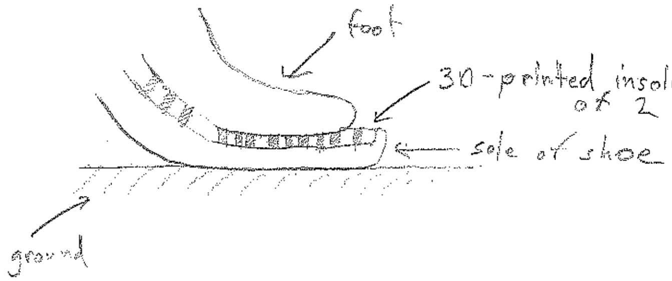
Overall Diagrams

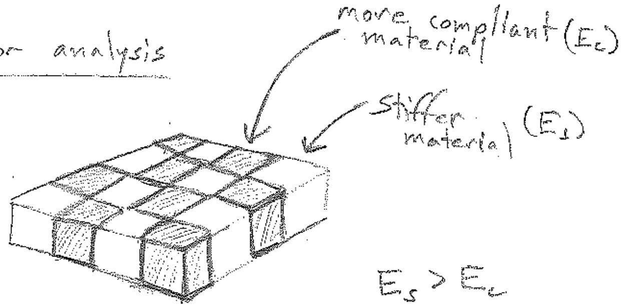


stiffer material (E_s)

more compliant material (E_c)

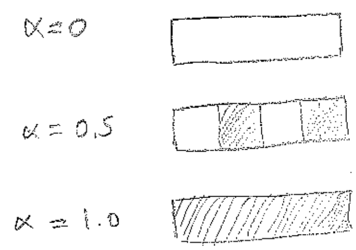
$E_s > E_c$

region for analysis

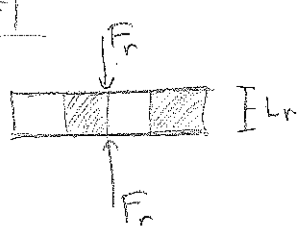


$E_s > E_c$

planar model of region

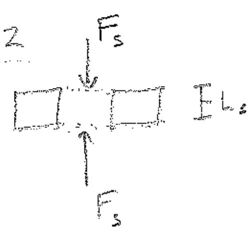


FBD #1



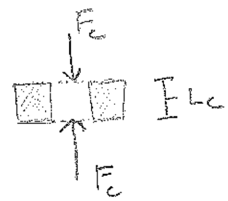
$$\delta_r = \frac{P_r L_r}{A_r E_r} = \frac{-F_r L_r}{A_r E_r}$$

FBD #2



$$\delta_s = \frac{P_s L_s}{A_s E_s} = \frac{-F_s L_s}{A_s E_s}$$

FBD #3



$$\delta_c = \frac{P_c L_c}{A_c E_c} = \frac{-F_c L_c}{A_c E_c}$$

interpret and check results

$$\boxed{E_r = \alpha E_c + (1-\alpha) E_s}$$

$\alpha = 0$, only stiffer material, $E_r = E_s$ ✓

$\alpha = 1$, only more compliant, $E_r = E_c$ ✓

$\alpha = ½$, half and half, $E_r = \frac{1}{2} E_c + \frac{1}{2} E_s$

this seems sensible!

E_r is between E_c and E_s and is more like the material with more cross-sectional area, if there is one.

$$F_r = F_s + F_c$$

$$\delta_r = \delta_s = \delta_c$$

$$\frac{-F_r L_r}{A_r E_r} = \frac{-F_s L_s}{A_s E_s} = \frac{-F_c L_c}{A_c E_c}$$

$L_r = L_s = L_c$, $A_s = (1-\alpha) A_r$, $A_c = \alpha A_r$

$$\frac{-F_r}{A_r E_r} = \frac{-F_s}{(1-\alpha) A_r E_s} = \frac{-F_c}{\alpha A_r E_c}$$

$$\frac{F_r}{E_r} = \frac{F_s}{(1-\alpha) E_s} = \frac{F_c}{\alpha E_c}$$

$$\frac{F_s}{A_s E_s} = \frac{F_c}{A_c E_c}$$

$$\frac{F_s}{F_c} = \frac{A_s E_s}{A_c E_c} = \frac{(1-\alpha) A_r E_s}{\alpha A_r E_c}$$

$$\frac{F_s}{F_c} = \left(\frac{1-\alpha}{\alpha}\right) \frac{E_s}{E_c}$$

solve for E_r

$$\frac{F_r}{E_r} = \frac{F_c}{\alpha E_c} \Rightarrow E_r = \frac{F_r}{F_c} \alpha E_c = \frac{F_s + F_c}{F_c} \alpha E_c = \left(\frac{F_s}{F_c} + 1\right) \alpha E_c$$

$$E_r = \left(\left(\frac{1-\alpha}{\alpha}\right) \frac{E_s}{E_c} + 1\right) \alpha E_c = (1-\alpha) E_s + \alpha E_c$$