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Viscous damping of toroidal angular momentum in tokamaks

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The Braginskii viscous stress tensor formalism was generalized to accommodate non-axisymmetric 3D magnetic fields in general toroidal flux surface geometry in order to provide a representation for the viscous damping of toroidal rotation in tokamaks arising from various "neoclassical toroidal viscosity" mechanisms. In the process, it was verified that the parallel viscosity contribution to damping toroidal angular momentum still vanishes even in the presence of toroidal asymmetries, unless there are 3D radial magnetic fields. © *2014 AIP Publishing LLC*.
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I. INTRODUCTION

Rotation in tokamak plasmas is an important topic of current research (e.g., the recent review of Ref. 1). Plasma rotation has been demonstrated to stabilize resistive wall modes (e.g., Ref. 2) and to correlate with energy confinement (e.g., Ref. 3), and rotation shear is widely believed to stabilize microinstabilities and thereby reduce diffusive energy transport (e.g., Refs. 4 and 5). Evidence of self-driven "intrinsic" rotation[6] provides hope that these benefits of rotation can be extended to future large tokamaks without the necessity of large external torques.

The early "classical" theories (e.g., Refs. 7–9) for rotation in tokamaks were developed on the basis of an assumed toroidal symmetry in a 2D magnetic field geometry. However, there have long been theoretical predictions of mechanisms (field ripple, error fields, magnetic instabilities, etc.) for the production of toroidal asymmetries in the magnetic field (e.g., Refs. 10–17). The damping of toroidal rotation by toroidal field ripple (e.g., Refs. 18 and 19) and the "magnetic braking" of toroidal rotation by a static n/m = 1/1 error field (Ref. 20) have been demonstrated experimentally. Such "classical" mechanisms for momentum damping have come to be collectively identified as "neoclassical toroidal viscosity" (NTV).

To the extent that the NTV mechanisms can be treated as "triggers" for parallel and perpendicular viscosity, the resulting viscosities can be calculated from the fluid rate of strain tensor using neoclassical and/or NTV viscosity coefficients. Thus, in order to make a fluid calculation of the effect of various NTV mechanisms on rotation in a tokamak, it is first necessary to generalize the fluid viscosity tensor representation to accommodate 3D magnetic fields with toroidal asymmetries. We have previously adapted the Braginskii stress tensor[21] from x-y-z geometry to a circular plasma toroidal flux surface geometry[7] to develop such a tokamak rotation theory,[22] which has been subsequently generalized to an elongated flux surface geometry representation with Shafranov shift.[23,24] These rotation theories have predicted measured rotation velocities relatively well,[22,24] but have not yet represented the NTV effects associated with the 3D toroidally asymmetric magnetic field geometry. The purpose of this paper is to further extend the Braginskii stress tensor to a generalized 3D, non-axisymmetric magnetic field geometry in order to provide a basis for the subsequent construction of a computational neoclassical plus NTV fluid rotation theory for tokamaks.

II. VISCOUS DAMPING OF TOROIDAL ANGULAR MOMENTUM

We define a general right hand orthogonal (ψ, p, ϕ) coordinate system with differential length elements $(dl_\psi = h_\psi d\psi, dl_p = h_p dp, dl_\phi = h_\phi d\phi)$, where ψ is a radial-like flux surface variable, p is a poloidal-like angular variable, and ϕ is the toroidal angle. In such a system, the viscous damping of toroidal angular momentum is represented by the flux surface average $\langle\rangle$ of the toroidal component of the viscous torque

$$\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\rangle = \frac{1}{V'}\frac{\partial}{\partial\psi}\left(V'\langle R^2\nabla\phi\cdot\Pi\cdot\nabla\psi\rangle\right) = \left\langle \frac{1}{Rh_p}\frac{\partial}{\partial l_\psi}\left(R^2 h_p \pi_{\psi\phi}\right) + B_p\frac{\partial}{\partial l_p}\left(\frac{R\pi_{p\phi}}{B_p}\right)\right\rangle, \tag{1}$$

where V' is the differential volume between flux surfaces.

