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A Composite Neoclassical Toroidal Viscosity Model Incorporating Torques from both Axisymmetric and Nonaxisymmetric Tokamak Magnetic Fields

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Abstract — *This paper combines the older neoclassical gyroviscous model for toroidal viscosity in the plasma core, which is based on an axisymmetric magnetic field and obtains reasonable agreement with experiment for toroidal rotation in the plasma core but not in edge plasma, with recent models for neoclassical toroidal viscosity (NTV) based on nonaxisymmetric "perturbation" magnetic field components present primarily in the edge plasma to obtain a composite toroidal viscosity model for toroidal velocity calculations in the tokamak core and edge plasma. This combination is facilitated by the fact that the same form of "drag frequency" representation of the viscous torque used in many of the new (NTV) torque models arising from toroidally nonaxisymmetric perturbation magnetic fields that are present mostly in the plasma edge can also be used to represent the old neoclassical toroidal viscous torques arising from toroidally axisymmetric magnetic fields.*

Keywords — *toroidal viscosity, tokamak, rotation.*

Note — *Some figures may be in color only in the electronic version.*

I. INTRODUCTION

Recognition of both the importance of plasma rotation to the stable magnetic confinement of plasmas in tokamaks (e.g., Ref. 1) and of the complexity of the damping of toroidal plasma rotation by nonaxisymmetric magnetic fields (e.g., Refs. 2 through 5) has increased significantly in recent years.

The older neoclassical viscosity theory (e.g., Refs. 6 through 10) was developed under the assumption that the tokamak was magnetically axisymmetric, i.e., there was no radial component nor toroidal asymmetry in the magnetic field (or at least that any such nonaxisymmetry was small and unimportant). A significant feature of magnetic axisymmetry is that the leading order parallel component of the neoclassical toroidal rotation viscous damping term vanishes identically,[7,11] leaving only the gyroviscous and perpendicular viscous terms to contribute to the damping of toroidal velocity.

It was estimated by one group[7,8] that the leading order surviving gyroviscosity was of the proper magnitude to explain the measured toroidal viscosity damping in tokamaks, but two other groups[9,10] ordered out the leading order gyroviscosity and retained only the neoclassical perpendicular viscosity, with the result that their estimated toroidal rotation was orders of magnitude larger than measured values. These and similar estimates that neglected gyroviscosity led to the widespread, but incorrect, opinion that axisymmetric neoclassical viscosity theory greatly underpredicts toroidal momentum damping in tokamaks.

When neoclassical gyroviscosity was retained in the neoclassical viscosity theory, the axisymmetric neoclassical viscosity theory[12–16] overpredicted measured toroidal rotation by less than an order of magnitude in the core region of tokamak plasmas, but agreement was much poorer in the edge plasma. A more accurate summary of

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the evidence is that (1) axisymmetric neoclassical perpendicular viscosity has been estimated to greatly overpredict measured toroidal rotation in tokamaks, but that (2) axisymmetric neoclassical gyroviscosity has been calculated to overpredict measured toroidal rotation by $\leq 20\%$ in the core of tokamak plasmas, but with much poorer agreement in the plasma edge.

However, the assumption of magnetic field axisymmetry is not justified in the edge plasma, and a nonaxisymmetric neoclassical fluid theory to represent neoclassical viscous torques arising from nonaxisymmetric magnetic fields in tokamaks (e.g., Refs. 2 through 5) has recently been developed. Many of the recently developed neoclassical viscous torques due to nonaxisymmetric magnetic fields are likely to be most important in the plasma edge—just where the gyroviscous neoclassical torque due to axisymmetric magnetic fields has the greatest difficulty matching experiments.

The neoclassical torques due to both axisymmetric and nonaxisymmetric magnetic fields can be written in the same "drag frequency" form, which facilitates the representation of both in a composite neoclassical viscous torque model for fluid rotation calculations. The purpose of this paper is to describe and discuss a form for the axisymmetric neoclassical gyroviscous torque that is compatible with the drag frequency form (e.g., in Refs. 3 and 4) used for the nonaxisymmetric neoclassical viscous torques that have been developed, enabling a combination of the two which hopefully will be able to represent toroidal rotation damping in both the tokamak core and edge.

