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Extensions of ion orbit loss theory

W. M. Stacey
Georgia Institute of Technology, Atlanta, Georgia 30332, USA

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Theoretical refinements to an existing model for the loss of ions by drifting across the last closed flux surface are presented. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4861612]

The collisionless loss of energetic ions on orbits through the X-region which carried them into the divertor seems to have been first suggested in Ref. 1 and confirmed by Monte Carlo analysis of measured particle and heat fluxes in JET.[2] Collisional loss of ions on banana orbits near the boundary was then explored as a cause for the H-mode transition,[3,4] and direct loss of ions on banana orbits that crossed the separatrix was discussed in Refs. 5–7. A theory was developed[8–10] for the L-H transition based on the bifurcation of the poloidal flow velocity due to the effect of ion orbit loss on viscosity, and later the experimental observation of large rotation velocities in MAST were explained in terms of the torques produced by return currents compensating the ion orbit loss of energetic beam ions.[11,12] Recently, the intrinsic rotation observed in DIII-D in the absence of an external torque has been explained by ion orbit loss of thermalized ions.[13–17]

There is a school of thought which holds that the physics of the edge plasma is determined in large part by ion orbit loss—the free-streaming of ions from inner flux surfaces along drift orbits that cross the last closed flux surface and are lost to the plasma. There are two different basic mechanisms for ion orbit loss in the edge plasma. The most general is the loss of ions on passing or banana-trapped orbits that leave the plasma by drifting outward across the last closed flux surface (as developed, e.g., in Refs. 7 and 13–15 or 18–22). Both thermalized plasma ions and energetic neutral beam ions (and fusion alpha particles) can be lost in this manner. This type of ion orbit loss will be referred to as "ion orbit loss." A second ion orbit loss mechanism ("X-loss," as developed, e.g., in Refs. 23–29) is the ion loss through the X-point in diverted plasmas produced by the fact that ions on orbits that pass near the X-point where the poloidal magnetic field is very small have a very small poloidal displacement in time and are essentially trapped in the poloidal vicinity of the X-point, where they are subject to vertical curvature and grad-B drifts which take them outward across the last closed flux surface and eventually into the divertor. Such ion orbit loss effects could significantly alter the results of most of the ongoing work on edge plasma physics experimental interpretation and prediction, but they are not yet routinely taken into account in such calculations.

We have recently developed a model[18–22] for (i) the calculation of the radially cumulative fractions of ion particles F_{orb}, momentum M_{orb}, and energy E_{orb} in a tokamak plasma flowing outward across an internal flux surface that are on drift orbits that would carry them immediately outward across the last closed flux surface (i.e., be ion orbit lost) and (ii) for the calculation of how these ion orbit losses reduce the corresponding radial particle Γ, momentum M, and energy Q fluxes that would be calculated in the absence of ion orbit loss from the respective radial particle (continuity), momentum and energy balance equations $(\hat{\Gamma} = (1 - F_{orb})\Gamma$, $\hat{M} = (1 - M_{orb})M$, $\hat{Q} = (1 - E_{orb})Q)$. Applications of this theory to the interpretation of edge plasma data from DIII-D[30] yield physically reasonable results, except perhaps just inside the separatrix where F_{orb} and E_{orb} become quite large, indicating the desirability of extending the calculation to take into account the possibility (i) that some ions on orbits which exit the plasma may return into the plasma on those same orbits, (ii) that some ions on orbits that exit the plasma may not cross the last closed flux surface (LCFS) because of scattering, and (iii) that a more accurate calculation of the effect of ion orbit loss would result from directly including the loss in the solution of the continuity and energy balance equations for $\hat{\Gamma}$ and $\hat{Q}$. Theoretically extending the ion orbit loss model of Refs. 18–22 to address these issues is the purpose of this Brief Communication.

