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# ***Conventional Wideband Beamforming***

**ECE 6279: Spatial Array Processing  
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Lecture 11**

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# Where We Are in J&D

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- **Lecture material drawn from:**
  - Sec. 4.4 (all), with some of Aaron's interpretation



# When is the Narrowband Model OK?

$$s^+(t) = s_b(t) \exp(j\omega_0 t)$$

- **Narrowband assumption** means that:

$$s_b(t - \tau) \approx s_b(t) \text{ for the largest possible } \tau$$

- For a plane wave with speed  $c$ :

$$\tau = \frac{1}{c} \max_m \{ |\vec{x}_m| \}$$

Aaron's idea

$$BW = O\left(\frac{1}{\tau}\right) = O\left(\frac{c}{\max_m \{ |\vec{x}_m| \}}\right)$$



# Inspiration (1)

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- **Recall basic delay-and-sum beamformer:**

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

- **Taking FT on both sides, we see the delay corresponds to a phase shift in Fourier space:**

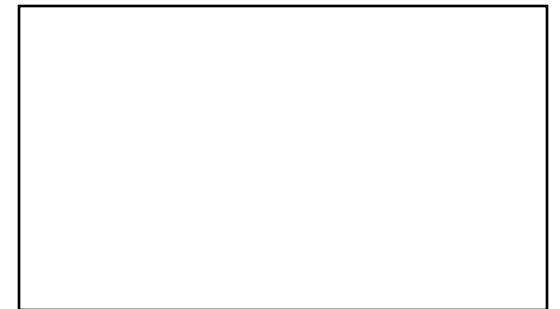
$$Z(\omega) = \sum_{m=0}^{M-1} w_m Y_m(\omega) \exp(-j\omega\Delta_m)$$



## Inspiration (2)

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- **Problem: computing  $Y_m(\omega)$  would require  $y_m(t)$  for **all** time**
- **Even if we had it, sources would move during the observation**



# Short-Time Fourier Transforms

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- **Solution: compute short-time FT**

$$Y_m(t, \omega) \equiv \int_t^{t+D} \tilde{w}(\tau - t) y_m(\tau) \exp(-j\omega\tau) d\tau$$

- **Substitute:**  $\tau' = \tau - t$   $\tau' + t = \tau$   $d\tau' = d\tau$

$$Y_m(t, \omega) = \int_0^D \tilde{w}(\tau) y_m(t + \tau) \exp\{-j\omega(t + \tau)\} d\tau$$

$$Y_m(t, \omega) e^{j\omega t} = \int_0^D \tilde{w}(\tau) y_m(t + \tau) \exp(-j\omega\tau) d\tau$$

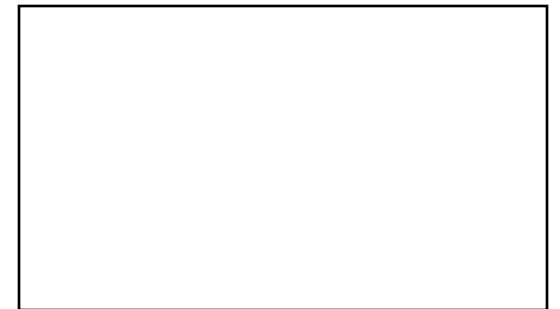


# Interpretation of the STFT

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$$Y_m(t, \omega) e^{j\omega t} = \int_0^D \tilde{w}(\tau) y_m(t + \tau) \exp(-j\omega\tau) d\tau$$

- $Y_m(t, \omega)$  “is a complex-valued lowpass signal that approximates the ‘local’ spectrum of the sensor’s output at time  $t$  and frequency  $\omega$ ” (J&D, p. 134)



# Frequency-Domain Beamforming

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- Do the delay-and-sum beamforming (equates to phase shift) **on the STFT:**

$$Z(\omega, t) \equiv \sum_{m=0}^{M-1} w_m Y_m(t, \omega) \exp(j\omega t) \exp(-j\omega \Delta_m)$$

(Usually, this is what we're interested in)





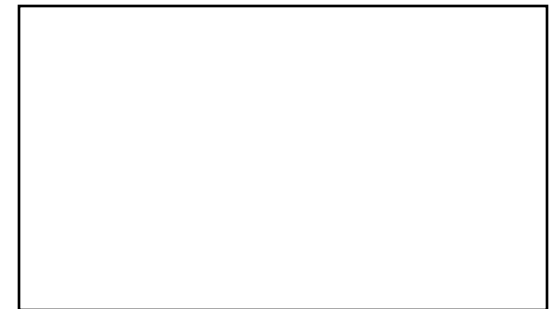
# Filtering in Time

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$$Z(\omega, t) \equiv \sum_{m=0}^{M-1} w_m(\omega) Y_m(t, \omega) \exp(j\omega t) \exp(-j\omega \Delta_m)$$

↑

- We can let the weights be a function of frequency
- Gives us frequency-domain counterpart of (temporal filter)-and-sum beamforming



# Beamforming for Plane Waves

$$\begin{aligned} Z(\omega, t) \exp(-j\omega t) &= \\ & \sum_{m=0}^{M-1} w_m Y_m(t, \omega) \exp(-j\omega \Delta_m) \\ &= \sum_{m=0}^{M-1} w_m Y_m(t, \omega) \exp\{j\vec{k}(\omega) \cdot \vec{x}_m\} \end{aligned}$$

$\uparrow$   
 $-\vec{\alpha} \cdot \vec{x}_m$

**We'll see the  $\exp(-j\omega t)$   
part isn't too important**



# Wideband Vector Notation

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$$\sum_{m=0}^{M-1} w_m Y_m(t, \omega) \exp(j \vec{k}(\omega) \cdot \vec{x}_m) \\ = \mathbf{e}^H(\vec{k}(\omega)) \mathbf{W} \mathbf{Y}(t, \omega)$$

$$\mathbf{W} \mathbf{Y} = \begin{bmatrix} w_0 Y_0(t, \omega) \\ \vdots \\ w_{M-1} Y_{M-1}(t, \omega) \end{bmatrix}$$

$$\mathbf{W} = \text{diag}(w_0, \dots, w_{M-1})$$



# Wideband Beamforming Strategy

- **Steered response power** (implicit integral over  $t$ ):

$$\begin{aligned} \int_{-\infty}^{\infty} |Z(\omega, t)|^2 d\omega &= \int_{-\infty}^{\infty} |Z(\omega, t) \exp(-j\omega t)|^2 d\omega \\ &= \int_{-\infty}^{\infty} \left| \mathbf{e}^H(\vec{k}(\omega)) \mathbf{W} \mathbf{Y}(t, \omega) \right|^2 d\omega \\ &= \int_{-\infty}^{\infty} \mathbf{e}^H(\vec{k}(\omega)) \mathbf{W} \underbrace{\mathbf{Y}(t, \omega) \mathbf{Y}^H(t, \omega)}_{\mathbf{R}(t, \omega)} \mathbf{W}^H \mathbf{e}(\vec{k}(\omega)) d\omega \end{aligned}$$

**spatial correlation matrix**

- **J&D drops dependence on  $t$ , but reintroduces it in Sec. 4.9**