This paper is organized as follows. First, the drag frequency representation of the gyroviscous torques of neoclassical theory in plasmas confined in an axisymmetric tokamak magnetic field $\mathbf{B}_0$ are presented, and the resulting toroidal rotation equations for axisymmetric tokamaks are summarized in Sec. II. Then, in Sec. III, following Refs. 3 and 4, we assume that the nonaxisymmetric magnetic field can be represented as a sum of small perturbations $\delta\mathbf{B}_{xx}$ plus an axisymmetric field $\mathbf{B}_0$, and that the individual toroidal φ viscous torques $\delta T_{xx\varphi} = R_0 nm\nu^{xx}_{visc} V_\varphi$ resulting from each magnetic field nonaxisymmetry (denoted xx) can be added to the gyroviscous torque resulting from the axisymmetric magnetic field to obtain a total neoclassical viscous stress for the toroidal momentum balance equations which can then be solved for the toroidal rotation velocities in both the plasma core and edge. (A similar procedure could, in principle, be followed for the poloidal rotation velocity.) An example representation for $\delta T_{xx\varphi} = R_0 nm\nu^{xx}_{visc} V_\varphi$ is discussed in Sec. III, and the results are discussed in Sec. IV.

II. DRAG REPRESENTATION OF NEOCLASSICAL GYROVISCOUS TORQUE IN AXISYMMETRIC TOKAMAKS

The toroidal (and poloidal) fluid rotation velocities are determined largely by the toroidal (and poloidal) components of the coupled momentum balance equations for the plasma ion species (e.g., Ref. 17).

The major issue in developing a solution for the rotation equations is the evaluation of the flux surface average (FSA) of the toroidal components of the divergence of the viscous torque tensors $\langle R\mathbf{n}_\varphi \cdot \nabla \cdot \Pi \rangle$. For a toroidally axisymmetric tokamak, the Braginskii rate-of-strain tensor,[6] extended to toroidal flux surface geometry[7] and arbitrary collisionality,[8] $\Pi_{Brag} = \Pi_{\|}(\eta_0) + \Pi_\Omega(\eta_\Omega) + \Pi_\perp(\eta_\perp)$, where $\eta_0 \gg \eta_\Omega \gg \eta_\perp$ are viscosity coefficients associated with flows parallel to, gyrating about, and perpendicular to the magnetic field, respectively. For toroidally axisymmetric magnetic fields[7,11] $\langle R\mathbf{n}_\varphi \cdot \nabla \cdot \Pi_{\|} \rangle \equiv 0$, leaving the gyroviscous torque as the leading order neoclassical torque, which may be represented[18] as a drag term with a drag frequency $\nu_{\Omega\varphi}$:

$$\langle R\mathbf{n}\varphi \cdot \nabla \cdot \Pi_\Omega \rangle = nmR_0\nu_{\Omega\varphi}V_\varphi,$$

$$\nu_{\Omega\varphi} \equiv \frac{1}{2}\eta_\Omega \frac{r}{R_0}\left(L_n^{-1} + L_T^{-1} + L_{v\varphi}^{-1}\right) \times \left[\left(4 + \widetilde{n}_j^c\right)\widetilde{V}^s_{\varphi j} + \left(1 - \widetilde{V}^c_{\varphi j}\right)\widetilde{n}_j^s\right], \quad (1)$$

where

$$L_X^{-1} \equiv -\frac{1}{X}\frac{\partial X}{\partial r}$$

$$\eta_\Omega = {nmT}/{eB}$$

$$R_0 = \text{FSA value of the major radius}$$

$$\text{subscript } j = \text{major plasma ion species,}$$

and where $\widetilde{X}^{s,c}$ are the $\sin\theta, \cos\theta$ components in a low-order Fourier expansion $X(r,\theta) = X_0(r)[1 + X_c\cos\theta + X_s\sin\theta]$ of ion density and toroidal velocity divided by $\varepsilon \equiv r/R_0$, and the quantities without the s or c subscript refer to FSA values. The calculation of these poloidal asymmetries is discussed in Refs. 15 and 16. (There has been some confusion about gyroviscosity in the literature, and this is discussed in the Appendix.)

The smaller perpendicular viscous torque term can be represented[18] as

$$\langle R\mathbf{n}_\varphi \cdot \nabla \cdot \Pi_\perp \rangle \simeq nmR_0 \nu_{\perp\varphi} V_\varphi ,$$
$$\nu_{\perp\varphi} \equiv \frac{\eta_\perp}{nm}\left[L_{\nu\varphi}^{-1}\left(\frac{1}{r} - L_{\eta\perp}^{-1}\right) - \frac{1}{V_\varphi}\frac{\partial^2 V_\varphi}{\partial r^2}\right] , \quad (2)$$

with $\eta_\perp \simeq (\Omega_j \tau_j)\eta_\Omega \ll \eta_\Omega$, where $\Omega_j \equiv e_j B / m_j$ is the gyrofrequency and τ_j is the collision frequency. This term remains small even when extended from this original collisional form used by Braginskii to a collision-less regime form.

