Spring 2011, Aaron Lanterman

ECE 6279: Spatial Array Processing
Homework 7

Due date: Friday 4/1/11 for on-campus students, Friday 4/8/11 for distance learning students. Homeworks are due at the *start* of class; homeworks turned in significantly later in the hour may be penalized at my discretion.

Late due date (30% penalty): Monday 4/4/11 for on-campus students; Monday 4/11/11 for distance learning students. (Again, if you need to use this late option, your homeworks are due at the *start* of class.)

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, **your solutions should be your own work and should be written up by yourself**; feel free to discuss things, but **don't be looking at someone else's paper when you are writing your solution**. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

Looking at solutions to homeworks and quizzes from previous offerings of ECE6279 is expressly forbidden. Look, here I am expressing how forbidden it is. Forbidden! Forbidden!!!

1 Required Problems

1. Suppose we have a two-element linear array lying along the x axis, separated by a distance Δ. We could apply the ESPRIT algorithm to data from this array if we only have one source. Since each subarray consists of a single element, the various steps of the ESPRIT algorithm simplify drastically.

   Suppose we collect some data and compute the usual unconstrained empirical covariance estimate $\hat{\mathbf{R}}$. Let $\mathbf{v}_0$ denote the "large eigenvector" and $\mathbf{v}_1$ denote the "small eigenvector" of $\hat{\mathbf{R}}$. Denote the elements of the eigenvector matrix as

$$\mathbf{V} = [\mathbf{v}_0 \quad \mathbf{v}_1] = \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix}$$

   Find the ESPRIT estimate of our usual polar angle ϕ in terms of the elements of the eigenvector matrix shown above. Note that in this simple case where each subarray is a single element, we do not need to invoke least squares or total least squares to find Ψ (see your notes from class); *simple division will suffice!*

2. On the website, you will find a file called `hw7_data11.mat`. This consists of a MATLAB array called `real_data` that has 128 snapshots from a linear array of 32 equally spaced elements (with half-wavelength spacing) along the x-axis, centered around the origin. The scene contains four sources, all with azimuthal angles $\theta = 0$; you may assume that the number of sources is known. The data was simulated using the usual independent, identically distributed electronic receiver noise.

   (a) Estimate the incident angles ϕ using root MUSIC.

   (b) Estimate the incident angles ϕ using total least squares ESPRIT.

   (Note: It wouldn't hurt to go ahead and estimate the angles using one of the techniques you've already implemented, such as regular MUSIC, the Eigenvalue method, MVDR, or whatever. This isn't required, but it might provide a "sanity check" on your answers.)

   As usual, please include listings of your programs.