Combining the requirements for conservation of canonical angular momentum, energy, and magnetic moment leads to an orbit constraint equation which may be used for the determination of the minimum speed for which an ion at some location on an internal flux surface, ψ_0, with initial speed V_0 and direction cosine ζ_0 with respect to the toroidal magnetic field can execute a drift orbit which passes through some point ψ_{LCFS} on the LCFS and exit the confined plasma

$$V_0^2\left[\left(\left|\frac{B_{LCFS}}{B_0}\right|\frac{f_{\varphi 0}}{f_{\varphi LCFS}}\zeta_0\right)^2 - 1 + (1-\zeta_0^2)\left|\frac{B_{LCFS}}{B_0}\right|\right] + V_0\left[\frac{2e(\psi_o - \psi_{LCFS})}{R_{LCFS}mf_{\varphi LCFS}}\left(\left|\frac{B_{LCFS}}{B_0}\right|\frac{f_{\varphi 0}}{f_{\varphi LCFS}}\zeta_0\right)\right] + \left[\left(\frac{e(\psi_0 - \psi_{LCFS})}{R_{LCFS}mf_{\varphi LCFS}}\right)^2 - \frac{2e(\phi_0 - \phi_{LCFS})}{m}\right] = 0, \qquad (1)$$

where $f_\varphi = |B_\varphi/B|$, R is the major radius, ϕ is the electrostatic potential, and ψ is the flux surface value. Equation (1) is quite general with respect to flux surface geometry representation of R, B, and the flux surfaces ψ. By specifying an initial "0" location for an ion with initial direction cosine ζ_0, and specifying a final location on the last closed flux surface (LCFS) ψ_{LCFS}, Eq. (1) can be used to determine if an ion with speed $V_0(\zeta_0)$ can cross the *LCFS* at that final location on the flux surface ψ_{LCFS} (i.e., if Eq. (1) has a physical solution). Thus, Eq. (1) can be solved repeatedly to determine the minimum ion energy necessary for an ion located on an internal flux surface to cross the last closed flux surface at a given location (or to strike the chamber wall at a given location, etc.). All of the ions at this location with this value of ζ_0 with speeds greater than this $V_{0\min}(\zeta_0)$ are able to cross the last closed flux surface.

The probability that an ion would not scatter before crossing the LCFS is $R^{scat}_{noloss}(V_0,\zeta_0) = \exp(-\int_0^{L_1}\Sigma(l)dl)$, where, for an ion with speed V_0 and direction cosine ζ_0, L_1 is the length of the ion orbit along l in the plasma between the "launch" point ψ_0 on the inner flux surface and the point of exit ψ_{LCFS} into the SOL (scrape off layer), and $\Sigma(l) = n_{p,o}(l)\sigma_{p.o}(l)$ is the macroscopic cross section for interactions with plasma ions/electrons (p, ionization) and neutrals (o, charge-exchange). This type of calculation could also in principle be applied to ions on orbit that did not cross the LCFS but which scattered onto orbits that did, but this would be a more involved calculation. The probability that this exiting ion then has an interaction with a plasma ion or neutral or the chamber wall, while traversing an orbit in the SOL that eventually returns back across the LCFS into the confined plasma is $R^{SOL}_{loss}(V_0,\zeta_0) = 1-\exp(-\int_0^{L_2}\Sigma_{sol}(l)dl)$, L_2 is the distance along the orbit in the SOL between exiting and returning across the LCFS into the confined plasma, and Σ_{SOL} is defined the same as Σ but with SOL parameters and has a very large value where the orbit traverses through the chamber wall. Thus, for an ion with speed V_0 and direction cosine ζ_0 that satisfies Eq. (1) for the "launch" point ψ_0 on the inner flux surface and the point of exit ψ_{LCFS} into the SOL, the total ion orbit loss probability is $R^{iol}_{loss}(V_0,\zeta_0) = R^{scat}_{noloss}(V_0,\zeta_0)\times R^{SOL}_{loss}(V_0,\zeta_0)$.

