Optimal Antenna Spacings in Interferometric SAR

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ABSTRACT

In practice, a synthetic aperture radar (SAR) reconstructs the complex reflectivity function of a scene, modulated by phase terms that capture 3-D imaging geometry. INSAR (interferometric SAR) attempts to obtain the geometric information by interfering two images (from two antennas) to cancel the same scene reflectivity and recover the scene topography transduced by the image-phase data. This approach, however, leads to a phase-unwrapping problem, which causes ambiguities in estimates of elevation. The phase-unwrapping problem can be solved in a pointwise fashion by using more than two antennas. This approach can effectively prevent error propagation which occurs in traditional phase-unwrapping algorithms. In this work, we study the optimal antenna spacings for pointwise terrain height estimation. In particular, we start from the maximum likelihood estimates of the phase using neighborhood pixels collected by any pair of antennas. The phase estimation noise is approximated as Gaussian with variance prescribed by the Cramer-Rao lower bound on the phase estimate. The ambiguous terrain height derived from any pair of antennas is modeled by a periodic waveform with each period having an approximately Gaussian shape. For multiple pairs of antennas, the corresponding functions describing the ambiguous elevation have different periods, which acts to help resolve the ambiguity. We derive and analyze the ML estimate of elevation at each scene point using multiple pairs of antennas. For the three-antenna case, by analyzing the tradeoff between cycle errors and measurement errors, a closed-form formula approximating the mean squared error (MSE) of the estimated terrain height is derived as a function of antenna spacing. By minimizing the MSE, we determine the optimal antenna spacing. The algorithm is tested with simulated data.

Keywords: SAR, INSAR, IFSAR, interferometry, phase unwrapping

1. INTRODUCTION

Interferometric processing has been widely used to derive terrain heights in 3-D radar mapping of land surfaces.[1] Traditionally, the data collection configuration uses two separated antennas to form two complex SAR images. The terrain height at a specified location is a linear function of the unwrapped phase difference of the pair of complex images. Since only the principle values of phase differences are provided by the complex image data, a phase unwrapping algorithm is required to calculate terrain heights. Numerous algorithms have been proposed for 2-D phase unwrapping. Many of them are based on the least-squares method,[2−5] under the assumption that terrain heights change slowly around neighboring pixels or that pixels along some path do not cross a discontinuous region. The least-squares approach to 2-D phase unwrapping can be implemented efficiently using the FFT,[3] however, it is easy to pick the wrong path when there exists a discontinuous gap in the terrain surface, and errors are propagated wherever the path leads. Another solution to the phase unwrapping problem is to use multiple baselines, instead of one, by inserting more antennas into the data collection apparatus. In this case, the terrain heights are usually estimated in a pointwise fashion. This prevents error propagation since the estimate in one location is independent of others. The idea of using three phase centers to resolve phase ambiguities was proposed by Jakowatz et al.[6] In their approach, a large baseline provides an elevation estimate without any phase wrapping but with a large measurement error, and a small baseline is used to accurately estimate the unwrapped phase position within the proper cycle number predicted by the estimate from the large baseline. This scheme can often solve the phase ambiguity problem and provide good quality elevation estimates. The exact maximum likelihood solution for unwrapped phase from multiple baseline data was provided by Lombardini,[7,8] which is an extension to the discussion in Jakowatz[1] for the case of one baseline. However, this solution requires a large computational search, and there is no explicit relation between the estimated phases and measurement data. Another approach used the Chinese Remainder Theorem for the case with integer periods,[9,10] but this scheme does not work for noisy data and non-integer periods.

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In our work, we study how to determine the optimal lengths of the large and small baselines to produce the best terrain height estimates in the multiple baseline scenario. We use an idea similar to that of Jakowatz,[6] that is, for multiple baselines, each pair of baselines provides a terrain height estimate, with the estimator variance determined by the Cramer-Rao lower bound. First we set up a proper model for the terrain height estimation, and find the maximum likelihood solution for terrain height under the model assumption. We then derive the mean-squared error (MSE) of our estimate for three phase centers, and find the best antenna spacing by minimizing the MSE. The antenna spacings depend only on the maximum terrain height and on the signal-to-noise ratio (SNR) of the system.

