
Pisarenko Harmonic Decomposition

**ECE 6279: Spatial Array Processing
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Lecture 18**

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Where We Are in J&D

- **Section 7.2.5**



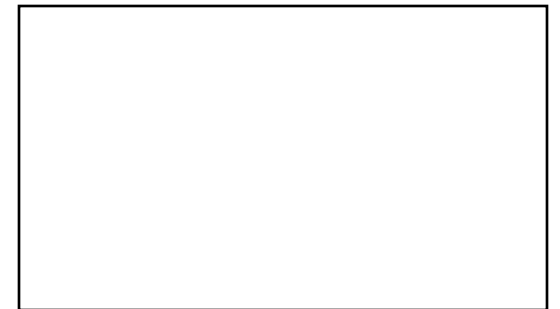
Previous “Minimum Variance” Techniques

- **General linear constraint**

$$\mathbf{w}_{\diamond} = \arg \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_y \mathbf{w} \text{ s.t. } \mathbf{C}\mathbf{w} = \mathbf{c}$$

- **“Distortionless Response” constraint**

$$\mathbf{w}_{\diamond} = \arg \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_y \mathbf{w} \text{ s.t. } \mathbf{e}^H (\vec{k}) \mathbf{w} = 1$$

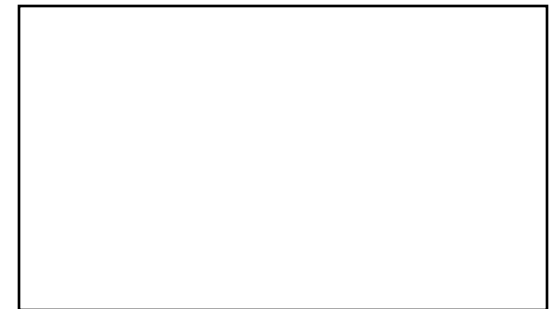


New “Minimum Variance” Technique

- **Power constraint on weights**

$$\mathbf{w}_{\diamond} = \arg \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_y \mathbf{w} \text{ s.t. } \mathbf{w}^H \mathbf{w} = 1$$

- **Philosophy: To find where signals are, look for where they aren't**



The Lagrangian

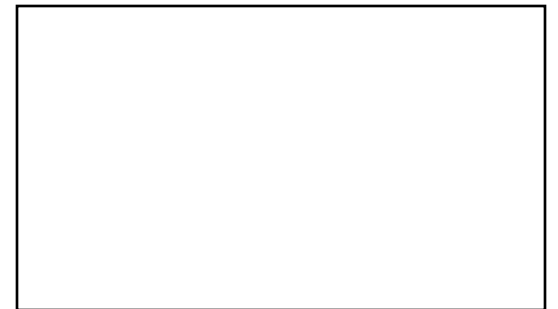
- **Power constraint on weights**

$$L(\mathbf{w}, \lambda) = \mathbf{w}^H \mathbf{R}_y \mathbf{w} + \lambda (\mathbf{w}^H \mathbf{w} - 1)$$

$$\nabla_{\mathbf{w}^*} L(\mathbf{w}, \lambda) = \mathbf{R}_y \mathbf{w} + \lambda \mathbf{w} = 0$$

$$\mathbf{R}_y \mathbf{w} = -\lambda \mathbf{w}$$

- **\mathbf{w} must be an eigenvector of \mathbf{R}_y**



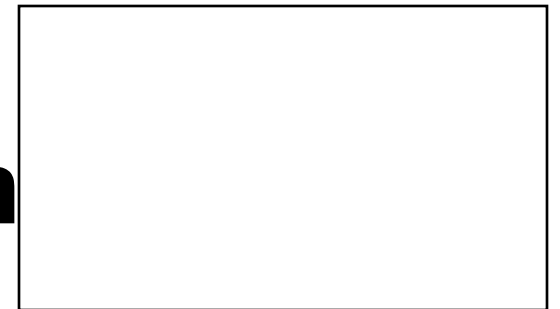
Eigenvector/Eigenvalue Analysis

$$\mathbf{w}^H \mathbf{R}_y \mathbf{w} = \sum_{m=1}^M \lambda_m \left| \mathbf{w}^H \mathbf{v}_m \right|^2$$