The toroidal component of the FSA of the inertial torque also has the form[18]:

$$\langle R\mathbf{n}_\varphi \cdot \nabla nm(\mathbf{V}\cdot\nabla)\mathbf{V}\rangle \simeq \frac{1}{2}\left[\frac{V_{rj}}{R_0}\left\{\varepsilon\left(1+\tilde{n}_j^c+\tilde{V}_{\varphi j}^c\right) - 2R_0 L_{V_{\varphi j}}^{-1}\right\} - \varepsilon\frac{V_{\theta j}}{R_0}\left\{\tilde{V}_{\varphi j}^s\left(1+\tilde{n}_j^c+\tilde{V}_{\theta j}^c\right) - \tilde{V}_{\theta j}^s\left(1+\tilde{V}_{\varphi j}^c\right) - \tilde{V}_{\varphi j}^c\tilde{n}_j^s\right\}\right] \times R_0 n_j m_j V_{\varphi j} \equiv R_0 nm\nu_{in} V_\varphi , \quad (3)$$

where ν_{in} is an inertial momentum transport frequency. (A similar term probably exists for the poloidal component of the inertial force, but we have not investigated this.)

As discussed in Sec. III, the representations of the FSA of the toroidal components of the new nonaxisymmetric nax viscosity torque tensor also have the form[3,4] $\langle R\mathbf{n}_\varphi \cdot \nabla \cdot \Pi_{nax}^{xx}\rangle \simeq nmR_0\nu_{nax\varphi}^{nax} V_\varphi$.

Using Eqs. (2) and (3), including a nonaxisymmetry contribution to be defined[3,4] and including also the charge-exchange ν_{cx} and ionization ν_{ion} momentum exchange terms, we obtain a composite momentum exchange frequency $\nu_d = (\nu_{\Omega\varphi} + \nu_{nax\varphi}) + \nu_{in} + \nu_{ion} + \nu_{cx}$ in terms of which the steady-state toroidal momentum balance equation for each ion species can be written in the form

$$n_j m_j(\nu_{jk} + \nu_{dj}) V_{\varphi j} - n_j m_j \nu_{jk} V_{\varphi k} = n_j e_j E_\varphi + e_j B_\theta \Gamma_{rj} + M_{\varphi j} \quad (4)$$

to obtain a coupled set of n equations for the FSA toroidal velocities, where n is the number of ion species/charge states, which can be solved for the toroidal rotation velocities of the main and impurity ions. The above equations are coupled to the equations for poloidal velocities and the poloidal (sin and cos) asymmetries in density, velocity, and electrostatic potential which can be reduced analytically to $3n$ coupled equations (e.g., Refs. 15 and 16).

A Lorentz-type representation has been used for the collisional friction between the main ions j and the one or more impurity ions/charge states k' (e.g., so $\nu_{jk} \equiv \sum_{k'\in k} \nu_{jk'}$ and $\nu_{jk} V_{\varphi k} \equiv \sum_{k'\in k} \nu_{jk'} V_{\varphi k'}$ are understood when more than one impurity charge state is being treated). The quantities E, B, Γ_r and M represent electric field, magnetic field, radial ion particle flux, and momentum input, respectively.

Several years ago we demonstrated[12,13] that Braginski's gyroviscous torque[6] (when extended to toroidal flux surface geometry[7] and arbitrary collisionality[8]) predicts the same order of magnitude toroidal rotation damping as observed in many tokamaks. Subsequent calculations[15,16] overpredicted measured toroidal velocities in DIII-D by less than an order of magnitude when a low-order Fourier expansion of the poloidal momentum balance and a simple circular flux surface model were used to calculate the poloidal asymmetries in Eq. (1). When the more accurate Miller model geometry[19] was used to calculate the poloidal asymmetries,[16] the gyroviscous model overpredicted measured toroidal velocities in the core of two DIII-D shots by only 10% to 20%, but there were much larger overpredictions in the edge, suggesting the need for an additional momentum dissipation mechanism [perhaps neoclassical toroidal viscosity (NTV)] in the edge. Another possible cause of the greater disagreement in the edge is the failure to date to take into account the effect of ion orbit loss,[20] which is large in the plasma edge, in the rotation calculations.