The methodology employed in Refs. 18–22 can be extended to calculate radially cumulative particle, momentum and energy ion orbit loss fractions of the outwardly flowing ions that include these ion orbit loss probabilities

$$\hat{F}_{orb} \equiv \frac{n_{loss}}{n_{tot}} = \frac{\int_{-1}^{1}\left[\int_{V_{0\min(\zeta_0)}}^{\infty} R^{iol}_{loss}(V_0,\zeta_0) f(V_0)V_0^2 dV_0\right]d\zeta_0}{\int_{-1}^{1}\left[\int_0^{\infty} f(V_0)V_0^2 dV_0\right]d\zeta_0} = \frac{1}{2}\int_{-1}^{1}\langle R^{iol}_{loss}(V_{0\min(\zeta_0)},\zeta_0)\rangle_n \left[\frac{\Gamma(3/2,\varepsilon_{\min(\zeta_0)})}{\Gamma(3/2)}\right]d\zeta_0$$

$$\equiv \frac{1}{2}\int_{-1}^{1}\left\{\frac{\left[\int_{V_{0\min(\zeta_0)}}^{\infty} R^{iol}_{loss}(V_0,\zeta_0) f(V_0)V_0^2 dV_0\right]}{\left[\int_{V_{0\min(\zeta_0)}}^{\infty} f(V_0)V_0^2 dV_0\right]}\right\}\left\{\frac{\left[\int_{V_{0\min(\zeta_0)}}^{\infty} f(V_0)V_0^2 dV_0\right]}{\left[\int_0^{\infty} f(V_0)V_0^2 dV_0\right]}\right\}d\zeta_0,$$

$$\hat{E}_{orb} \equiv \frac{E_{loss}}{E_{tot}} = \frac{\int_{-1}^{1}\left[\int_{V_{0\min(\zeta_0)}}^{\infty} R^{iol}_{loss}(V_0,\zeta_0)\left(\frac{1}{2}mV_0^2\right) f(V_0)V_0^2 dV_0\right]d\zeta_0}{\int_{-1}^{1}\left[\int_0^{\infty}\left(\frac{1}{2}mV_0^2\right) f(V_0)V_0^2 dV_0\right]d\zeta_0} = \frac{1}{2}\int_{-1}^{1}\langle R^{iol}_{loss}(V_{0\min(\zeta_0)},\zeta_0)\rangle_E \left[\frac{\Gamma(5/2,\varepsilon_{\min(\zeta_0)})}{\Gamma(5/2)}\right]d\zeta_0$$

$$\equiv \frac{1}{2}\int_{-1}^{1}\left\{\frac{\left[\int_{V_{0\min(\zeta_0)}}^{\infty} R^{iol}_{loss}(V_0,\zeta_0)\left(\frac{1}{2}mV_0^2\right) f(V_0)V_0^2 dV_0\right]}{\left[\int_{V_{0\min(\zeta_0)}}^{\infty}\left(\frac{1}{2}mV_0^2\right) f(V_0)V_0^2 dV_0\right]}\right\}\left\{\frac{\left[\int_{V_{0\min(\zeta_0)}}^{\infty}\left(\frac{1}{2}mV_0^2\right) f(V_0)V_0^2 dV_0\right]}{\left[\int_0^{\infty}\left(\frac{1}{2}mV_0^2\right) f(V_0)V_0^2 dV_0\right]}\right\}d\zeta_0$$

$$\hat{M}_{orb} \equiv \frac{M_{loss}}{M_{tot}} = \frac{\int_{-1}^{1}\zeta_0\left[\int_{V_{0\min(\zeta_0)}}^{\infty} R^{iol}_{loss}(V_0,\zeta_0)(mV_0) f(V_0)V_0^2 dV_0\right]d\zeta_0}{\int_{-1}^{1}\left[\int_0^{\infty}(mV_0) f(V_0)V_0^2 dV_0\right]d\zeta_0} = \frac{1}{2}\int_{-1}^{1}\zeta_0\langle R^{iol}_{loss}(V_{0\min(\zeta_0)},\zeta_0)\rangle_M \left[\frac{\Gamma(2,\varepsilon_{\min(\zeta_0)})}{\Gamma(2)}\right]d\zeta_0$$