The Braginskii decomposition of the viscous stress[21] has been generalized to toroidal flux surface geometry[7,24] in the case of 2D toroidally symmetric magnetic fields. We now further generalize it to the above general flux surface geometry with 3D toroidally non-symmetric magnetic field geometry by writing the elements of the general rate of strain tensor of fluid theory[21,25]

$$W_{\alpha\beta} \equiv \hat{n}_\alpha\cdot\nabla\mathbf{V}\cdot\hat{n}_\beta + \hat{n}_\beta\cdot\nabla\mathbf{V}\cdot\hat{n}_\alpha - \frac{2}{3}\delta_{\alpha\beta}\nabla\cdot\mathbf{V} = \left(\frac{\partial V_\beta}{\partial l_\alpha} + \sum_k \Gamma^\alpha_{\beta k}V_k\right) + \left(\frac{\partial V_\alpha}{\partial l_\beta} + \sum_k \Gamma^\beta_{\alpha k}V_k\right) - \frac{2}{3}\delta_{\alpha\beta}\nabla\cdot\mathbf{V}, \tag{2}$$

where the Christoffel symbols are defined in terms of the metric elements of the coordinate system

$$\Gamma^{\alpha}_{\beta k} \equiv \frac{1}{h_\beta h_k}\left(\frac{\partial h_\beta}{\partial \xi_k}\delta_{\alpha\beta} - \frac{\partial h_k}{\partial \xi_\beta}\delta_{\alpha k}\right), \quad (\xi_1 = \psi, \xi_2 = p, \xi_3 = \phi). \tag{3}$$

Braginskii[21] showed that the viscous stress tensor elements could be decomposed into "parallel," "gyroviscous," and "perpendicular" components with viscosity coefficients which differed by several orders of magnitude $(\eta_0 \gg \eta_{3,4} \gg \eta_{1,2})$

$$\begin{aligned}\pi_{\alpha\beta} &= -\eta_0 W^0_{\alpha\beta} + [\eta_3 W^3_{\alpha\beta} + \eta_4 W^4_{\alpha\beta}] - [\eta_1 W^1_{\alpha\beta} + \eta_2 W^2_{\alpha\beta}] \\ &\equiv \pi^0_{\alpha\beta} + \pi^{34}_{\alpha\beta} + \pi^{12}_{\alpha\beta},\end{aligned} \tag{4}$$

where denoting $f_\alpha \equiv B_\alpha/|B|$ and using the Einstein summation convention, the $W^n_{\alpha\beta}$ are

$$\begin{aligned}
W^0_{\alpha\beta} &\equiv \frac{3}{2}\left(f_\alpha f_\beta - \frac{1}{3}\delta_{\alpha\beta}\right)\left(f_\mu f_\nu - \frac{1}{3}\delta_{\mu\nu}\right)W_{\mu\nu},\\
W^1_{\alpha\beta} &\equiv \left(\delta^\perp_{\alpha\mu}\delta^\perp_{\beta\nu} + \frac{1}{2}\delta^\perp_{\beta\nu}f_\mu f_\nu\right)W_{\mu\nu},\\
W^2_{\alpha\beta} &\equiv \left(\delta^\perp_{\alpha\mu}f_\beta f_\nu + \delta^\perp_{\alpha\beta}f_\alpha f_\mu\right)W_{\mu\nu},\\
W^3_{\alpha\beta} &= \frac{1}{2}\left(\delta^\perp_{\alpha\mu}\varepsilon_{\beta\gamma\nu} + \delta^\perp_{\beta\nu}\varepsilon_{\alpha\gamma\mu}\right)W_{\mu\nu},\\
W^4_{\alpha\beta} &= \frac{1}{2}(f_\alpha f_\mu \varepsilon_{\beta\gamma\nu} + \varepsilon_{\alpha\gamma\mu}f_\beta f_\nu)W_{\mu\nu},
\end{aligned} \tag{5}$$

with $\varepsilon_{\alpha\beta\gamma}$ being the antisymmetric unit tensor, $\delta_{\alpha\beta}$ being the Kroneker delta function, and $\delta^\perp_{\alpha\beta} \equiv \delta_{\alpha\beta} - f_\alpha f_\beta$.

Working out the leading order parallel viscosity contributions $\pi^0_{\psi\phi} = -\eta_0 W^0_{\psi\phi}$ and $\pi^0_{p\phi} = -\eta_0 W^0_{p\phi}$ for the two components of the rate-of-strain tensors yields

$$W^0_{\psi\phi} = \frac{3}{2}f_\psi f_\phi H^0, \quad W^0_{p\phi} = \frac{3}{2}f_p f_\phi H^0, \text{ where}$$

