

Apertures, Part II

ECE 6279: Spatial Array Processing
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Lecture 5
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Where We Are in J&D

- Lecture material drawn from:
 - Sec. 3.3, 3.3.1-3.3.2
- Might be a good idea to read through Sec. 3.2
 - Reviews sampling theory



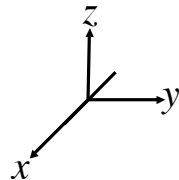
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Last Time: Filled Linear Aperture

$$b(x) = \begin{cases} 1, & |x| \leq D/2 \\ 0, & \text{otherwise} \end{cases}$$

$$w(\vec{x}) = b(x)\delta(y)\delta(z)$$



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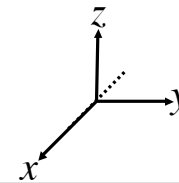


Regularly Sampled Linear Aperture

- Odd number of sensors M , w/spacing d
- Total length $D=Md$

$$b(x) = \sum_{m=-(M-1)/2}^{(M-1)/2} \delta(x - md)$$

$$w(\vec{x}) = b(x)\delta(y)\delta(z)$$



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Aperture Smoothing Function (1)

$$\begin{aligned}
 W(k_x) &= \int_{-\infty}^{\infty} b(x) \exp(\oplus jk_x x) dx \\
 &= \int_{-\infty}^{\infty} \sum_{m=-(M-1)/2}^{(M-1)/2} \delta(x - md) \exp(jk_x x) dx \\
 &= \sum_{m=-(M-1)/2}^{(M-1)/2} \exp(jk_x md) \times \left[\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right) \right] \\
 &= \frac{\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)}{\boxed{}}
 \end{aligned}$$

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Aperture Smoothing Function (2)

$$\sum_{m=-(M-1)/2}^{(M-1)/2} \exp(jk_x md) \times \left[\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right) \right]$$

• Terms from first set:

$$\left\{ a + \frac{1}{2} \right\} \left\{ \cancel{(a-1) + \frac{1}{2}} \right\} \cdots \left\{ \cancel{-(a-1) + \frac{1}{2}} \right\} \left\{ \cancel{-a + \frac{1}{2}} \right\}$$

• Subtract terms from second set:

$$\left\{ \cancel{a - \frac{1}{2}} \right\} \left\{ \cancel{(a-1) - \frac{1}{2}} \right\} \cdots \left\{ \cancel{-(a-1) - \frac{1}{2}} \right\} \left\{ -a - \frac{1}{2} \right\}$$

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Our Old Friend, the Sinc

$$\begin{aligned}
 W(k_x) &= \frac{\exp\left(\frac{jk_x Md}{2}\right) - \exp\left(-\frac{jk_x Md}{2}\right)}{\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)} \\
 &= \frac{\sin\left(\frac{k_x Md}{2}\right)}{\sin\left(\frac{k_x d}{2}\right)} \quad W(0) \equiv M \\
 &= \frac{\sin\left(\frac{k_x Md}{2}\right)}{\sin\left(\frac{k_x d}{2}\right)} \quad \boxed{}
 \end{aligned}$$

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In Polar Wavenumber Coordinates

$$\frac{\sin\left(\frac{k_x Md}{2}\right)}{\sin\left(\frac{k_x d}{2}\right)} = \frac{\sin\left(\frac{k_r Md \sin k_\phi \cos k_\theta}{2}\right)}{\underbrace{\sin\left(\frac{k_r d \sin k_\phi \cos k_\theta}{2}\right)}_{W(k_r, k_\phi, k_\theta)}}$$

$$k_x = k_r \sin k_\phi \cos k_\theta \quad \boxed{}$$

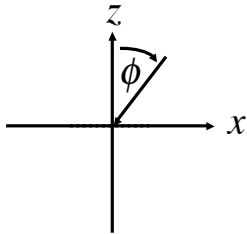
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Limiting to the X-Z Plane

- Suppose $k_\theta = 0$ and let $\phi = -k_\phi$

$$W\left(\frac{2\pi}{\lambda}, -\phi, 0\right) = \frac{\sin\left(\frac{Md\pi}{\lambda} \sin\phi\right)}{\sin\left(\frac{d\pi}{\lambda} \sin\phi\right)}$$

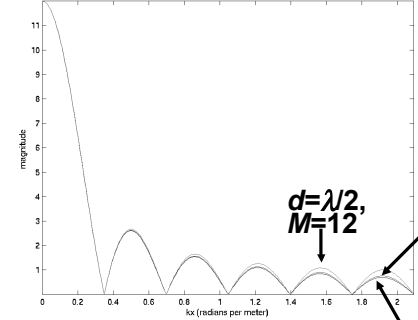
$W(\phi)$



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Comparison (Wavenumber)