$$\equiv \frac{1}{2}\int_{-1}^{1}\zeta_0\left\{\frac{\left[\int_{V_{0\min(\zeta_0)}}^{\infty} R^{iol}_{loss}(V_0,\zeta_0)(mV_0) f(V_0)V_0^2 dV_0\right]}{\left[\int_{V_{0\min(\zeta_0)}}^{\infty}(mV_0) f(V_0)V_0^2 dV_0\right]}\right\}\left\{\frac{\left[\int_{V_{0\min(\zeta_0)}}^{\infty}(mV_0) f(V_0)V_0^2 dV_0\right]}{\left[\int_0^{\infty}(mV_0) f(V_0)V_0^2 dV_0\right]}\right\}d\zeta_0. \quad (2)$$

For the purpose of numerical evaluation, the local plasma ion distribution function has been assumed to be a Maxwellian at the local ion temperature T_i, leading to the reduced energy $\varepsilon_{\min(\zeta_0)} \equiv (1/2)mV^2_{0\min(\zeta_0)}/kT$, and the Γ in Eq. (2) refer to the gamma functions and incomplete gamma functions. An appropriate average, as discussed in Refs. 18 and 19, over the different poloidal "launch" and "exit" locations is implied in the above definitions of the $\langle R_{loss}^{iol}(V_{0\min(\zeta_0)},\zeta_0)\rangle_{F,E,M}$.

The thermal ion orbit losses (and the similarly calculated beam ion orbit losses) can be incorporated directly into the solution of the particle continuity and energy balance equations for the radial ion and energy fluxes

$$\frac{\partial\hat{\Gamma}_{ri}}{dr} = -\frac{\partial n_i}{\partial t} + N_{nbi}\left(1-2f_{nbi}^{iol}\right) + n_e\nu_{ion} - 2\frac{\partial F_{orbi}}{\partial r}\hat{\Gamma}_{ri},$$
$$\frac{\partial\hat{Q}_{ri}}{dr} = -\frac{\partial}{\partial t}\left(\frac{3}{2}n_iT_i\right) + q_{nbi}^{i}f_{nbi}^{iol} - q_{ie} - n_in_o^c\langle\sigma\upsilon\rangle_{cx}\frac{3}{2}\left(T_i - T_o^c\right) - \frac{\partial E_{orbi}}{\partial r}\hat{Q}_{ri},$$
$$\frac{\partial\hat{Q}_{re}}{dr} = -\frac{\partial}{\partial t}\left(\frac{3}{2}n_eT_e\right) + q_{nbi}^{e}f_{nbi}^{iol} + q_{ie} - n_en_z\langle\sigma\upsilon\rangle_{cx}L_z(T_e), \quad (3)$$

where the carat indicates that the radial particle and heat fluxes are calculated including the effects of ion orbit loss (and return current) represented by (cumulative in radius) thermal ion particle and energy loss fractions $\hat{F}_{orbi}$ and $\hat{E}_{orbi}$ and by the (local) fast beam ion loss fraction f_{nbi}^{iol}. The factor of 2 in the first of Eq. (3) arises from taking into account the inward main ion particle fluxes from the SOL which are necessary to maintain charge neutrality in the presence of the beam and thermal plasma ion orbit losses.[19]

