ECE 6605
Information Theory
Exam #2 Fall 99

Name:

Directions:

Answer all questions, all parts.
4 problems total.
Each problem is worth 15 points.
Exam duration is 2 hours.
Calculators allowed.

You may use any result given in class **without proof** unless specifically told to do so.

(15 points)

1) Let a source emit IID random variables X from an alphabet A={1,2,3,4,5} with respective probabilities 1/8, 1/2, 1/4, 1/16, 1/16. Assume that you must encode only one source output at a time.

a) Design the Shannon-Fano-Elias code for this source and find its average length.
b) How does this code compare to the source entropy?
c) Can one design a better code? If so design it.

(15 points)
2) Prove that the Shannon Fano Elias code is prefix free. Assume that the source is discrete, has a known probability mass function p(x) and that the source letters are IID. (Give as precise and concise a proof as possible. Wordy, nonmathematical arguments are likely to receive less than full credit. You need not reprove anything already proved in class).

(15 points)

3) In this problem you will find the capacity of a channel (the reverse erasure channel) with probability transition matrix P.

$$P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$$

Assume that X is the input to the channel and Y is the output. Assume that $p(X=0) = p_0$,

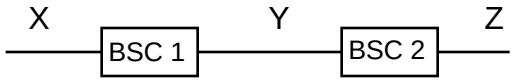
$p(X=1) = p_1$ and $p(X=2) = p_2$.

Compute the capacity of the channel. You may do it however you like. I suggest you follow the approach in a)-d) below.

a) Show that $I(X;Y) = H(Y) - p_1$ (3 points)
b) Find $P(Y=0)$ and $P(Y=1)$ in terms of p_0 , p_1 and p_2 (3 points)
c) Use facts about maximizing $H(Y)$ to find the input distribution p_0 , p_1 and p_2 that maximizes a). (5 points)
d) Compute the capacity and give the maximizing input distribution. (4 points)

15 points

4) In this problem you will ***set up, but not necessarily solve*** the problem of computing the capacity for the following channel (input X, output Z):



Assume that channel 1 is a binary symmetric channel (BSC) with crossover probability p and channel 2 is a BSC with crossover probability q. The channels are cascaded so that the output of BSC 1 is fed directly to BSC 2. Let

$\pi_0 = P(X=0)$ $\varphi_0 = P(Y=0)$
$1-\pi_0 = P(X=1)$ $1-\varphi_0 = P(Y=1)$

Compute the capacity of this channel. Clearly define the problem and give the expression(s) for solving for the capacity of the cascaded channels. If the solution to this problem is easy then state the result. If the solution is not so easy, then describe the optimization to be performed in detail. You need not perform the actual final optimization. Specifically, write the quantities involved in the optimization using as few parameters (such as π_0) as possible and describe the procedure for determining capacity. Solutions that are not well organized will not receive much credit. Be specific.