2. MODEL DESCRIPTION

In the standard interferometric SAR scenario shown in Fig 1, the terrain height at a fixed location is proportional to the unwrapped phase difference at that location, whose principle value θ_1 is estimated from a pair of complex images reconstructed from two antennas A and B[1]:

$$h = a_1(\theta_1 + 2\pi k_1) \tag{1}$$

$$\text{with} \quad a_1 = \frac{\lambda \cos\psi}{4\pi\Delta\psi} \tag{2}$$

where h, θ_1 and k_1 are the actual terrain height, wrapped phase and the unknown integer that unwraps the phase respectively. We use a subscript 1 in all quantities to indicate that we are, for now, considering a single baseline A-B. a_1 will be referred to as the baseline scale factor (BSF) in this paper.

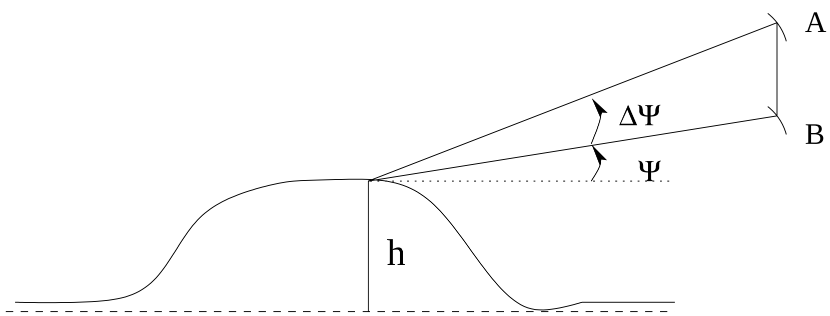

Figure 1. Configuration of Interferometric SAR with two phase centers

In practice, we must estimate θ_1. Let

$$\hat{\theta}_1 = \theta_1 + n_1, \tag{3}$$

where $\hat{\theta}_1$ is the estimate of θ_1 and n_1 is the estimation error in $\hat{\theta}_1$. In general, $\hat{\theta}_1$ is not estimated as a pointwise phase difference between images reconstructed by antennas A and B. Instead, to reduce phase noise, $\hat{\theta}_1$ is taken to be the maximum likelihood (ML) estimate based on local neighborhood pixels within the complex SAR images, i.e.

$$\hat{\theta}_1 = \angle \sum_{(i,j)\in\aleph} a^*_{i,j} b_{i,j} \tag{4}$$

Since all ML estimates are asymptotically Gaussian, the estimator noise n_1 can be approximately modeled as Gaussian. We shall assume a variance prescribed by the Cramer-Rao lower bound[1]:

$$\sigma^2 = \frac{1}{N \cdot SNR} \tag{5}$$

where, N is the number of neighborhood pixels in Eq. 4 and SNR is the clutter-to-receiver noise ratio of the SAR images. The principle value of a phase difference is the original phase difference wrapped onto a 2π period, so the distribution of $\hat{\theta}_1$ should be Gaussian-like with tails wrapped into the $[-\pi + \theta_1, \pi + \theta_1)$ interval, which can be approximated as a Gaussian without the tails if the standard deviation is small compared to π.

Denoting $\hat{h}_1 = a_1\hat{\theta}_1$, we have

$$h = a_1(\hat{\theta}_1 - n_1 + 2\pi k_1) = \hat{h}_1 + 2\pi a_1 k_1 - a_1 n_1 \tag{6}$$

In our discussion, the conditional probability density function (pdf) of the ambiguous terrain height h given the estimate $\hat{h}_1$ is modeled as a periodic waveform extending across some $[h_{min}, h_{max}]$ with each period having an approximately Gaussian shape, due to the unknown integer k_1. A smaller baseline (smaller $\Delta\psi$) results in a larger linear term a_1, which amplifies the estimation noise n_1 but also produces less ambiguity in h. For a_1 sufficiently large such that

$$h_{max} \leq 2\pi a_1, \tag{7}$$

there is no phase ambiguity, where, h_{max} is the maximum terrain height.