Beam Pattern (lambda = 3 m, D = 6 * lambda = 18 m)



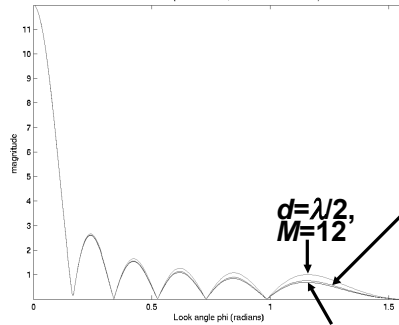
To study changes in overall shape, all plotted curves are artificially normalized to have gain 12 at angle 0

- 100 MHz wave
- 18 meter long aperture

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Comparison (Phi)

Beam Pattern (lambda = 3 m, D = 6 * lambda = 18 m)



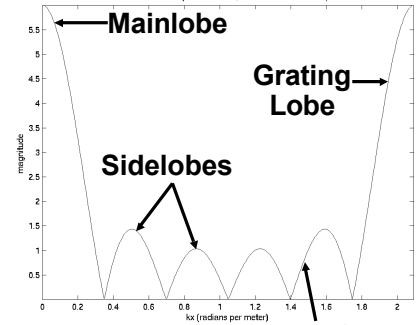
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Grating Lobes

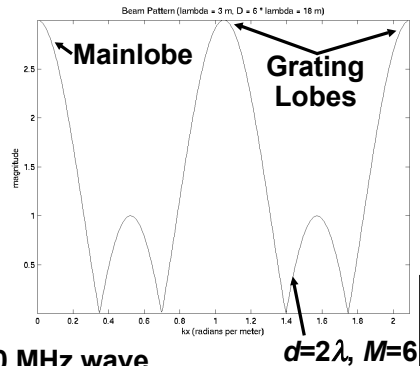
Beam Pattern (lambda = 3 m, D = 6 * lambda = 18 m)



- 100 MHz wave
- 18 meter long aperture

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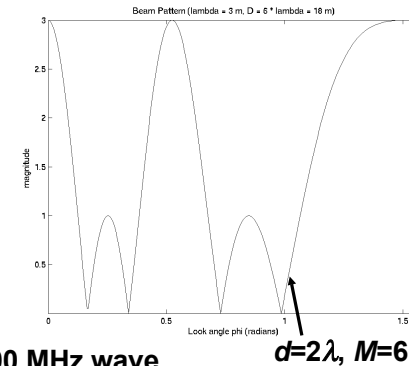
More Grating Lobes



- 100 MHz wave
- 18 meter long aperture

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Stretching of Grating Lobes



- 100 MHz wave
- 18 meter long aperture

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Sampling Issues

- When designing a uniform linear array, generally try to pick $d \leq \lambda/2$
- Can sometimes get away $d > \lambda/2$ with if you have side information
 - Detect a target with a low frequency
 - Long wavelength
 - Then track it with a high frequency
 - Short wavelength
 - Use knowledge from detector to disambiguate aliases
- Most papers will assume $d \leq \lambda/2$
 - Usually assume $d = \lambda/2$

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2-D & 3-D Apertures

- Covered in detail by VT-IV, Chapter 4
- Can solve “cone of ambiguity” problem
 - Careful: some 2-D apertures have their own kinds of cones of ambiguity
 - i.e. circular disk
- Sampling issues get trickier to analyze
 - Theory from ECE6258: Digital Image Processing applies here
 - Hexagonal sampling, etc.
 - Discussed in Sec. 3.2 of J&D

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Windowing

- **By weighting an aperture by a window, we can**
 - Reduce sidelobe levels...
 - ...at the expense of a wider mainlobe
- **All your favorites from ECE4270 apply**
 - Bartlett, Dolph-Chevshv, Hamming, Hann, Kaiser, etc.
 - Just use them in space (ECE6279) instead of time (ECE4270)

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Integrating Across Apertures

- Here's one way aperture smoothing functions show up
- Typically integrate across the aperture

$$z(t) = \int_{-\infty}^{\infty} w(\vec{x}) f(\vec{x}, t) d\vec{x}$$

- “Input” a monochromatic plane wave to the “system”

$$f(x, t) = \exp\{j(\omega_0 t - \vec{k}^0 \cdot \vec{x})\}$$

$$z(t) = \exp(j\omega_0 t) \underbrace{\int_{-\infty}^{\infty} w(\vec{x}) \exp(-j\vec{k}^0 \cdot \vec{x}) d\vec{x}}_{W(-\vec{k}^0)}$$

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