**Spring 2011, Aaron Lanterman**

# ECE 6279: Spatial Array Processing
# Homework 8

**Due date:** Friday 4/22/11 for on-campus students, Friday 4/29/11 for distance learning students. Homeworks are due at the *start* of class; homeworks turned in significantly later in the hour may be penalized at my discretion.

**Late due date (30% penalty):** Monday 4/25/11 for class for on-campus students; Monday, 5/2/11 for distance learning students. (Again, if you need to use this late option, your homeworks are due at the *start* of class.)

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, **your solutions should be your own work and should be written up by yourself**; feel free to discuss things, but **don't be looking at someone else's paper when you are writing your solution**. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

**Looking at solutions to homeworks and quizzes from previous offerings of ECE6279 is expressly forbidden. Look, here I am expressing how forbidden it is. Forbidden! Forbidden!!!**

## 1 Required Problems

1. In class, we derived an expression for the optimal weights derived from constrained optimization algorithms that employ uncertainty constraints on the signal model:

$$\mathbf{w}_\diamond = -\lambda_1 \left( \mathbf{R} - \frac{\lambda_1^2}{\lambda_2} \mathbf{I} \right)^{-1} \mathbf{e} \tag{1}$$

However, we did not fully explore the difficulties of explicitly finding the Lagrange multipliers associated with this problem.

(a) Show that $\mathbf{w}_\diamond^H \mathbf{R} \mathbf{w}_\diamond = -\lambda_1$, thereby indicating that this multiplier is negative. (Hint: it's easier to use the gradients of the Lagrangian and the constraints than it is to use equation (1).)

(b) Assuming that the spatial correlation matrix has the simple form $\mathbf{R} = \sigma^2 \mathbf{I}$, use part (a) and equation (1) to show that

$$M\sigma^2 = \left( \sigma^2 - \frac{\lambda_1^2}{\lambda_2} \right)^2 / (-\lambda_1).$$

(c) Use the constraints and the gradients of the Lagrangian to show that under the same conditions as in part (b), the other simultaneous equation for the

Lagrange multipliers is

$$\lambda_1 = -\sigma^2 \epsilon^2 \left( \frac{\lambda_2^2}{\lambda_1^2} \right).$$

2. Let's play some more with the 9-element array we used in Homeworks 4 and 6; hence, your the code from those homeworks may come in handy here.

   To make sure there's not too much variability from solution to solution, let's use 300 snapshots to form our covariance estimate. (There might still be some variability, so you may want to run the experiment a few times to watch overall trends and make sure you didn't get just one "lucky" case. You need only turn in one set of plots, though.)

   Let's have two sources, one at $\phi = 45°$ and $\theta = 30°$ (call it Source A), and another at $\phi = 45°$ and $\theta = 60°$ (call it Source B). Let's let each have power 1/4 (so you'll want to multiply your steering vectors by 1/2 when making the "seen by array" variables), and let's make the noise have power 2 (as in the last assignment). Notice we're really pushing our algorithms hard now; the signals are pretty weak compared to the noise.

   To avoid a big complicated multidimensional optimization problem, we'll plot log-likelihoods keeping most of the parameters fixed.

   So that we can make "apples to apples" comparisons, do each of the problems below using the same simulated data set. That makes sure that any difference you observe between, say, the graph you make in Part (a) and the graph you make in Part (b) is due to different techniques, and not due to variations in the data.

   As usual, please provide printouts of your code. (Don't bother including a separate printout for every little parameter change).

   (a) For the "deterministic signal" Gaussian model given in Lecture 27, the single-source case reduces to conventional beamforming. We already know how the conventional beamformer acts on this data set, having experimented with it in Homework 4.

   So, let's play with the case where we assume *two* sources.

      i. Fix the parameters of one source to be $\phi = 45°$ and $\theta = 30°$ (corresponding to Source A), and then plot the trace on the second-to-last slide ("Maximum Likelihood Procedure") of Lecture 27 as a function of the second source azimuth $\theta$ for $0 \leq \theta < 360°$ (with $\phi$ of the second source fixed at $\phi = 45°$).

      ii. Repeat part (a), except change the fixed source parameters to be $\phi = 45°$ and $\theta = 60°$ (corresponding to Source B).

      iii. Repeat part (a), except change the fixed source parameters to be $\phi = 45°$ and $\theta = 45°$. Note this doesn't correspond with either source! This experiment lets us get a feel for what the loglikelihood surface looks like for one source when we get the position of the other source *wrong*.

(b) Repeat Part (a), except here plot the expression on the last slide of Lecture 28 ("Asymptotic ML Procedure").

(c) Plot the expression on the last slide of Lecture 28 assuming a *single-target* model, as a function of $\theta$ for $0 \leq \theta < 360°$, for $\phi = 45°$.

Looking at the results of your experiments, **comment on any interesting phenomena you observe.**