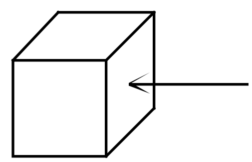
ECE 6605
Information Theory
Exam #1 Fall 2001

Name:

Directions: Answer all questions, all parts. Point values for each part are given in parentheses. Exam duration is 2 hours. You may use any result given in class **without proof** unless specifically told to do so. None of the problems require a great number of calculations or equations. If you find yourself writing a long answer, you are probably doing the problem the long (or wrong) way.

1. (10 points) Consider one roll of a die (a six-sided cube labeled with the numbers 1-6). Let X be a random variable indicating the number facing upwards. Let Y be the number facing sideways toward the North and Z be the number facing South



1) Find H(X), H(Y)
2) Find I(X;Y)
3) Find H(Y,Z) and I(Y;Z)

2) (10 points) **Uncertainty in being served.** You arrive at the front of a line, and a coin is flipped to determine if you are to be served or not. Assume p(head)=*p*. When a 'head' appears, you will be served, when a 'tail' appears you are not served and need to wait until the next coin flip.

Let X denote the number of flips required to be served, find H(X) and evaluate for p=1/2.

Hint: recall $\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$; $\sum_{n=1}^{\infty} nr^n = \frac{r}{(1-r)^2}$

3) (10 points) Let $X_{-n},\ldots, X_0,\ldots, X_n$ be a stationary (not necessarily Markov) random process.

   a) Show $H(X_1) \geq \frac{H(X_1,X_2)}{2}$. Under what conditions is equality achieved?
   b) Show $\frac{1}{3}H(X_1, X_2, X_3,) \geq H(X_3 \mid X_2, X_1)$
   c) Does $H(X_n| X_0) = H(X_{-n}| X_0)$?
   d) Is $H(X_n| X_1,\ldots, X_{n-1},X_{n+1})$ nonincreasing in n?

4) (10 points) In general, do the codewords produced by the Huffman algorithm satisfy the Kraft Inequality? If so, argue why. If not, give a counterexample.