Using these radial particle and heat fluxes evaluated with the experimental density and temperature, the experimental thermal diffusivities interpreted from the measured data can be calculated from the heat conduction relation

$$\chi_{i,e}^{\exp} = \frac{q_{i,e}}{-n_{i,e}^{\exp}\left(\frac{\partial T_{i,e}^{\exp}}{\partial r}\right)} = \frac{\left(\hat{Q}_{ri,e} - 1.5T_{i,e}\hat{\Gamma}_{ri,e}\right)}{-n_{i,e}^{\exp}\left(\frac{\partial T_{i,e}^{\exp}}{\partial r}\right)}. \quad (4)$$

[1]F. L. Hinton and M. Chu, Nucl. Fusion **25**, 345 (1985).
[2]M. A. Kovanen, R. Reichle, E. Lazzaro, D. D. R. Summers, and T. S. Taylor, Nucl. Fusion **32**, 863 (1992).
[3]S. Itoh and K. Itoh, Nucl. Fusion **29**, 1031 (1989).
[4]K. C. Shaing and E. C. Crume, Phys. Rev. Lett. **63**, 2369 (1989).
[5]S. Putvinskij, Plasma Phys. Controlled Fusion **35**, 219 (1993)
[6]A. Chankin and G. McCracken, Nucl. Fusion **33**, 1459 (1993).
[7]K. Miyamoto, Nucl. Fusion **36**, 927 (1996).
[8]K. C. Shaing, E. C. Crume, and W. A. Houlberg, Phys. Fluids B **2**, 1492 (1990).
[9]K. C. Shaing, Phys. Fluids B **4**, 171 (1992).
[10]K. C. Shaing, Phys. Plasmas **9**, 1 (2002).
[11]P. J. Helander, R. J. Akers, and L.-G. Eriksson, Phys. Plasmas **12**, 112503 (2005).
[12]K. G. McClements and A. Thyagaraja, Phys. Plasmas **13**, 042503 (2006).
[13]J. S. deGrassie, J. E. Rice, K. H. Burrell, R. J. Groebner, and W. M. Solomon, Phys. Plasmas **14**, 056115 (2007).
[14]J. S. deGrassie, R. J. Groebner, K. H. Burrell, and W. M. Solomon, Nucl. Fusion **49**, 085020 (2009).
[15]J. S. deGrassie, S. H. Mueller, and J. A. Boedo, Nucl. Fusion **52**, 013010 (2012).
[16]W. M. Stacey, J. A. Boedo, T. E. Evans, B. A. Grierson, and R. J. Groebner, Phys. Plasmas **19**, 112503 (2012).
[17]W. M. Stacey and B. A. Grierson, "Interpretation of rotation and momentum transport in DIII-D edge plasma and comparison with neoclassical theory," Phys. Plasmas (submitted).
[18]W. M. Stacey, Phys. Plasmas **18**, 102504 (2011).
[19]W. M. Stacey, Nucl. Fusion **53**, 063011 (2013).
[20]W. M. Stacey, "Structure in the edge plasma profiles in tokamaks," Contrib. Plasma Phys. (to be published).
[21]W. M. Stacey, Phys. Plasmas **20**, 092508 (2013).
[22]W. M. Stacey, Phys. Plasmas **20**, 112503 (2013).
[23]C. S. Chang, S. Kue, and H. Weitzner, Phys. Plasmas **9**, 3884 (2002).
[24]C. S. Chang, S. Kue, and H. Weitzner, Phys. Plasma **11**, 2649 (2004).
[25]H. Weitzner and C. S. Chang, Phys. Plasmas **11**, 3060 (2004).
[26]C. S. Chang, Phys. Plasmas **11**, 5626 (2004).
[27]S. H. Hahn, S. Ku, and C. S. Chang, Phys. Plasmas **12**, 102501 (2005).
[28]C. S. Chang and S. Ku, Phys. Plasmas **15**, 062510 (2008).
[29]W. M. Stacey, Phys. Plasmas **18**, 122504 (2011).
[30]J. Luxon, Nucl. Fusion **42**, 614 (2002).