Given more than two phase centers, each pair of antennas provides such a periodic waveform for the conditional pdf of the ambiguous height h. For the case of three antennas labelled A, B and C, we assume that baseline between A and C is longer than the baseline between A and B. The number of independent measurements is two, provided by A-B and A-C. There might exist some phase information in the B-C pair, since the phase estimates in Eq. 4 were not calculated in a pointwise fashion, but any such information is likely to be exceedingly small and will not be considered. The actual terrain height h can be written in forms of two baselines:

$$h = a_1(\hat{\theta}_1 - n_1 + 2\pi k_1) = a_1(\hat{\theta}_1 + 2\pi k_1) - a_1 n_1 \tag{8}$$

$$h = a_2(\hat{\theta}_2 - n_2 + 2\pi k_2) = a_2(\hat{\theta}_2 + 2\pi k_2) - a_2 n_2 \tag{9}$$

where, a_1 is the large BSF corresponding to the small baseline, and a_2 is the small BSF. n_1 and n_2 are Gaussian with variances given by Eq. 5. Our purpose is to select a_1 and a_2 to guarantee the best estimate of h. For a system of N antennas, the number of independent equations would be $N - 1$.

3. ML ESTIMATE AND MSE APPROXIMATION

3.1. ML estimate

Given the model described in the last section, we now can derive the ML estimate of the terrain height h, and the mean squared error of the estimate. The pair of baselines that reaches the global minimum of the MSE is optimal. Assume that noises n_1 and n_2 are uncorrelated. The conditional pdf of $\hat{\theta}_1$ and $\hat{\theta}_2$ given h and the unknown integers k_1 and k_2 are:

$$p(\hat{\theta}_1|h, k_1, k_2) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{(\hat{\theta}_1 - (\frac{h}{a_1} - 2\pi k_1))^2}{2\sigma_1^2}\right) \tag{10}$$

$$p(\hat{\theta}_2|h, k_1, k_2) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{(\hat{\theta}_2 - (\frac{h}{a_2} - 2\pi k_2))^2}{2\sigma_2^2}\right). \tag{11}$$

The ML estimate of h is the value of h that maximizes

$$p(\hat{\theta}_1, \hat{\theta}_2|h, k_1, k_2) = p(\hat{\theta}_1|h, k_1, k_2)p(\hat{\theta}_2|h, k_1, k_2). \tag{12}$$

Thus, the ML estimate is

$$\hat{h} = \arg_h \max_{h,k_1,k_2} p(\hat{\theta}_1|h, k_1, k_2)p(\hat{\theta}_2|h, k_1, k_2) \tag{13}$$

$$= \arg_h \min_{h,k_1,k_2} \frac{(h - a_1(\hat{\theta}_1 + 2\pi k_1))^2}{2a_1^2\sigma^2} + \frac{(h - a_2(\hat{\theta}_2 + 2\pi k_2))^2}{2a_2^2\sigma^2} \tag{14}$$

$$= \arg_h \min_h \min_{k_1,k_2} \frac{(h - a_1(\hat{\theta}_1 + 2\pi k_1))^2}{2a_1^2\sigma^2} + \frac{(h - a_2(\hat{\theta}_2 + 2\pi k_2))^2}{2a_2^2\sigma^2} \tag{15}$$

$$= \arg_h \min_h \frac{(h - a_1(\hat{\theta}_1 + 2\pi \hat{k}_1))^2}{2a_1^2\sigma^2} + \frac{(h - a_2(\hat{\theta}_2 + 2\pi \hat{k}_2))^2}{2a_2^2\sigma^2} \tag{16}$$

with the $\hat{k}_i$'s determined as:

$$\frac{h - a_i\hat{\theta}_i}{a_i 2\pi} - \frac{1}{2} < \hat{k}_i \leq \frac{h - a_i\hat{\theta}_i}{a_i 2\pi} + \frac{1}{2} \qquad i = 1, 2 \tag{17}$$

