Spring 2007, Aaron Lanterman

ECE 6279: Spatial Array Processing Homework 7

Due: Wed 4/25/07 (campus) or Wed 5/2/07 (video)

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, your solutions should be your own work and should be written up by yourself; feel free to discuss things, but don't be looking at someone else's paper when you are writing your solution. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

For on-campus students, homework is due at the *start* of class; for video students, please have your homework submitted by noon on the day it's due (that gives time for the distance learning staff to get it logged and on its way to me that day.)

1 Required Problems

1. In lecture on April 20, we will derive an expression for the Cramér-Rao bound on the parameter γ under the "deterministic" Gaussian model, where γ is the "electrical angle" specified by $\gamma = (2\pi d/\lambda)\sin\phi$ (where $\theta = 0$) for the case of an equally-spaced linear array of M elements with spacing d lying along the x-axis. Since I'm posting this homework before give that lecture, I will go ahead and give the result here:

$$CRB(\gamma) = \frac{6}{SNR}\frac{1}{M(M^2-1)}$$

SNR denotes the signal power to noise power ratio. The expression acts as you might expect; achieving a better SNR or adding more elements gives you a lower CRB. Interestingly, that the CRB on the electrical angle is not actually a function of the true electrical angle.

In this problem, we'll consider an alternate formulation where we take $\phi = \pi/2$, and consider the azimuth angle θ. Hence, in this problem, we will let $\gamma = (2\pi d/\lambda)\cos\theta$.

(a) We'd like a CR bound in terms of θ instead of γ. At the end of the lecture on April 18, I showed you two formulas for computing the CR bound on a parameter θ (I happened to use ϕ in the example in lecture, but the same idea applies) that's defined by a functional mapping $\theta = f(\gamma)$, given the CR bound on γ. Using these formulas, compute the CR bound on θ using the CR bound on γ given above for an M-element linear array. Try both formulas, and make sure you get the same answer with each!

(b) What happens to the CRB on θ as $\theta \to 0°$?

(c) Suppose I told you that there are estimators for θ *that beat the CRB.* (If you think about your answer for part (b) a bit, and realize that the greatest error you can ever have in estimating an angle is 180°, you'll see that this must be true.) What does this tell you about those estimators? (I'm looking for a short answer here - something like two or three words.) Re-watch the initial couple of minutes of my discussion of the CRB in the video for April 16 if you get stuck on this.

2. Let's pull out our 9-element cross array one last time, the one you used in Homeworks 3, 4, and 6. So as usual, you can probably take code you already have and just tweak it to do this problem.

   Let's use 150 snapshots to form our covariance estimate. (There will still be some variability, so you may want to run your experiments a few times to watch overall trends and make sure you didn't get just one "lucky" case. You need only turn in one set of plots per subpart, though.)

   Let's have two sources, one at $\phi = 45°$ and $\theta = 30°$, and another $\phi = 45°$ and $\theta = 60°$. Let's let each have power 1/4 (so you'll want to multiply your steering vectors by 1/2 when making the "seen by array" variables), and let's make the noise have power 2. Notice we're really pushing our algorithms hard now; the signals are pretty weak compared to the noise.

   Let's experiment with the model order estimation procedure that I clarified in the lecture on April 16. As usual, please provide printouts of your code. (You need not provide separate printouts for minor tweaks - i.e. when you change the noise level, don't bother printing out a whole other page just to show the code with that one change).

   (a) On the same graph, plot (1) the loglikelihood given in class for the model order estimation problem (the expression that has the geometric mean in the numerator and the artihmetic mean in the denominator), (2) the penalized likelihood using the AIC penalty, (3) the penalized likelihood using the MDL penalty, all as a function of N_s, the assumed number of sources. For what N_s do the different curves take their maximum? Did AIC estimate the correct model order? What about MDL?

   (b) If your model order estimators got the correct model order in part (a), slowly jack up the noise power from 2 until you start regularly seeing errors in the model order estimates. At what noise power do the criteria start to have problems? (There isn't an exact number here that's the "right" answer. Just experiment until you get something in the "ballpark.") Give a plot like in part (a) for one of these cases where both AIC and MDL get the model order wrong. Do they overestimate or underestimate the number of sources?