

A. Basic Conservation and Ion Orbit Loss Calculations

1. The basic IOL calculation is for the minimum speed (energy) of an ion at a given point (ψ_0, θ_0) on an internal flux surface with a given direction cosine ζ_0 w/respect to \mathbf{B}_ϕ to be able to reach a given point $(\psi_{sep}, \theta_{sep})$ on an external flux surface (the separatrix in our model). Conservation of canonical angular momentum, magnetic moment and energy are invoked to derive a quadratic equation for this minimum speed (energy) $V_{0min}(\zeta_0)$. All ζ_0 – directed ions in the thermalized ion distribution at location (ψ_0, θ_0) with $V_0(\zeta_0) \geq V_{0min}(\zeta_0)$ are assumed to be lost. This calculation is carried out for 22 ζ_0 at each of 8 θ – locations on an internal flux surface for loss at each of 8 θ – locations on the separatrix [PoP18, 102504, 2011].
2. The IOL model is applied to an outflowing thermalized ion distribution to calculate radially cumulative IOL fractions of particles (F_{orb}), momentum (M_{orb}) and energy (E_{orb}) averaged over successive internal flux surfaces (ψ_0) in the plasma edge. Averaging procedures [PoP18, 102504, 2011; PoP22, 042504, 2015] take into account that the outflowing ions spiral over the flux surface a large number of times in the time taken to be radially transported a small radial distance outward and assume there is no scattering change in particle direction cosine ζ_0 .
3. Flux Surface Geometry—The earlier work [PoP18, 102504, 2011] made use of a circular $R = R_0(1 + \varepsilon \cos \theta)$ flux surface model, but more recent work [PoP23, 122505, 2016] has shown that realistic flux surface geometry representation effects are important.
4. Ion orbit loss of *neutral beams* is treated in [PoP23, 122505, 2016].
5. X-loss and X-transport---In the vicinity of the X-point B_θ is very small and ions that are “stuck” (poloidally) in this X-region will gradB- and curvature-drift radially inward or outward until they can ExB θ –drift out of this low B_θ region. Modeling of this “X-transport” phenomenon is treated in [PoP18, 122504, 2011; PoP23, 122505, 2016].

B. Inclusion of Ion Orbit Loss in Fluid Theory

1. Initially, the radial particle Γ_r and energy Q_r fluxes were calculated w/o taking into account IOL and then reduced $\Gamma(r) = \Gamma(r)[1 - F_{orb}(r)]$, $Q(r) = Q(r)[1 - E_{orb}(r)]$ [PoP18, 102504, 2011]. However, this was found to over-predict the effect when the loss fractions are large, which they are in the very edge of the plasma. A more accurate procedure is to include loss terms $-\left(\frac{\partial F_{orb}(r)}{\partial r}\right)\Gamma(r)$ in the particle continuity equation and $-\left(\frac{\partial E_{orb}(r)}{\partial r}\right)Q(r)$ directly in the energy balance equation. [PoP23, 012508, 2016; NF57, 066034 & 119501, 2017].
2. The momentum ion orbit loss could be treated in the same manner as for particles and energy if we actually integrated the momentum conservation equations. However, with the rotation model that we use, it is more convenient to calculate a fluid rotation velocity from the momentum balance equations and then add to that a co-current intrinsic velocity due to the preferential ion orbit loss of ctr-current ions. [PoP23, 012508, 2016].
3. The effect of ion return currents required to maintain charge neutrality is represented in the fluid continuity equation [NF57, 066034, 2017].