Math 2551 Final Review Fall 2022

1. Find $T, N$, and the curvature $\kappa$ for $r(t) = 3\sin(t)i + 3\cos(t)j + 4tk$.

2. Write acceleration in terms of its tangential and normal components for $r(t) = (t + 1, 2t, t^2)$ at $t = 1$.

3. Find the limit: $\lim_{(x,y)\to(0,0)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2}$.

4. Find all second-order partial derivatives for $w = x\sin(x^2y)$.

5. Evaluate $\frac{dw}{dt}$ for $w = 2ye^x - \ln(z)$ if $x(t) = \ln(t^2 + 1), y = \arctan(t), z = e^t$ at $t = 1$.

6. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the surface $z^3 - xy + yz + y^3 - 2 = 0$ at the point $(1, 1, 1)$.

7. Find the derivative of the function $f(x, y) = 2xy - 3y^2$ in the direction $4i + 3j$ at the point $(5, 5)$.

8. Find the tangent plane and normal line at $(2, 0, 2)$ of the surface $2z - x^2 = 0$.

9. Find all local maxima, local minima, and saddle points for $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.

10. Find the absolute maximum and minimum of $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular region bounded by the lines $x = 0, y = 2$, and $y = 2x$.

11. Find the maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$.

12. Write an iterated integral using (a) vertical cross-sections and (b) horizontal cross-sections whose region of integration is the region bounded by $y = \sqrt{x}, y = 0, x = 9$.

13. Sketch the region of integration and evaluate the integral $\int_{0}^{1} \int_{0}^{x^2} 3y^3e^{xy} \, dx \, dy$.

14. Change to polar coordinates and evaluate the integral $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \, dy \, dx$.

15. Integrate the function $f(x, y) = 3 - 4x$ over the region below $z = 4 - xy$ and above the rectangle $0 \leq x \leq 2, 0 \leq y \leq 1$ in the $xy$-plane.

16. Find the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cylinder $x^2 + y^2 = 1$.

17. Use the change of coordinates $x(u, v) = \frac{u}{v}, y(u, v) = uv$ to evaluate the integral $\iint_{R} \left( \sqrt{\frac{y}{x}} + \sqrt{\frac{x}{y}} \right) \, dA$, where $R$ is the region in the first quadrant bounded by $xy = 1, xy = 0, y = x$, and $y = 4x$.

18. Evaluate the line integral $\int_{C} (xy + y + z) \, ds$ along the curve $r(t) = 2ti + tj + (2 - 2)k$ for $0 \leq t \leq 1$.

19. Find the flow of the field $\mathbf{F} = (-4xy, 8y, 2)$ along the curve $r(t) = (t, t^2, 1), 0 \leq t \leq 2$.

20. Find the counterclockwise circulation and the outward flux of the field $\mathbf{F} = xi + yj$ around/through the unit circle centered at the origin.

21. Find the potential function for $\mathbf{F} = ey^{2z}(i + xj + 2zk)$.

22. Use Green’s Theorem to find counterclockwise circulation and outward flux of the field $\mathbf{F} = (y^2 - x^2, x^2 + y^2)$ for the curve $C$ enclosing the region bounded by $y = 0, x = 3$, and $y = x$.

23. Use a parameterization to write a double integral for the area of the surface $S$ which is the portion of the cone $z = 2\sqrt{x^2 + y^2}$ between the planes $z = 2$ and $z = 6$. 
24. Evaluate \( \iint_S 2y \, d\sigma \) over the surface \( S \) which is the part of the cylinder \( y^2 + z^2 = 4 \) between \( x = 0 \) and \( x = 3 - z \).

25. Let \( S \) be the surface that consists of the part of the paraboloid \( z = 4 - x^2 - y^2 \) above the \( xy \)-plane and below the cone \( z = 3\sqrt{x^2 + y^2} \).
   
   (a) Sketch \( S \).
   
   (b) Find a parameterization of \( S \).
   
   (c) Calculate the area of \( S \).

26. Let \( S \) be the surface consisting of the top half \( z \geq 0 \) of the sphere \( x^2 + y^2 + z^2 = 9 \), together with the disk \( x^2 + y^2 \leq 9, \ z = 0 \), its base in the \( xy \)-plane. Use the divergence theorem to evaluate

\[
\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma,
\]

where \( \mathbf{F}(x, y, z) = 3xy^2 \mathbf{i} + 3x^2y \mathbf{j} + z^3 \mathbf{k} \).