A similar solution can be derived for more than two baselines by adding more terms in the formula to be minimized. This approach finds the weighted minimum least squares estimate from multiple measurements by putting less weight on smaller baseline (bigger BSF) data, i.e., the data are less trusted when the two antennas are closer. This is an intuitively appealing approach even if we are not certain about the noise model, and the noises n_1 and n_2 are somewhat correlated. Solution (16) can be interpreted pictorially. As we stated before, given the estimate $\hat{\theta}_i$, each model of the conditional pdf of h has ambiguity on the noise and the integer number of 2π. Therefore, each $\hat{\theta}_i$ provides an ambiguous periodic waveform for the conditional pdf of h, and our estimate $\hat{h}$ from multiple-baseline data is the point where the product of those curves is maximum. See Fig. 2. The solid broad curve corresponds to the conditional pdf of h centered at $\hat{h}_1$ using the pair of data collected by the smallest baseline, i.e., the largest BSF. The dashed and dotted curves present the conditional pdf of h with the small BSFs, while the thinnest dotted curve has the largest baseline so that the noise variance is the smallest, but it also has more ambiguous periods. This optimization situation is an integer programming problem. There is no closed-form solution in general.

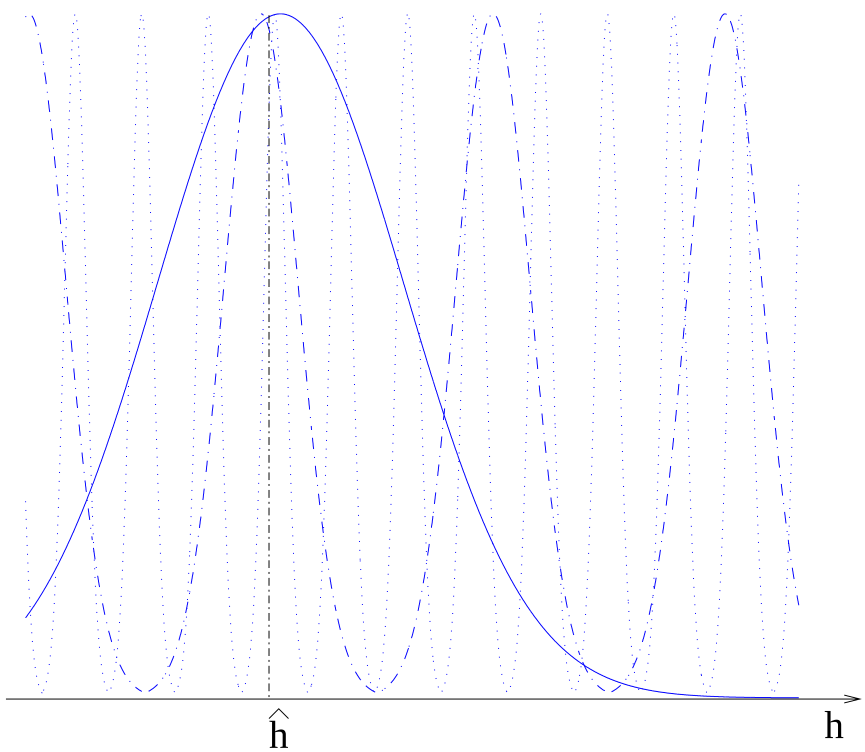

Figure 2. Ambiguous waveforms of h given measurements $\hat{\theta}_i$

3.2. MSE approximation and optimal baselines

We deal with the two-baseline case, where h was derived in Eq. 15. One observation is that one of the baselines must be short enough to prevent phase wrapping, otherwise the MSE can be infinitesimal. We fix the large BSF as $a_1 = \frac{h_{max}}{2\pi}$. We assume the noise variances $\sigma_1 = \sigma_2 = \sigma$. The estimation problem then becomes:

$$\hat{h} = \arg\min_h \min_{k_2} \frac{(h - a_1\hat{\theta}_1)^2}{a_1^2\sigma^2} + \frac{(h - a_2(\hat{\theta}_2 + 2\pi k_2))^2}{a_2^2\sigma^2} \tag{18}$$

$$= \arg\min_h \min_{k_2} \left(\frac{1}{a_1^2\sigma^2} + \frac{1}{a_2^2\sigma^2}\right)\left(h - \frac{\frac{\hat{\theta}_1}{a_1} + \frac{\hat{\theta}_2 + 2\pi k_2}{a_2}}{\frac{1}{a_1^2} + \frac{1}{a_2^2}}\right)^2 + \frac{1}{(a_1^2 + a_2^2)\sigma^2}(a_1\hat{\theta}_1 - a_2(\hat{\theta}_2 + 2\pi k_2))^2 \tag{19}$$