$$H^0 \equiv \left[\begin{array}{l}
\left(f_\psi f_\psi - \frac{1}{3}\right)\left\{\frac{4}{3}\frac{\partial V_\psi}{\partial l_\psi} - \frac{2}{3}\left(\frac{\partial V_p}{\partial l_p} + \frac{\partial V_\phi}{\partial l_\phi}\right) + 2\left(\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial l_p}V_p + \frac{1}{h_\psi}\frac{\partial h_\psi}{\partial l_\phi}V_\phi\right)\right\} + \\
\left(f_p f_p - \frac{1}{3}\right)\left\{\frac{4}{3}\frac{\partial V_p}{\partial l_p} - \frac{2}{3}\left(\frac{\partial V_\psi}{\partial l_\psi} + \frac{\partial V_\phi}{\partial l_\phi}\right) + 2\left(\frac{1}{h_p}\frac{\partial h_p}{\partial l_\psi}V_\psi + \frac{1}{h_p}\frac{\partial h_p}{\partial l_\phi}V_\phi\right)\right\} + \\
\left(f_\phi f_\phi - \frac{1}{3}\right)\left\{\frac{4}{3}\frac{\partial V_\phi}{\partial l_\phi} - \frac{2}{3}\left(\frac{\partial V_p}{\partial l_p} + \frac{\partial V_\psi}{\partial l_\psi}\right) + 2\left(\frac{1}{h_\phi}\frac{\partial h_\phi}{\partial l_\psi}V_\psi + \frac{1}{h_\phi}\frac{\partial h_\phi}{\partial l_p}V_p\right)\right\} + \\
2f_\psi f_p\left\{\frac{\partial V_\psi}{\partial l_p} + \frac{\partial V_p}{\partial l_\psi} - \left(\frac{1}{h_p}\frac{\partial h_p}{\partial l_\psi}V_p + \frac{1}{h_\psi}\frac{\partial h_\psi}{\partial l_p}V_\psi\right)\right\} + \\
2f_p f_\phi\left\{\frac{\partial V_\phi}{\partial l_p} + \frac{\partial V_p}{\partial l_\phi} - \left(\frac{1}{h_p}\frac{\partial h_p}{\partial l_\phi}V_p + \frac{1}{h_\phi}\frac{\partial h_\phi}{\partial l_p}V_\phi\right)\right\} + \\
2f_\psi f_\phi\left\{\frac{\partial V_\phi}{\partial l_\psi} + \frac{\partial V_\psi}{\partial l_\phi} - \left(\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial l_\phi}V_\psi + \frac{1}{h_\phi}\frac{\partial h_\phi}{\partial l_\psi}V_\phi\right)\right\}
\end{array}\right]. \tag{6}$$

The presence of radial magnetic field components is represented in these expressions by the $f_\psi = B_\psi/|B|$ terms, and non-axisymmetry is represented by the $\partial()/\partial l_\phi$ terms, where () is any such quantity so appearing in Eq. (6).

Clearly, $W^0_{\psi\phi} \equiv 0$ in the absence of a 3D (radial) component of the magnetic field (i.e., for $f_\psi \equiv B_\psi/|B| = 0$), so the first, $\pi^0_{\phi\psi}$ term in Eq. (1) vanishes for, $f_\psi \equiv B_\psi/|B| = 0$ but would survive for $f_\psi \equiv B_\psi/|B| \neq 0$.

The flux surface average of the second, $\pi^0_{p\phi}$ term in Eq. (1) is

$$\left\langle B_p \frac{\partial}{\partial l_p}\left(\frac{R\pi^0_{p\phi}}{B_p}\right)\right\rangle = \frac{\oint\int_0^{2\pi} d\phi\left\{B_p\frac{\partial}{\partial l_p}\left(\frac{R\pi^0_{p\phi}}{B_p}\right)\right\}\frac{dl_p}{B_p}}{\oint\int_0^{2\pi} d\phi\frac{dl_p}{B_p}}. \tag{7}$$

The term in the numerator of Eq. (7) is a perfect differential and must vanish from the requirement of single-valuedness; i.e., Eq. (7) vanishes, and there is no contribution of the leading order parallel viscosity to the toroidal angular momentum viscous damping from this term even if $\pi^0_{p\phi}$ is toroidally asymmetric. So, only if there are 3D fields such that $f_\psi \equiv B_\psi/|B| \neq 0$, can there be a parallel viscosity contribution to toroidal angular momentum damping. Thus, if $f_\psi \equiv B_\psi/|B| = 0$, the largest remaining contributions to toroidal angular momentum damping are due to the gyroviscous $\pi^{34}_{\psi\phi}$ and $\pi^{34}_{p\phi}$ terms[7,22] with viscosity coefficients $\eta_{34} \ll \eta_0$.

III. SUMMARY

The Braginskii fluid rate-of-strain tensor formalism for the calculation of the toroidal angular momentum damping rate arising from various neoclassical parallel, perpendicular, and gyro viscosity mechanisms has been extended to generalized tokamak flux surface geometry in order to represent the viscous damping of toroidal rotation by various non-axisymmetric 3D magnetic fields. In the process, it was shown that neoclassical parallel viscosity does not contribute to toroidal angular momentum damping unless there are 3D magnetic fields with radial components, even in the presence of toroidal asymmetries.

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