Conventional Wideband Beamforming

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Where We Are in J&D

- **Lecture material drawn from:**
  - Sec. 4.4 (all), with some of Aaron's interpretation

When is the Narrowband Model OK?

$$s^{+}(t) = s_b(t)\exp(j\omega_0 t)$$

- **Narrowband assumption means that:**

$s_b(t-\tau) \approx s_b(t)$ **for the largest possible** τ

- **For a plane wave with speed** c**:**

$$\tau = \frac{1}{c}\max_m\left\{\left|\vec{x}_m\right|\right\}$$

Aaron's idea

$$BW = O\left(\frac{1}{\tau}\right) = O\left(\frac{c}{\max_m\left\{\left|\vec{x}_m\right|\right\}}\right)$$

Inspiration (1)

- **Recall basic delay-and-sum beamformer:**

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t-\Delta_m)$$

- **Taking FT on both sides, we see the delay corresponds to a phase shift in Fourier space:**

$$Z(\omega) = \sum_{m=0}^{M-1} w_m Y_m(\omega)\exp(-j\omega\Delta_m)$$

Inspiration (2)

- **Problem: computing $Y_m(\omega)$ would require $y_m(t)$ for all time**
- **Even if we had it, sources would move during the observation**

Short-Time Fourier Transforms

- **Solution: compute short-time FT**

$$Y_m(t,\omega) \equiv \int_t^{t+D} \tilde{w}(\tau - t) y_m(\tau) \exp(-j\omega\tau) d\tau$$

- **Substitute:** $\tau' = \tau - t \quad \tau' + t = \tau \quad d\tau' = d\tau$

$$Y_m(t,\omega) = \int_0^D \tilde{w}(\tau) y_m(t+\tau) \exp\{-j\omega(t+\tau)\} d\tau$$

$$Y_m(t,\omega) e^{j\omega t} = \int_0^D \tilde{w}(\tau) y_m(t+\tau) \exp(-j\omega\tau) d\tau$$

Interpretation of the STFT

$$Y_m(t,\omega) e^{j\omega t} = \int_0^D \tilde{w}(\tau) y_m(t+\tau) \exp(-j\omega\tau) d\tau$$

- $Y_m(t,\omega)$ **"is a complex-valued lowpass signal that approximates the 'local' spectrum of the sensor's output at time t and frequency ω" (J&D, p. 134)**

Frequency-Domain Beamforming

- **Do the delay-and-sum beamforming (equates to phase shift) on the STFT:**

$$Z(\omega,t) \equiv$$

$$\sum_{m=0}^{M-1} w_m Y_m(t,\omega) \exp(j\omega t) \exp(-j\omega\Delta_m)$$

(Usually, this is what we're interested in)

Filtering in Time

$$Z(\omega,t) \equiv \sum_{m=0}^{M-1} w_m(\omega) Y_m(t,\omega)\exp(j\omega t)\exp(-j\omega\Delta_m)$$

- We can let the weights be a function of frequency
- Gives us frequency-domain counterpart of (temporal filter)-and-sum beamforming

Beamforming for Plane Waves

$$Z(\omega,t)\exp(-j\omega t) = \sum_{m=0}^{M-1} w_m Y_m(t,\omega)\exp(-j\omega\Delta_m)$$

($\omega\Delta_m = -\vec{\alpha}\cdot\vec{x}_m$)

$$= \sum_{m=0}^{M-1} w_m Y_m(t,\omega)\exp\{j\vec{k}(\omega)\cdot\vec{x}_m\}$$

We'll see the exp(-*jωt*) part isn't too important

Wideband Vector Notation

$$\sum_{m=0}^{M-1} w_m Y_m(t,\omega)\exp(j\vec{k}(\omega)\cdot\vec{x}_m) = \mathbf{e}^H(\vec{k}(\omega))\mathbf{WY}(t,\omega)$$

$$\mathbf{WY} = \begin{bmatrix} w_0 Y_0(t,\omega) \\ \vdots \\ w_{M-1} Y_{M-1}(t,\omega) \end{bmatrix}$$

$$\mathbf{W} = diag(w_0,\ldots,w_{M-1})$$

Wideband Strategy

- **Steered response power** (implicit integral over t):

$$\int_{-\infty}^{\infty} |Z(\omega,t)|^2 d\omega = \int_{-\infty}^{\infty} |Z(\omega,t)\exp(-j\omega t)|^2 d\omega$$

$$= \int_{-\infty}^{\infty} \left|\mathbf{e}^H(\vec{k}(\omega))\mathbf{WY}(t,\omega)\right|^2 d\omega$$

$$= \int_{-\infty}^{\infty} \mathbf{e}^H(\vec{k}(\omega))\mathbf{W}\underbrace{\mathbf{Y}(t,\omega)\mathbf{Y}^H(t,\omega)}_{\mathbf{R}(t,\omega)}\mathbf{W}^H\mathbf{e}(\vec{k}(\omega))d\omega$$

$\mathbf{R}(t,\omega)$ ← spatial correlation matrix

- J&D drops dependence on *t*, but reintroduces it in Sec. 4.9