where, $\hat{\theta}_1$ is the phase value in $[0 \;\; 2\pi)$. Now we derive the best value a_2 for the second BSF. There is a tradeoff in selecting a_2. Decreasing a_2 causes a smaller error variance for h, which is $a_2^2\sigma^2$ if the correct cycle, i.e., correct k_2

is known, but also increases the possibility of cycle errors, i.e., a wrong estimate of k_2. The best value of a_2 should balance these two kinds of errors.

Clearly, Eq. 19 is minimized when we pick the integer k_2 to minimize the second term, and choose h to zero out the first term. Therefore, the ML estimate of h is

$$\hat{h} = \frac{\frac{\hat{\theta}_1}{a_1} + \frac{\hat{\theta}_2 + 2\pi \hat{k}_2}{a_2}}{\frac{1}{a_1^2} + \frac{1}{a_2^2}} \tag{20}$$

$$= \frac{a_2^2(a_1\hat{\theta}_1) + a_1^2 a_2(\hat{\theta}_2 + 2\pi\hat{k}_2)}{a_1^2 + a_2^2}, \tag{21}$$

with $\hat{k}_2$ calculated as the minimizer of the second term of Eq. 19:

$$\hat{k}_2 = int[\frac{a_1\hat{\theta}_1 - a_2\hat{\theta}_2}{2\pi a_2} + \frac{1}{2}], \tag{22}$$

where *int* means the integer part of, and the resulting $\hat{k}_2$ makes $a_2(\hat{\theta}_2 + 2\pi\hat{k}_2)$ closest to $a_1\hat{\theta}_1$.

The MSE of the estimate $\hat{h}$ can be obtained numerically. We have

$$E[(\hat{h} - h)^2] = E[(\frac{a_2^2(a_1\hat{\theta}_1) + a_1^2 a_2(\hat{\theta}_2 + 2\pi\hat{k}_2(\hat{\theta}_1, \hat{\theta}_2))}{a_1^2 + a_2^2} - h)^2] \tag{23}$$

$$= \frac{1}{(a_1^2 + a_2^2)^2} E[(a_2^2 a_1(\hat{\theta}_1 - \theta_1) + a_1^2 a_2(\hat{\theta}_2 - \theta_2 + 2\pi(\hat{k}_2 - k_2)))^2] \tag{24}$$

$$= \frac{1}{(a_1^2 + a_2^2)^2}\{a_2^4 a_1^2 E[(\hat{\theta}_1 - \theta_1)^2] + a_1^4 a_2^2 E[(\hat{\theta}_2 - \theta_2)^2] + 4\pi^2 a_1^4 a_2^2 E[(\hat{k}_2 - k_2)^2] + 2a_1^3 a_2^3 E[(\hat{\theta}_1 - \theta_1)(\hat{\theta}_2 - \theta_2)] + 4\pi a_1^3 a_2^3 E[(\hat{\theta}_1 - \theta_1)(\hat{k}_2 - k_2)] + 4\pi a_1^4 a_2^2 E[(\hat{\theta}_2 - \theta_2)(\hat{k}_2 - k_2)]\} \tag{25}$$

$$= \frac{1}{(a_1^2 + a_2^2)^2}\{a_2^4 a_1^2 \sigma^2 + a_1^4 a_2^2 \sigma^2 + 4\pi^2 a_1^4 a_2^2 E[(\hat{k}_2 - k_2)^2] + 4\pi a_1^3 a_2^3 E[(\hat{\theta}_1 - \theta_1)(\hat{k}_2 - k_2)] + 4\pi a_1^4 a_2^2 E[(\hat{\theta}_2 - \theta_2)(\hat{k}_2 - k_2)]\} \tag{26}$$

$$= \frac{1}{(a_1^2 + a_2^2)^2}\{a_2^4 a_1^2 \sigma^2 + a_1^4 a_2^2 \sigma^2 + 4\pi^2 a_1^4 a_2^2 T_1 + 4\pi a_1^3 a_2^3 T_2 + 4\pi a_1^4 a_2^2 T_3\}. \tag{27}$$

