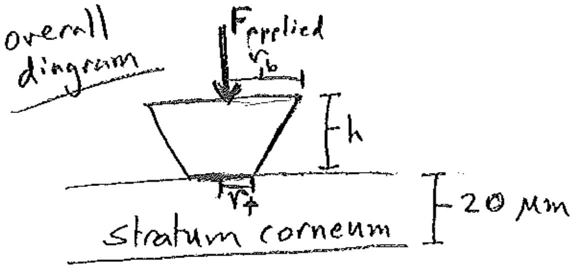
* *Davis SP, Landis BJ, Adams ZH, Allen MG, Prausnitz MR. Insertion of microneedles into skin: measurement and prediction of insertion force and needle fracture force. J Biomech. 2004 Aug;37(8):1155-63.*

Based on the information on the previous page, model the microneedle and find values for r_t and h that will result in the smallest microneedle that will not fracture and will penetrate the stratum corneum. You will be graded on the clarity, correctness, and thoroughness of the presentation of your findings. To earn full credit, you must show all of your work and fully justify your answer with clear reasoning. To earn full credit, you must also communicate visually using figures that help communicate your reasoning (e.g., diagrams and graphs).

FBD



overall diagram

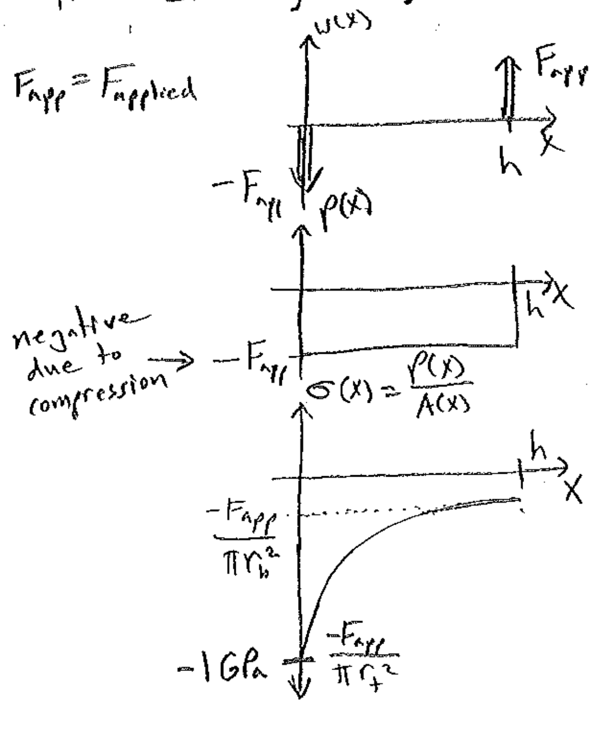


In order to fully penetrate the stratum corneum and deliver a substance $h > 20\,\mu m$ and $F_{applied} \geq F_{penetrate}$.

In order to have the minimum size and avoid fracture, we set $\boxed{h = 20\,\mu m + \epsilon}$, which is just a bit taller than the stratum corneum, and $\boxed{F_{applied} = \frac{1}{2}\pi 10^9 r_t^2 + \frac{1}{2}\pi 10^{-3}\,N}$, which is the smallest force that will result in the microneedle penetrating the skin.

The stress, $\sigma(x)$, in our microneedle model will then be

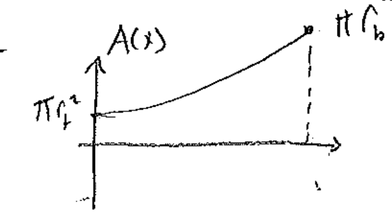
$F_{app} = F_{applied}$



$A(x) = \pi\, r(x)^2$

$A(0) = \pi\, r_t^2$

$A(h) = \pi\, r_b^2$



$$\sigma(x) = \frac{P(x)}{A(x)} = \frac{-F_{app}}{\pi\, r(x)^2}$$

$\sigma(x)$ has the largest magnitude at $x=0$. This is because the internal axial load $P(x)$ is constant and the cross-sectional area $A(x)$ is smallest at the tip, $x=0$.

Since the largest compressive stress is at the tip, the material is most likely to fail at the tip, $x=0$. We need to pick a tip radius, r_t, that is large enough to not fracture.

$\sigma(x=0) \geq -1\,GPa$ will keep the tip from fracturing (notice that due to compression the stress is negative)

$$\sigma(x=0) = \frac{-F_{app}}{\pi r_t^2} \geq -1\,GPa$$

$$\frac{-\frac{1}{2}\pi 10^9 r_t^2 - \frac{1}{2}\pi 10^{-3}}{\pi r_t^2} \geq -10^9$$

$$-\tfrac{1}{2}10^9 - \tfrac{1}{2}10^{-3} r_t^{-2} \geq -10^9$$

$$-\tfrac{1}{2}10^{-3} r_t^{-2} \geq -10^9 + \tfrac{1}{2}10^9$$

$$-\tfrac{1}{2}10^{-3} r_t^{-2} \geq -\tfrac{1}{2}10^9$$

$$\tfrac{1}{2}10^{-3} r_t^{-2} \leq \tfrac{1}{2}10^9$$

$$r_t^{-2} \leq \frac{10^9}{10^{-3}}$$

$$r_t^{-2} \leq 10^{12}$$

$$10^{-12} \leq r_t^2$$

$$r_t^2 \geq 10^{-12}$$

$$r_t > 0 \rightarrow r_t \geq 10^{-6} \text{ meters}$$

So, the minimum size for the tip is a radius of $1\,\mu m$, $\boxed{r_t = 1\,\mu m}$

Note: The cited paper uses more complex models and different values. For example, $F_{penetrate}$ has the same form but different constants.