Spring 2007, Aaron Lanterman

ECE 6279: Spatial Array Processing Homework 2

Due: Wed 2/7/07 (campus) or Wed 2/14/07 (video)
Late due date (20% penalty): Fri 2/9/07 (campus) or Fri 2/16/07 (video)

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, your solutions should be your own work and should be written up by yourself; feel free to discuss things, but don't be looking at someone else's paper when you are writing your solution. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

For on-campus students, homework is due at the *start* of class; for video students, please have your homework submitted by noon on the day it's due (that gives time for the distance learning staff to get it logged and on its way to me that day.)

A 20% penalty will be assessed on late homeworks (even homeworks turned in later the day it is due); I will distribute solutions shortly after class on Friday, so I will not accept solutions after that. The "late" turn in is available as an emergency buffer in case you get overwhelmed with life in general, but try not to rely on it. If you cannot make a class, please make arrangements to get your homework to me ahead of time. (A mechanism will be developed to distribute solutions to the on-campus and video students separately at appropriate times; in this sense, they are considered separate "classes.")

1 Required Problems

1. Suppose we have a circularly symmetric aperture defined by $w(x,y,z) = o(x,y)\delta(z)$, where
$$o(x,y) = \begin{cases} 1, & R_2 \leq \sqrt{x^2+y^2} \leq R_2 \\ 0, & \text{otherwise} \end{cases}$$
Of course, $0 < R_1 < R_2 < \infty$. Find its aperture smoothing function W as a function of $k_{xy} = \sqrt{k_x^2 + k_y^2}$. (Hint: Use *linearity* and the result in the box on p. 64.)

2. Suppose we have an array consisting of M (M is odd) elements evenly spaced along the x-axis, centered at the origin, with spacing d. The twist here is that each element, instead of being a point, is actually a small linear segment of length L along the x-axis (where $L < d/2$, so the segments don't overlap), centered at the element locations. Using ideas from p. 131, find the wavenumber-frequency response of the resulting filter-and-sum beamformer if the sensor delays are adjusted to steer the beam to look for plane waves propagating with a slowness vector $\vec{\alpha}$. (On this problem, and actually on all the homeworks, feel free to quote results from the lectures whenever you find them useful; you do not need to rederive things we've already established in the slides.)

3. Consider an array of seven sensors located at $(x, y) = (-1, -1)$, $(1, -1)$, $(0, -2)$, $(0, 0)$, $(0, 2)$, $(-1, 1)$ and $(1, 1)$ with uniform shading coefficients ($w_m = 1$).

   (a) Compute the array pattern as a function of k_x, k_y, and k_z. Express your answer as a sum of cosine terms (i.e. I don't want to see any complex exponentials left in the answer).

   (b) Compute the array pattern as a function of the angles of incidence ϕ and azimuth θ and wavelength λ.

4. Do Problem 4.12 on p. 192 of J & D.

5. Verify the formula in the box on p. 119 of J & D.