

Delay-and-Sum Beamforming for Spherical Waves

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Lecture 7

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Where We Are in J&D

- Lecture material drawn from:
 - Sec. 4.1.3
 - Sec. 4.2.1 (“Point Focusing” part on p. 123)
- Please read section 4.1.1 on your own

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Delay-and-Sum Beamforming

- Array of M sensors at positions $\vec{x}_0 \dots \vec{x}_{M-1}$
- For convenience, put the phase center at the origin

$$\frac{1}{M} \sum_{m=0}^{M-1} \vec{x}_m = \vec{0}$$

- Delay-and-sum beamforming

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

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Spherical Waves at the Sensors

Source location
↓

$$f(\vec{x}, t) = \frac{s(t - |\vec{x} - \vec{x}^0|/c)}{|\vec{x} - \vec{x}^0|}$$

$$y_m(t) = \frac{s(t - |\vec{x}_m - \vec{x}^0|/c)}{|\vec{x}_m - \vec{x}^0|} = \frac{s(t - r_m^0/c)}{r_m^0}$$

Distance between source and sensor m

where $r_m^0 = |\vec{x}_m - \vec{x}^0|$

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Delay-and-Sum for Spherical Waves

$$y_m(t) = \frac{s(t - r_m^0/c)}{r_m^0}$$

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

$$= \sum_{m=0}^{M-1} w_m \frac{s(t - \Delta_m - r_m^0/c)}{r_m^0}$$

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Matched Delays (1)

$$z(t) = \sum_{m=0}^{M-1} w_m \frac{s(t - \Delta_m - r_m^0/c)}{r_m^0}$$

- If we pick $\Delta_m = (r^0 - r_m^0)/c$ then we get
- Distance between source and origin

$$z(t) = \sum_{m=0}^{M-1} w_m \frac{s(t - r^0/c)}{r_m^0}$$

$$= \frac{s(t - r^0/c)}{r^0} \left[\sum_{m=0}^{M-1} w_m \frac{r^0}{r_m^0} \right]$$

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Matched Delays (2)

$$z(t) = \underbrace{\frac{s(t - r^0/c)}{r^0}}_{\text{Original signal, delayed and attenuated}} \left[\sum_{m=0}^{M-1} w_m \frac{r^0}{r_m^0} \right]_{\text{Approaches sum of weights as } r^0, r_m^0 \rightarrow \infty}$$

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Mismatched Delays

$$z(t) = \sum_{m=0}^{M-1} w_m \frac{s(t - \Delta_m - r_m^0/c)}{r_m^0}$$

Distance between assumed source and origin

Distance between assumed source and sensor m

- In general, if we pick $\Delta_m = (r - r_m^0)/c$ then we get

$$z(t) = \sum_{m=0}^{M-1} \frac{w_m}{r_m^0} s\left(t - \frac{r - (r_m^0 - r^0)}{c}\right)$$

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Beamforming Strategy

$$\Delta_m = (r - r_m) / c$$

$$z(t) \equiv \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

$$= \sum_{m=0}^{M-1} w_m y_m\left(t - \frac{r - r_m}{c}\right)$$

- Find the x which gives the most energy in $z(t)$

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Time Signal Seen at the Origin

$$s_0(t) = \frac{s(t - r^0 / c)}{r^0}$$

$$S_0(\omega) = \frac{1}{r^0} S(\omega) \exp(-j\omega r^0 / c)$$

$$r^0 S_0(\omega) \exp(j\omega r^0 / c) = S(\omega)$$

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Output in Fourier Domain (1)

$$z(t) = \sum_{m=0}^{M-1} \frac{w_m}{r_m^0} s\left(t - \frac{r - (r_m - r_m^0)}{c}\right)$$

Take FT of both sides

$$Z(\omega) = \sum_{m=0}^{M-1} \frac{w_m}{r_m^0} S(\omega) \times$$

$$\exp\left[-j\omega \frac{r - (r_m - r_m^0)}{c}\right]$$

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Output in Fourier Domain (2)

$$Z(\omega) = \sum_{m=0}^{M-1} \frac{w_m}{r_m^0} S(\omega) \exp\left[-j\omega \frac{r - (r_m - r_m^0)}{c}\right]$$

$$S(\omega) = r^0 S_0(\omega) \exp(j\omega r^0 / c)$$

$$Z(\omega) = \sum_{m=0}^{M-1} w_m \frac{r^0}{r_m^0} S_0(\omega) \times$$

$$\exp\left[j\omega \left\{ \frac{(r^0 - r) - (r_m^0 - r_m)}{c} \right\}\right]$$

$$k = \omega / c$$

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Array-Pattern-Like Thing

$$Z(\omega) = S_0(\omega) \widehat{W}(k, \vec{x}, \vec{x}^0)$$

Where $\widehat{W}(k, \vec{x}, \vec{x}^0) \equiv$ $k = \omega/c$

$$\sum_{m=0}^{M-1} w_m \frac{r^0}{r_m} \exp[jk\{(r^0 - r) - (r_m^0 - r_m)\}]$$

$\widehat{W}(k, \vec{x}, \vec{x}^0)$ plays a role analogous to $W(\vec{\omega}^0 \vec{\alpha} - \vec{k}^0)$ in previous lecture on beamforming for plane waves

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When Perfectly Focused

- If $\vec{x} = \vec{x}^0$, i.e., $r^0 = r$ and $r_m^0 = r_m$

$$\widehat{W}(k, \vec{x}, \vec{x}^0) \equiv$$

$$\sum_{m=0}^{M-1} w_m \frac{r^0}{r_m} \exp[jk\{(r^0 - r^0) - (r_m^0 - r_m^0)\}]$$

$$= \sum_{m=0}^{M-1} w_m \frac{r^0}{r_m} \longrightarrow z(t) = s_0(t) \left[\sum_{m=0}^{M-1} w_m \frac{r^0}{r_m} \right]$$

We copy the signal exactly - no filtering!

Now show movie

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