- To minimize $\mathbf{w}^H \mathbf{R}_y \mathbf{w}$, pick the eigenvector \mathbf{v}_{\min} corresponding to the smallest eigenvalue of \mathbf{R}_y

$$\mathbf{w} = \mathbf{v}_{\min}$$

- In practice, must use an estimate $\hat{\mathbf{R}}_y$



Strategy

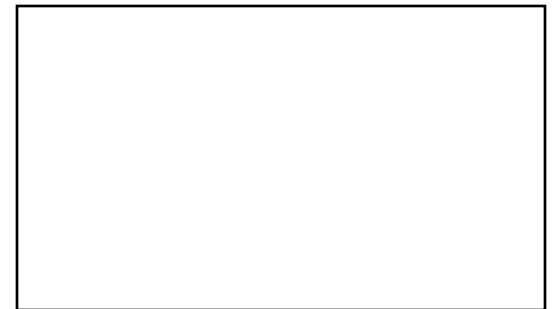
$\left| \mathbf{v}_{\min}^H \mathbf{e}(\vec{k}) \right|$ is small when \vec{k}
corresponds to an actual signal

$\frac{1}{\left| \mathbf{v}_{\min}^H \mathbf{e}(\vec{k}) \right|}$ is large when \vec{k}
corresponds to
an actual signal



Warnings

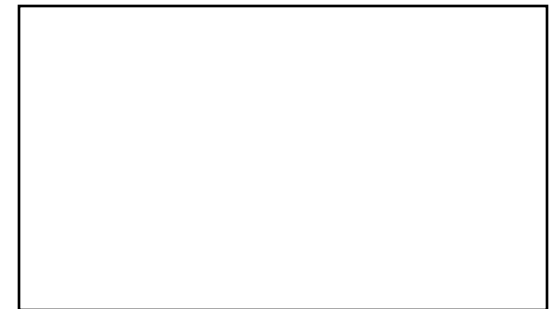
- **Spurious peaks**
- **Peak values have little to do with actual signal amplitudes**
- **Resolution depends on quality of estimate of R_y**
- **Helps to know the number of targets ahead of time**



Rank One Approximations

$$\mathbf{R}_y = \sum_{m=1}^M \lambda_m \mathbf{v}_m \mathbf{v}_m^H \approx \lambda_{\max} \mathbf{v}_{\max} \mathbf{v}_{\max}^H$$

$$\mathbf{R}_y^{-1} = \sum_{m=1}^M \frac{1}{\lambda_m} \mathbf{v}_m \mathbf{v}_m^H \approx \frac{1}{\lambda_{\min}} \mathbf{v}_{\min} \mathbf{v}_{\min}^H$$



Comparisons and Interpretations

$$P^{CONV}(\vec{k}) \equiv \mathbf{e}^H(\vec{k}) \mathbf{R}_y \mathbf{e}(\vec{k})$$

$$P^{MV}(\vec{k}) \equiv \left[\mathbf{e}^H(\vec{k}) \mathbf{R}_y^{-1} \mathbf{e}(\vec{k}) \right]^{-1}$$

$$P^P(\vec{k}) \equiv \left| \mathbf{v}_{\min}^H \mathbf{e}(\vec{k}) \right|^{-2}$$

$$= \left[\mathbf{e}^H(\vec{k}) \underbrace{\mathbf{v}_{\min} \mathbf{v}_{\min}^H}_{\text{rank-1}} \mathbf{e}(\vec{k}) \right]^{-1}$$

proportional to a rank-
one approximation of \mathbf{R}_y^{-1}

