# Statistical radar imaging of diffuse and specular targets using an expectation-maximization algorithm

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# Models for Received Radar Data

- Received data $\mathbf{r} = \mathbf{\Gamma}^H \mathbf{c} + \mathbf{w}$, where
  — $\mathbf{c}$ = complex target reflectance
  — $\mathbf{\Gamma}$ = linear observation mechanism
  — $\mathbf{w} \sim CN(0, N_0 I)$

- Examples of $\mathbf{\Gamma}$:
  — Simple range profiling: Samples of transmitted waveform (representing convolution)
  — Delay-doppler imaging: Transmitted waveform multiplied by sinusoids
  — Tomographic imaging: Transmitted waveform with partial Radon transform

- In target detection, $\mathbf{c}$ is often treated as random vector. In target imaging, $\mathbf{c}$ is more often treated as an unknown deterministic parameter. Here we explore random models for $\mathbf{c}$ for imaging purposes.

# Models for Target Reflectance

From Shapiro:

- Diffuse/speckle: $\mathbf{c}_d \sim CN(0, \mathbf{\Sigma})$
  — $\mathbf{\Sigma}$ is a diagonal covariance
  — $\mathbf{s} = diag(\mathbf{\Sigma})$ called the *scattering function*
  — Goal: estimate $\mathbf{s}$

- Specular/glint: $\mathbf{c}_s = \mathbf{b} \times \exp[j\theta]$
  — $\theta \sim$ i.i.d. uniform over $[0, 2\pi)$
  — $\mathbf{b}$ is a deterministic *glint reflection coefficient*
  — $\times$ is elementwise multiplication
  — Goal: estimate $\mathbf{b}$

- Mixed model: $\mathbf{c} = \mathbf{c}_d + \mathbf{c}_s$
  — Goal: estimate $\mathbf{s}$ and $\mathbf{b}$
  — Tends to be overparameterized

# EM Algorithm for Diffuse Imaging

- For diffuse imaging, $\mathbf{r} \sim CN(0, \mathbf{\Gamma}^H \mathbf{\Sigma} \mathbf{\Gamma} + N_0 \mathbf{I})$, yielding a structured covariance estimation problem

- No obvious closed-form formula for ML estimate

- Iterative EM algorithm by Snyder-O'Sullivan-Miller:

  $$\sigma_i^{new} = \sigma_i^{old} - (\sigma_i^{old})^2 [\mathbf{\Gamma} \mathbf{K}^{-1} \mathbf{\Gamma}^H - \mathbf{\Gamma} \mathbf{K}^{-1} \mathbf{r} \mathbf{r}^H \mathbf{K}^{-1} \mathbf{\Gamma}^H]_{ii},$$

  where

  $$\mathbf{K} = \mathbf{\Gamma}^H \mathbf{\Sigma}^{old} \mathbf{\Gamma} + N_0 \mathbf{I}$$

- Enjoys usual properties of EM algorithms
  —— Likelihood increases at each iteration
  —— Iterates guaranteed to be nonnegative

# What About Specular Imaging?

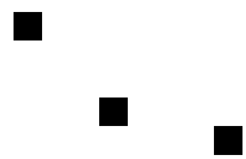
- Specular $\mathbf{r}$ is vastly more complicated
  —— Not aware of a closed form for the density on $\mathbf{r}$

- If the columns of $\mathbf{\Gamma}$ have a sufficient non-zero entries:
  —— $\mathbf{r}$ consists of sums of indep. 0-mean random variables
  —— By CLT, *marginals* on $\mathbf{r}$ approx. 0-mean Gaussian
  —— $\mathbf{r}$ "almost Gaussian" in the spirit of Mallows

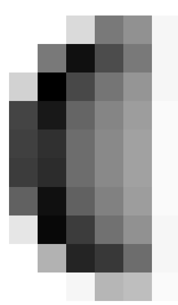
- Motivates trying the diffuse EM algorithm on the specular case

# Phantoms for Simulations

- Three point scatterers:

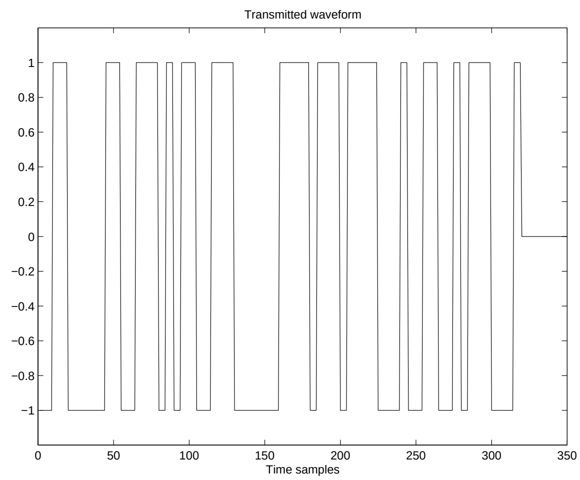


- Rotating sphere:



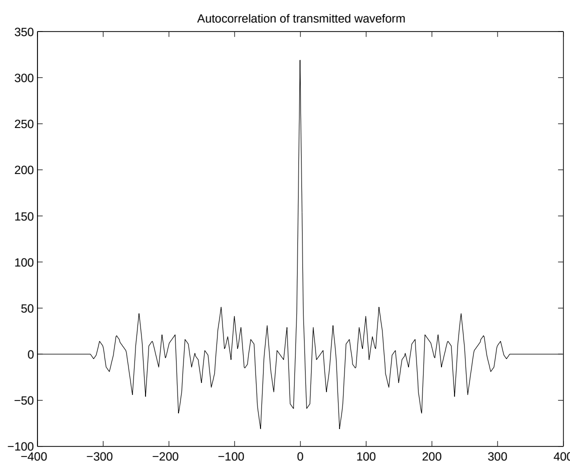
# Transmitted Waveform

- Specular realization:



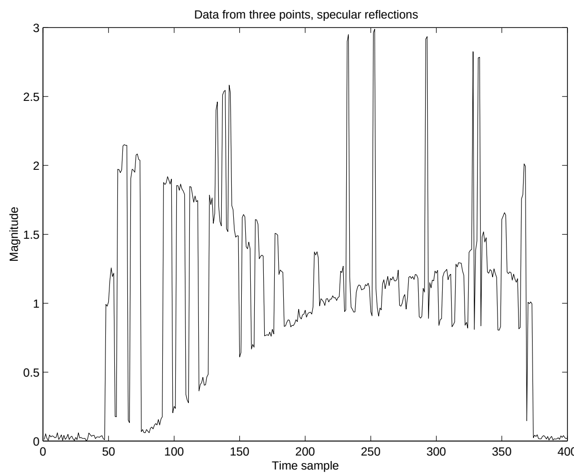

x

- Autocorrelation:



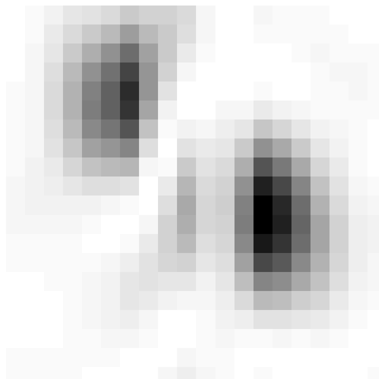
# Data from Three Point Scatterer

- Data from three point scatterer:

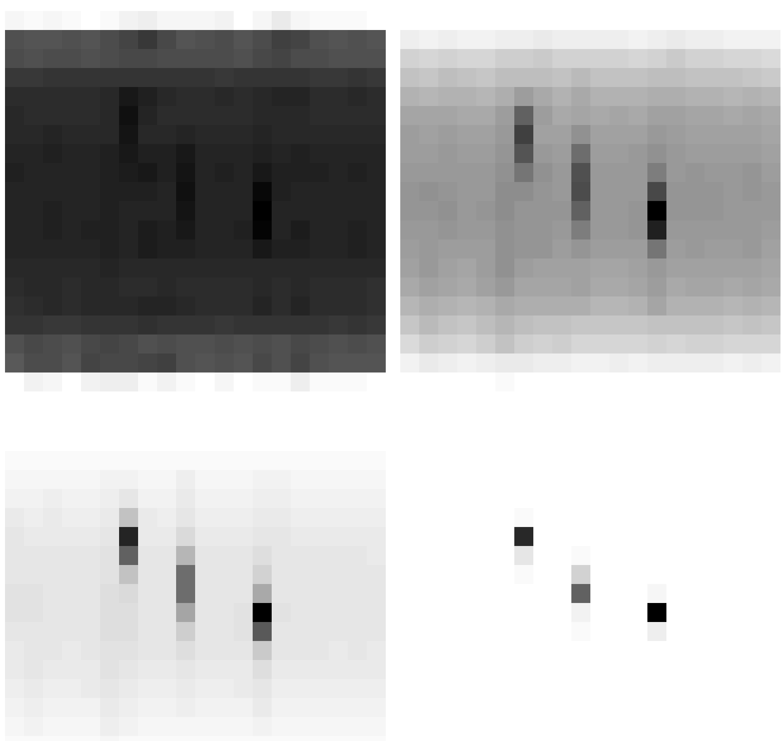


# Results for Three Point Scatterer

- Matched filter output:

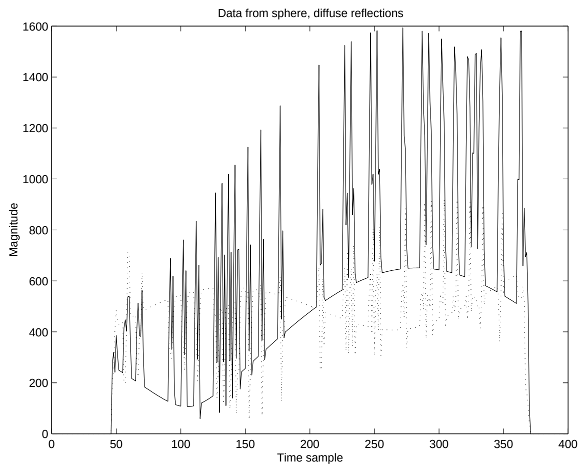


- At 1, 5, 10 and 20 EM iterations:

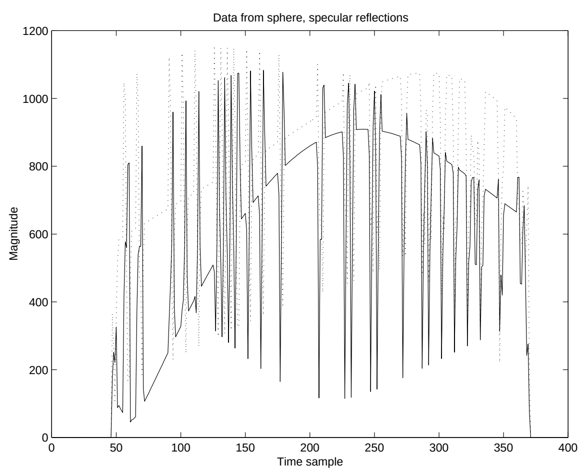


# Data for the Sphere

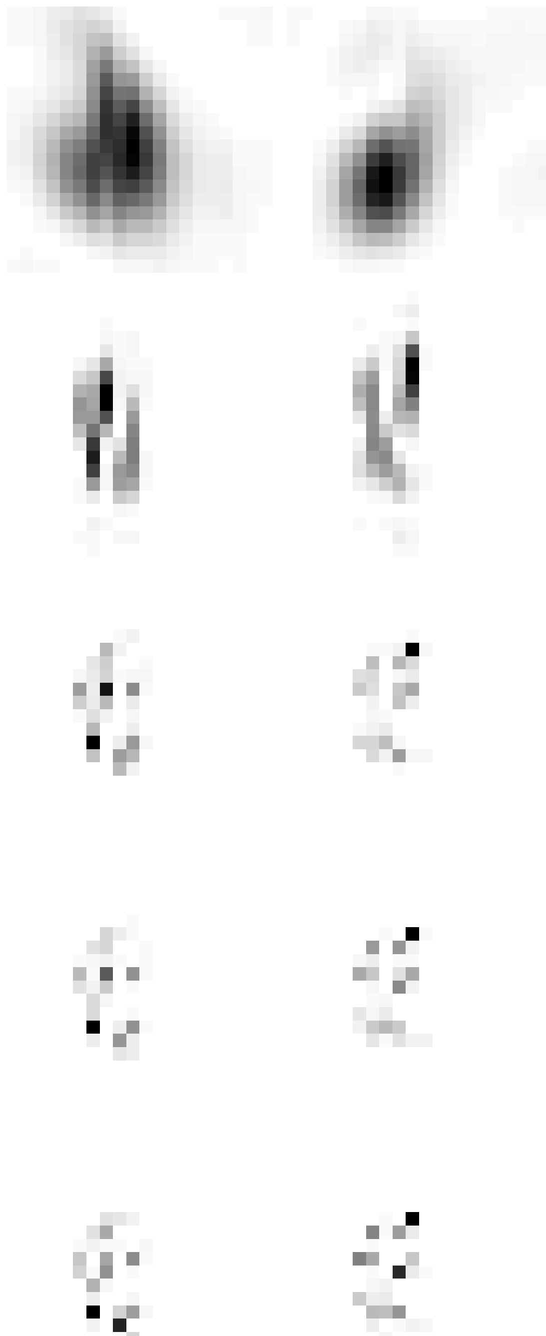
- Two diffuse realizations:



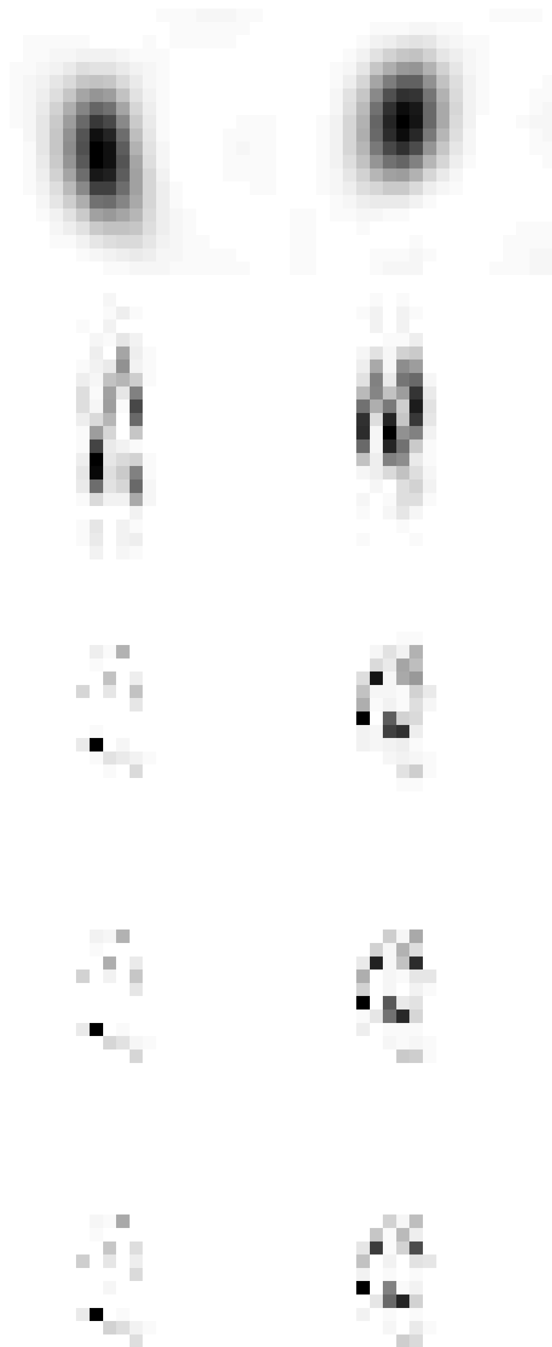

- Two specular realizations:



# Results for Diffuse Sphere



# Results for Specular Sphere



# Regularization Techniques

- Grenander's Method of Sieves
  — B-spline basis for $f$ (Moulin 92)
  — Wavelet basis for $\log(f)$ (Moulin 93)

- Penalized likelihood methods
  — Subtract penalty from likelihood

$$P(r|f) = L(r|f) - \alpha\Phi(f)$$

  — Good's roughness penalty:

$$\Phi_G(f) = \int [\frac{d}{dx}\sqrt{f(x)}]^2 dx$$

  — Good's is equivalent to O'Sullivan's I-divergence penalty
  — Silverman's roughness penalty:

$$\Phi_S(f) = \int [\frac{d}{dx}\log f(x)]^2 dx$$

  — Simple modification of EM algorithm produces penalized likelihood estimates; amounts to nonlinearly smoothing the result of the maximization step at each iteration
  — Admits a Bayesian interpretation

# Expectation-Maximization-Smoothing Algorithms

- Suggested by Silverman for emission tomography
- Try different kinds of *ad hoc* smoothing steps
- A particular choice of smoothing may not correspond to any particular penalized likelihood method
- Good performance shown in emission tomography
- However, it's hard to prove whether such algorithms converge, and even harder to show what they converge to

# Directions for Future Work

- Implementation and comparison of various regularization techniques

- Current execution time of MATLAB implementation on Sun Enterprise 3500:

| Image size | Total time | Time for inverse |
|---|---|---|
| 20 x 20 | 15 seconds | 6 seconds |
| 32 x 32 | 8 minutes | 4 minutes |
| 40 x 40 | 32 minutes | 13 minutes |

- Improve computation time
  —— Must find fast way of doing matrix inverse (or avoiding an explicit inverse altogether)
  —— Speed up multiplies by $\mathbf{\Gamma}$
  —— Fast EM Variants (SAGE, etc.)

- Statistical formulation provides criteria for radar waveform design (via Cramer-Rao bounds, etc.)

- Other applications
  —— Radar astronomy
  —— Direction finding?

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