27. Let \( S \) be the part of the surface \( z = 4x^2 + y^2 - 4 \) beneath the plane \( z = 5 \). Let \( C \) be the bounding curve of \( S \) in the plane \( z = 5 \), traversed counterclockwise and suppose \( S \) is oriented accordingly (normals towards the \( z \)-axis). Let \( \mathbf{F}(x, y, z) = (2y, 4x, e^z) \). Use Stokes’ Theorem to evaluate the curl integral

\[
\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, d\sigma.
\]

28. Let \( S \) be the surface of the cylinder defined by \( y^2 + z^2 = 4 \) between the planes \( x = -1 \) and \( x = 3 \). Let \( \mathbf{F}(x, y, z) = e^y \mathbf{i} + e^x \mathbf{j} + e^z \mathbf{k} \).

   (a) Sketch \( S \).
   
   (b) Find a parameterization of \( S \).
   
   (c) Let \( \mathbf{n} \) be an outward pointing unit normal for \( S \). Evaluate

\[
\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma
\]

by direct calculation (do not use the Divergence Theorem).
Math 2551 Final Review Fall 2022 ANSWERS

1. \( \mathbf{T} = \langle 3 \cos(t)/5, -3 \sin(t)/5, 4/5 \rangle, \mathbf{N} = -\sin(t)i - \cos(t)j, \kappa = \frac{3}{25} \)

2. \( a(1) = \frac{4}{3} \mathbf{T} + \frac{2\sqrt{5}}{3} \mathbf{N} \)

3. 5/2

4. \( w_{xx} = 6xy \cos(x^2y) - 4x^3y^2 \sin(x^2y), w_{yy} = -x^5 \sin(x^2y), w_{xy} = w_{yx} = 3x^2 \cos(x^2y) - 2x^4y \sin(x^2y) \)

5. \( \frac{dw}{dt} = 4t \arctan(t) + 1 \bigg|_{t=1} = \pi + 1 \)

6. \( \frac{\partial z}{\partial x} = \frac{1}{4}, \frac{\partial z}{\partial y} = -\frac{3}{4} \)

7. -4

8. \( 2x - z - 2 = 0, \mathbf{r}(t) = \langle 2, 0, 2 \rangle + t(-4, 0, 2) \)

9. \( f(-3, 3) = -5 \) is a minimum

10. \( f(0, 0) = 1 \) is the abs. max, \( f(1, 2) = -5 \) is the abs. min

11. 39

12. Regions of integration are a) \( 0 \leq x \leq 9, 0 \leq y \leq \sqrt{x} \), b) \( 0 \leq y \leq 3, y^2 \leq x \leq 9 \).

13. \( e - 2 \)

14. \( \frac{\pi}{2} \)

15. \( \int_0^2 \int_0^1 f^{4-x^2y}(3 - 4x) \) \( dz \) \( dy \) \( dx \) = \( -\frac{17}{3} \)

16. \( \frac{4\pi}{3} \)

17. \( \int_1^2 \int_1^3 \frac{(u+v)2u}{v} \) \( du \) \( dv \) = \( 8 + \frac{52}{3} \ln(2) \)

18. 13/2

19. 48

20. Circulation = 0, Flux = \( 2\pi \)

21. \( f(x, y, z) = x e^{y^2 z} + C \)

22. Circulation = 9, Flux = -9

23. \( \int_0^{2\pi} \int_1^3 r \sqrt{5} \) \( dr \) \( d\theta \)

24. \( \mathbf{r}(x, \theta) = \langle x, 2 \sin(\theta), 2 \cos(\theta) \rangle, \)

\[ \iint_R f(\mathbf{r}(x, \theta))|\mathbf{r}_x \times \mathbf{r}_\theta| \) \( dx \) \( d\theta \) = \( 0 \)

25. (b) \( \mathbf{r}(r, \theta) = \langle r \cos(\theta), r \sin(\theta), 4 - r^2 \rangle, 1 \leq r \leq 2, 0 \leq \theta \leq 2\pi \) (c) \( \frac{\pi}{6}(17^{3/2} - 5^{3/2}) \)

26. \( \frac{1458}{5}\pi \)

27. -27\pi

28. (b) \( \mathbf{r}(x, \theta) = \langle x, 2 \cos(\theta), 2 \sin(\theta) \rangle, -1 \leq x \leq 3, 0 \leq \theta \leq 2\pi \), (c) \( 8\pi(e^3 - e^{-1}) \)