where

$$T_1 = E[(\hat{k}_2 - k_2)^2] \tag{28}$$

$$T_2 = E[(\hat{\theta}_1 - \theta_1)(\hat{k}_2 - k_2)] \tag{29}$$

$$T_3 = E[(\hat{\theta}_2 - \theta_2)(\hat{k}_2 - k_2)]. \tag{30}$$

T_1 represents the MSE of the estimate of k_2, and T_2 and T_3 are cross terms. T_1 is the dominant term since it is the cyclic error of the estimate.

To calculate T_1, first consider Fig. 3. The broad curve denotes the distribution of $h_1 = a_1\hat{\theta}_1$ with mean $a_1\theta_1 = h$ being the true terrain height, and variance $a_1^2\sigma^2$. The narrower curve is the distribution of $h_2 = a_2\hat{\theta}_2 + 2\pi k_2$ with mean $a_2\theta_2 + 2\pi k_2 = h$, and variance $a_2^2\sigma^2$. So,

$$P(|\hat{k}_2 - k_2| = 1) = P\{\pi a_2 < |h_1 - h_2| < 3\pi a_2, |h_2 - h| < \pi a_2\} \tag{31}$$

When σ is small compared to π,

$$P(|\hat{k}_2 - k_2| = 1) \approx P\{\pi a_2 < |h_1 - h_2| < 3\pi a_2\} \tag{32}$$

$$= P\{\pi a_2 < |\Delta_h| < 3\pi a_2\}, \tag{33}$$

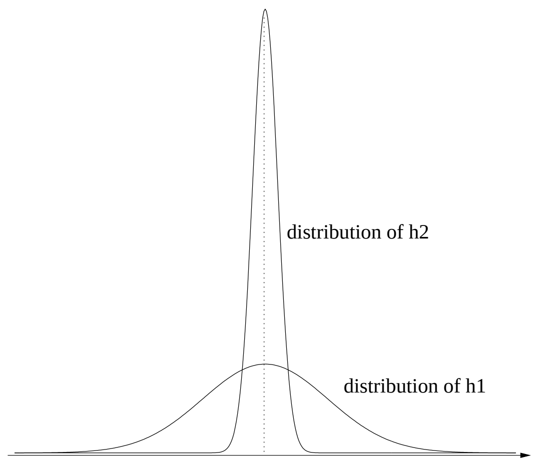

Figure 3. Distribution of h_1 and h_2

where Δ_h is a Gaussian random variable with mean 0 and variance $(a_1^2 + a_2^2)\sigma^2$. Thus,

$$P(|\hat{k}_2 - k_2| = 1) \approx 2[Q(\frac{\pi a_2}{\sqrt{(a_1^2 + a_2^2)\sigma^2}}) - Q(\frac{3\pi a_2}{\sqrt{(a_1^2 + a_2^2)\sigma^2}})] \tag{34}$$

where, $Q(x)$ is defined as

$$Q(x) = \int_x^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt \tag{35}$$

Similarly, $P(|\hat{k}_2 - k_2| = 2, 3, 4, ...)$ can be obtained in the same manner, so that

$$T_1 = E[(\hat{k}_2 - k_2)^2] \approx \sum_{i=1}^{\infty} 2[Q(\frac{(2i-1)\pi a_2}{\sqrt{(a_1^2 + a_2^2)\sigma^2}}) - Q(\frac{(2i+1)\pi a_2}{\sqrt{(a_1^2 + a_2^2)\sigma^2}})]i^2 \tag{36}$$