III. REPRESENTATION OF NONAXISYMMETRIC MAGNETIC FIELD EFFECTS ON TOROIDAL ROTATION CALCULATIONS FOR TOKAMAKS

The magnetic flux surfaces in real tokamaks are not axisymmetric for a variety of reasons, e.g., the discrete toroidal field coils, field errors due to manufacturing shortcomings, the intentional introduction of nonaxisymmetry by resonant magnetic perturbation coils for edge-localized mode control and other coils, magnetohydrodynamic instability production of currents in the plasma, etc. Reference 5 provides an excellent general discussion of magnetic field nonaxisymmetries and their effects upon toroidal plasma confinement, and Ref. 3 discusses the neoclassical plasma viscosity theory which represents the toroidal viscous torques associated with various nonaxisymmetric magnetic fields in the form of a drag frequency representation of the neoclassical toroidal viscous torque:

$$\langle R\mathbf{n}_\varphi \cdot \nabla \cdot \Pi_{nax}^{xx}\rangle \simeq nmR_0\nu_{nax\varphi}^{xx} V_\varphi ,$$

with the nonaxisymmetric toroidal viscous damping frequency given by

$$\nu^{xx}_{nax\varphi} = \mu^{xx}_{nax\varphi}(\delta B_{xx}/B_0)^2 \ ,$$

where $\delta B_{xx}(r,\theta)$ is the nonaxisymmetric magnetic field deviation of type xx from the symmetric magnetic field B_0 and the $\mu^{xx}_{nax\varphi}$ are tabulated as $\mu_{\|}$ in Table 1 of Ref. 3 for various types, xx, of field nonaxisymmetries. For example, the toroidal field ripples $\delta B_{rip}(r,\theta)$, which are present in all tokamaks, produce FSA neoclassical toroidal viscous torques

$$\langle R\mathbf{n}_\varphi \cdot \nabla \cdot \Pi^{rip}_{nax\varphi}\rangle \simeq nm\, R_0 \nu^{rip}_{nax\varphi} V_\varphi \ ,$$

where

$$\begin{aligned}\nu^{rip}_{nax\varphi,j} &= \mu^{rip}_{nax\varphi}(\delta B_{rip}/B_0)^2 \\ &= (B_\theta/B_0)^2(\delta B_{rip}/B_0)^{3/2}\left(V_{thj}/R_0\right)^2/\nu_{jj} \ , \qquad (5)\end{aligned}$$

and where V_{thj} is the thermal velocity and ν_{jj} is the self-collision frequency.

It is not our purpose to summarize the many different types of magnetic field asymmetries that have been worked out and reviewed in Ref. 3 and, in greater detail, in Ref. 4. Rather, we are making the point that these toroidal magnetic field nonaxisymmetries can be represented as additive FSA toroidal viscous torques

$$\langle R\mathbf{n}_\varphi \cdot \nabla \cdot \Pi^{xx}_{nax}\rangle \simeq nmR_0\nu^{xx}_{nax\varphi}V_\varphi$$

in the toroidal momentum balance [i.e., $\nu^{xx}_{nax\varphi} = \mu^{xx}_{nax\varphi}(\delta B_{xx}/B_0)^2$ can be included in the ν_{dj} term just above Eq. (4)].

It stands to reason that a similar representation of the FSA of the neoclassical poloidal viscous force

$$\langle \mathbf{n}_\theta \cdot \nabla \cdot \Pi^{xx}_{nax}\rangle = nm\nu^{xx}_{nax\theta}V_\theta \ ,$$

which arises in the poloidal rotation equations, can be developed along the same lines as discussed in Refs. 3 and 4. (This has not been done to our knowledge.)

IV. DISCUSSION

The rationale for the composite axisymmetric and nonaxisymmetric neoclassical viscous torque model put forward in this paper for plasma rotation calculations is based on three observations: (1) the old neoclassical gyroviscous torque model does reasonably well in predicting measured toroidal rotation velocities within the core plasma but not in the plasma edge[15,16]; (2) the new neoclassical viscous torque effects due to nonaxisymmetric magnetic fields[3,4] (e.g., toroidal field ripples, control coil fields, field errors, fields due instabilities) should be most important in the edge plasma where the old neoclassical gyroviscous torques due to axisymmetric magnetic fields provide viscous damping that disagrees the most with experimental rotation[15,16]; and (3) the newly developed nonaxisymmetric neoclassical viscosity theory[2–4] uses the same drag frequency representation of the toroidal viscous torque

$$\langle R\mathbf{n}_\varphi \cdot \nabla \cdot \Pi^{xx}_{nax}\rangle \simeq nmR_0\nu^{xx}_{nax\varphi}V_\varphi \ ,$$

arising from different three-dimensional magnetic field nonaxisymmetries xx as is used to represent the axisymmetric neoclassical toroidal gyroviscous torque in the old axisymmetric neoclassical gyroviscosity theory for magnetically axisymmetric tokamaks.[15,16] Thus, a combination of the nonaxisymmetric viscous effects,[3,4] such as would be most important in the edge, with the axisymmetric gyrovisous effects[15,16] which should be most important in the core, would seem to be imperative in order to properly model experiments which depend on core and edge effects.

