

Spring 2011, Aaron Lanterman

ECE 6279: Spatial Array Processing Homework 1

Due date: Friday 1/28/11 for on-campus students, Friday 2/4/11 for distance learning students. Homeworks are due at the *start* of class; homeworks turned in significantly later in the hour may be penalized at my discretion.

Late due date (30% penalty): Monday 1/31/11 for on-campus students; Monday 2/7/11 for distance learning students. (Again, if you need to use this late option, your homeworks are due at the *start* of class.)

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, **your solutions should be your own work and should be written up by yourself**; feel free to discuss things, but **don't be looking at someone else's paper when you are writing your solution**. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

Looking at solutions to homeworks and quizzes from previous offerings of ECE6279 is expressly forbidden. Look, here I am expressing how forbidden it is. Forbidden! Forbidden!!!

A 30% penalty will be assessed on late homeworks (even homeworks turned in later the day it is due); I will distribute solutions to students in the different sections (on-campus or distance learning) shortly after class on the associated "late option" due date, so I will not accept solutions after that. The "late option" turn in is available as an emergency buffer in case you get overwhelmed with life in general, but try not to rely on it. If you cannot make a class, try to make arrangements to get your homework to me ahead of time. (If you need to give your homework to a friend to deliver, please exercise judgement and caution; I have had problems in the past when the "friend" tasked with delivery directly copied from the homework they were tasked to deliver, without the original source's knowledge. This sort of thing tends to break up friendships.)

1 Required Problems

1. Problem 2.2 from J & D, with the following changes and observations:
 - Don't bother to "Show that indeed the formula given previously does correspond to a propagating signal." We already more-or-less showed that in class.
 - The "sketching" parts (d) and (e) are both asking for top-down, 2-D sketches of the (x, y) plane.
 - As Dan Fuhrmann noted when I took his array signal processing class at Washington Univ., part (e) is "a little ambiguous since a square pulse cannot propagate spherically in a way that the radial function $p(t, r)$ is flat at all times t . However we can assume that $s(r, t)$ is derived from a well-defined pulse, as described at the bottom p. 17."

2. Problem 2.3 from J & D, with the following hints and tweaks:

- To keep our notation consistent, denote the sinusoidal plane wave as $s(\vec{x}, t) = \cos(\omega_0 t - \vec{k} \cdot \vec{x} + \varphi)$. We've mostly been talking about complex exponentials in class, but don't let the real sinusoid scare you. This problem is just intended to get you thinking about sampling waves and to develop some intuition; you don't need any super-fancy theory from class to reason your way through it.
- On part (c): Show that you can now bound $|k_x|$, but there are two possible k_y for most choices of k_x .
- Hint on part (d): You may find it convenient to draw lines on a $k_x - \varphi$ plane, and see where lines intersect.
- Another hint on part (d): There are *four* possible directions of propagation.
- For part (e), if you don't like the cosine, you can try a complex exponential plane wave (as from class) instead, and meditate upon $s_1 s_2^*$; I think the solution is a little clearer that way, but your mileage may vary.

3. Problem 2.4 from J & D, with the following observations:

- In part (b), it's impossible to find a closed form solution. Show how you could find solutions graphically by arranging your necessary condition so that you have kr on one side of the equation and a cotangent on the other; you could then plot the cotangent and the line, and see where they intersect. (I'm not asking for any actual numeric values; just show me how you could do it.)
- In part (c), be careful about interpreting the critical points; notice that the maxima and minima alternate. You may need to spend some time meditating upon the nature of the cotangent function, and where the line intersects it for large r .