

Conventional Wideband Beamforming

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Where We Are in J&D

- Lecture material drawn from:
 - Sec. 4.4 (all), with some of Aaron's interpretation



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When is the Narrowband Model OK?

$$s^+(t) = s_b(t) \exp(j\omega_0 t)$$

- Narrowband assumption means that:
 $s_b(t - \tau) \approx s_b(t)$ for the largest possible τ
- For a plane wave with speed c :

Aaron's idea $\tau = \frac{1}{c} \max_m \{ |\vec{x}_m| \}$

$$BW = O\left(\frac{1}{\tau}\right) = O\left(\frac{c}{\max_m \{ |\vec{x}_m| \}}\right)$$



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Inspiration (1)

- Recall basic delay-and-sum beamformer:

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

- Taking FT on both sides, we see the delay corresponds to a phase shift in Fourier space:

$$Z(\omega) = \sum_{m=0}^{M-1} w_m Y_m(\omega) \exp(-j\omega \Delta_m)$$

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Inspiration (2)

- **Problem:** computing $Y_m(\omega)$ would require $y_m(t)$ for all time
- **Even if we had it, sources would move during the observation**



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Short-Time Fourier Transforms

- **Solution:** compute short-time FT

$$Y_m(t, \omega) \equiv \int_t^{t+D} \tilde{w}(\tau - t) y_m(\tau) \exp(-j\omega\tau) d\tau$$

- **Substitute:** $\tau' = \tau - t$ $\tau' + t = \tau$ $d\tau' = d\tau$

$$Y_m(t, \omega) = \int_0^D \tilde{w}(\tau) y_m(t + \tau) \exp\{-j\omega(t + \tau)\} d\tau$$

$$Y_m(t, \omega) e^{j\omega t} = \int_0^D \tilde{w}(\tau) y_m(t + \tau) \exp(-j\omega\tau) d\tau$$

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Interpretation of the STFT

$$Y_m(t, \omega) e^{j\omega t} = \int_0^D \tilde{w}(\tau) y_m(t + \tau) \exp(-j\omega\tau) d\tau$$

- $Y_m(t, \omega)$ “is a complex-valued lowpass signal that approximates the ‘local’ spectrum of the sensor’s output at time t and frequency ω ” (J&D, p. 134)



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Frequency-Domain Beamforming

- **Do the delay-and-sum beamforming (equates to phase shift) on the STFT:**

$$Z(\omega, t) \equiv$$

$$\sum_{m=0}^{M-1} w_m Y_m(t, \omega) \exp(j\omega t) \exp(-j\omega\Delta_m)$$

(Usually, this is what we’re interested in)



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Filtering in Time

$$Z(\omega, t) \equiv \sum_{m=0}^{M-1} w_m(\omega) Y_m(t, \omega) \exp(j\omega t) \exp(-j\omega \Delta_m)$$

- We can let the weights be a function of frequency
- Gives us frequency-domain counterpart of (temporal filter)-and-sum beamforming

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Beamforming for Plane Waves

$$\begin{aligned} Z(\omega, t) \exp(-j\omega t) &= \sum_{m=0}^{M-1} w_m Y_m(t, \omega) \exp(-j\omega \Delta_m) \\ &= \sum_{m=0}^{M-1} w_m Y_m(t, \omega) \exp\{j\vec{k}(\omega) \cdot \vec{x}_m\} \end{aligned}$$

We'll see the $\exp(-j\omega t)$ part isn't too important

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Wideband Vector Notation

$$\begin{aligned} \sum_{m=0}^{M-1} w_m Y_m(t, \omega) \exp(j\vec{k}(\omega) \cdot \vec{x}_m) \\ = \mathbf{e}^H(\vec{k}(\omega)) \mathbf{W} \mathbf{Y}(t, \omega) \end{aligned}$$

$$\mathbf{W} \mathbf{Y} = \begin{bmatrix} w_0 Y_0(t, \omega) \\ \vdots \\ w_{M-1} Y_{M-1}(t, \omega) \end{bmatrix}$$

$$\mathbf{W} = \text{diag}(w_0, \dots, w_{M-1})$$

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Wideband Strategy

- Steered response power (implicit integral over t):

$$\begin{aligned} \int_{-\infty}^{\infty} |Z(\omega, t)|^2 d\omega &= \int_{-\infty}^{\infty} |Z(\omega, t) \exp(-j\omega t)|^2 d\omega \\ &= \int_{-\infty}^{\infty} \left| \mathbf{e}^H(\vec{k}(\omega)) \mathbf{W} \mathbf{Y}(t, \omega) \right|^2 d\omega \\ &= \int_{-\infty}^{\infty} \mathbf{e}^H(\vec{k}(\omega)) \underbrace{\mathbf{W} \mathbf{Y}(t, \omega) \mathbf{Y}^H(t, \omega) \mathbf{W}^H}_{\mathbf{R}(t, \omega)} \mathbf{e}(\vec{k}(\omega)) d\omega \end{aligned}$$

spatial correlation matrix

- J&D drops dependence on t , but reintroduces it in Sec. 4.9

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