Both the old axisymmetric gyroviscous model and the new nonaxisymmetric viscosity models have met with enough success in predicting rotation features observed in the core and edge, respectively, of experiments to encourage their combination into a composite toroidal viscosity such as is outlined in this paper. However, as in all theories, a number of ad hoc mathematical approximations and physical arguments have been made (most importantly that[11] $B_r = 0, \partial/\partial\varphi = 0$) to arrive at this drag frequency representation of the viscous stress, and the rigorous representation of the viscous torque for magnetically nonaxisymmetric tokamaks could be considerably more complex.[11] Nevertheless, we believe that it is important to merge the magnetically nonaxisymmetric neoclassical theory of Refs. 3 and 4 with the magnetically axisymmetric neoclassical theory of Refs. 15 and 16 and to test the combined theory against experimental rotation measurements.

APPENDIX

GYROVISCOSITY

There has been some confusion in the literature about the retention and importance of gyroviscosity in the representation of the viscous torque in neoclassical rotation theory. Braginskii[6] developed his fluid rotation theory, which included gyroviscosity associated with gyromotion about field lines,[17] from a moments approximation to drift kinetic theory under the implicit assumption of strong fluid rotation velocity $|\mathbf{V}| \approx V_{th}$. Shortly thereafter, Mikhailovskii and Tsypin[21] (M-T) introduced a similar fluid rotation theory, but developed from a multimoment approximation of the Boltzmann transport equation. The M-T approach also took into account the contribution to the viscosity contribution of the heat flux as well as the momentum flux, which is significant when $|\mathbf{V}| \ll V_{th}$ but not in the strong rotation case when $|\mathbf{V}| \approx V_{th}$, which is the case of interest in present tokamaks and in the present paper. Claassen et al.[22] demonstrated that the M-T–type corrections strongly affected the Braginskii predictions for ohmic plasmas with $|\mathbf{V}| \ll V_{th}$, but observed that "as toroidal velocities are often observed to be of the order of the sound speed, the scaling relations underlying the theory (of their paper) may have to be modified." Catto and Simakov[23] (C-S) followed this line of investigation and confirmed that the magnitude of the gyroviscosity depended on the magnitude of the up-down asymmetries [the sine components with superscript s in Eq. (1)], which they estimated to be small but acknowledged "up-down asymmetry can increase the momentum relaxation rate, perhaps making it competitive with anomalous relaxation" by which they meant the measured rotation relaxation. Thus, it would appear that Braginskii focused on the strong rotation $|\mathbf{V}| \approx V_{th}$ situation, while M-T, Classen et al., and C-S were focused on the weak rotation situation $|\mathbf{V}| \ll V_{th}$. This distinction is not clear in the discussion of the matter in the literature nor in the minds of many researchers in the field.

In summary, those authors who evaluated gyroviscous momentum damping in the weak rotation ordering $|\mathbf{V}| \ll V_{th}$ (Refs. 21, 22, and 23) found momentum damping consistent with that seen in ohmic plasmas without external momentum sources (in which they were interested), while those authors who evaluated gyroviscous momentum damping in the strong rotation ordering $|\mathbf{V}| \approx V_{th}$ (Refs. 12 through 16) found a momentum damping consistent with that seen in strongly rotating plasmas with external moment sources (in which they were interested).

A second source of confusion was the absence of gyroviscosity in two contemporary derivations[9,10] of neoclassical rotation theory which actually found gyroviscosity but neglected it relative to the perpendicular viscosity based on a faulty gyroradius ordering argument that neglected terms $O(\delta\eta_\Omega)$ relative to terms $O(\eta_\perp)$ as being of higher order in the gyroradius ordering parameter $\delta \equiv r_{larmor}/r \simeq 10^{-2}$, even though $\delta\eta_\Omega \gg \eta_\perp$ because $\eta_\Omega \simeq 10^4\eta_\perp$. This "gyroradius ordering argument" neglect of gyroviscosity is found even in more recent derivations of neoclassical rotation theory, e.g., Ref. 24. We found[12–16] by direct numerical evaluation that Eq. (1) is the FSA of the largest neoclassical torque for a magnetically axisymmetric tokamak and that the resulting gyroviscous momentum confinement times are comparable with measured momentum damping rates in the core of a large number of strongly rotating tokamak discharges,[12–16] but not in the plasma edge.

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