Now let's calculate the other two terms. Notice T_2 is always positive, since $\hat{k}_2 > k_2$ implies that it is almost impossible for $\hat{\theta}_1$ to be smaller than its mean value θ_1, and vise versa. We can calculate the error probability similar to before. Write $\hat{\theta}_1 - \theta_1$ as $\frac{h_1 - h}{a_1}$. Then

$$T_2 = E[(\hat{\theta}_1 - \theta_1)(\hat{k}_2 - k_2)] \approx 2\int_{h_2} p_2(h_2) \int_{h_2+\pi a_2}^{h_2+3\pi a_2} p_1(h_1)\frac{h_1 - h}{a_1} \cdot 1 dh_1 dh_2 \tag{37}$$

$$+2\int_{h_2} p_2(h_2) \int_{h_2+3\pi a_2}^{h_2+5\pi a_2} p_1(h_1)\frac{h_1 - h}{a_1} \cdot 2 dh_1 dh_2 + ... \tag{38}$$

$$= 2\int_{h_2} p_2(h_2)\frac{\sigma}{\sqrt{2\pi}}[e^{-\frac{(h_2 - h + \pi a_2)^2}{2a_1^2\sigma^2}} + e^{-\frac{(h_2 - h + 3\pi a_2)^2}{2a_1^2\sigma^2}} + ...]dh_2 \tag{39}$$

$$= \frac{2}{2\pi a_2}\sqrt{\frac{2\pi\sigma^2 a_1^2 a_2^2}{a_1^2 + a_2^2}}(e^{-\frac{a_2^2\pi^2}{2\sigma^2(a_1^2 + a_2^2)}} + e^{-\frac{a_2^2 3\pi^2}{2\sigma^2(a_1^2 + a_2^2)}} +) + ... \tag{40}$$

$$= \frac{2a_1\sigma}{\sqrt{2\pi(a_1^2 + a_2^2)}}\sum_{i=1}^{\infty} e^{-\frac{a_2^2(2i-1)\pi^2}{2\sigma^2(a_1^2 + a_2^2)}} \tag{41}$$

Similarly, T_3 can be approximated as

$$T_3 \approx -\frac{2a_2\sigma}{\sqrt{2\pi(a_1^2 + a_2^2)}}\sum_{i=1}^{\infty} e^{-\frac{a_1^2(2i-1)\pi^2}{2\sigma^2(a_1^2 + a_2^2)}}. \tag{42}$$

Substitute all three terms in Eq. 27, the expression for the MSE of the terrain height estimate. Here, the dominant term is T_1 when σ is small compared to π. Furthermore, assuming that a_2 is much smaller than a_1, the MSE can be approximated as

$$E[(\hat{h}-h)^2] \approx a_2^2\sigma^2 + 4\pi^2 a_2^2 T_1 \quad (43)$$

$$\approx a_2^2\sigma^2 + 8\pi^2 a_2^2 Q(\frac{\pi a_2}{a_1\sigma}). \quad (44)$$

Given the expression for the MSE of our estimate, we can search for the optimal BSF a_2.

4. SIMULATION RESULTS

In our simulation, we assume a maximum height of 3141.6. The true terrain height is set to be 1000, and the measurements θ_1 and θ_2 are independent zero-mean Gaussian random variables with standard deviation $\sigma = 0.25$ (14.3^o). The optimal large BSF is calculated as $a_1 = \frac{h_{max}}{2\pi} = 500$. We determine the optimal a_2 by minimizing the MSE in Eq. 27. Figure 4 shows how the RMS error varies as a function of a_2. In this example, the minimum error occurs around $a_2 = 153$. Figure 4 contains three different curves. One was computed from the exact formula, Eq. 27. The second curve used the approximate error expression, Eq. 44. The third curve is the same as the first one except we omitted the last two cross terms T_2 and T_3. All three curves have a minimum around the same point. The third curve is almost coincident with the first one, which indicates that the two cross terms T_2 and T_3 are very small and can be neglected. Notice that the curves change more slowly for $a_2 > 153$. This occurs because when a_2 is large, there are almost no cycle errors. For large a_2, $E[(\hat{h}-h)^2]$ is mostly determined by the variance of $a_1 n_1$ and $a_2 n_2$, but when a_2 is small, the error variance depends more on cycle errors, which creates a larger slope.

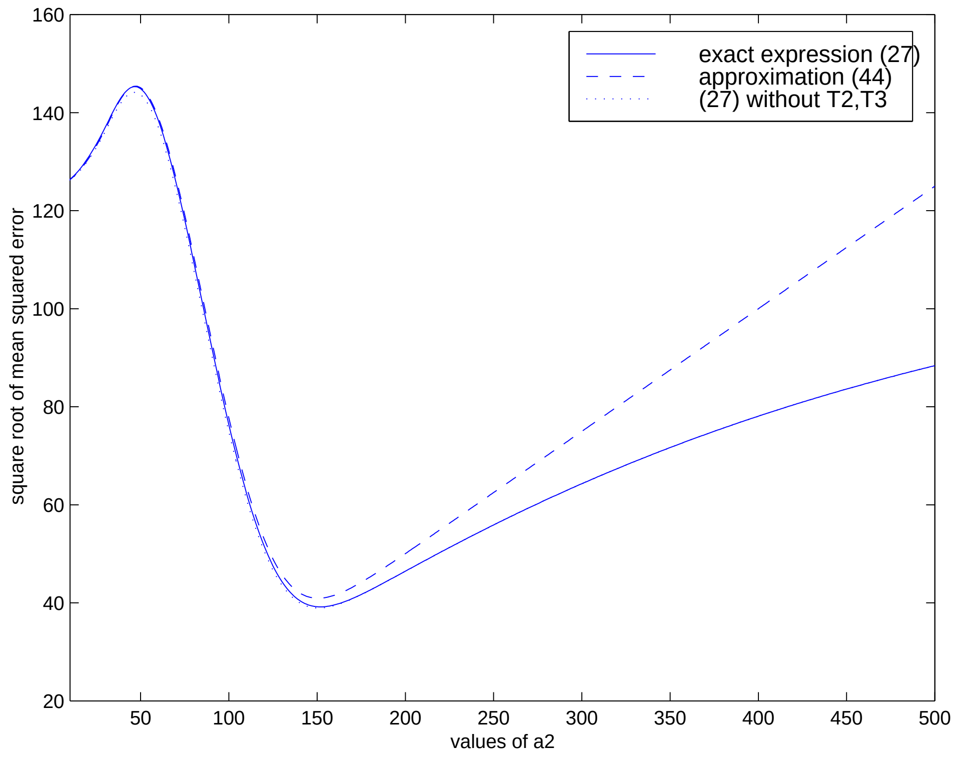

Figure 4. Root of mean squared (RMS) error of the estimate of h vs different a_2

The following table presents simulation results for the same setup as above with $h_{max} = 3141.6, a_1 = 500, \sigma = 0.25$. We calculated the true θ_is using Eq. 1 and produced random estimates $\hat{\theta}_i$ from Eq. 3. Then we found the maximum likelihood estimate $\hat{h}$ defined by Eq. 21. Each statistical result was obtained by running the program for 5000 independent trials, and calculating the sample mean and RMS error of our estimate corresponding to different values of a_2. The results in Table 1 match Fig. 4 very well, and the a_2 that provides the minimum RMS error is 153 as predicted from our formulas. Table 1 shows that the error is largest for small a_2, and there is little difference in RMS error for a_2 somewhat larger than 153.

Table 1. Comparison of RMS error of the estimate of h w.r.t. a_2

a_2	50	87	123	143	153	171	286	400
mean	1003.1	1003.4	1000.9	1000.7	1000.6	1000.7	998.48	999.62
RMS	142.03	100.57	45.84	41.44	37.08	40.38	61.45	78.21

For three or more baselines, the baseline optimization problem is more complicated. For more than two antennas, the number of independent baselines increases linearly. For a few baselines, a good procedure might be to iteratively repeat the process above, adding one new baseline at each iteration. This would guarantee a small cycle error corresponding to the largest baseline, but this surely would not be optimal since the problem is not separable. Probably, when we have many baselines, there would be no need to design them carefully, since a very rough design would provide a good maximum likelihood estimate using the formula in Lombardini and Lombardo.[8]

5. SUMMARY

In this paper, we studied optimal baselines for INSAR data collection. After introducing a statistical model for the terrain height estimation problem, we derived a closed-form expression for the maximum-likelihood estimate of the terrain height, for two baselines. The best baseline design minimizes the mean squared error in the elevation estimate. For the two-baseline case, we derived an approximate MSE curve by analyzing the tradeoff between the cycle error and measurement error. The optimal second baseline was then found as the minimizer of the MSE curve. The accuracy of this approach was confirmed by computer simulation.

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