Fall 2013, Aaron Lanterman

ECE 6279: Spatial Array Processing
Homework 5

Due date: Tuesday, October 22, at the start of class.

Late due date (30% penalty): Thursday, October 24. (Again, if you need to use this late option, your homeworks are due at the *start* of class.)

You are welcome to discuss approaches to the problems and solutions to difficulties you encounter with one another and with others outside the class. You can and should learn from each other as much as, and even more than, you learn from me. However, **your solutions should be your own work and should be written up by yourself**; feel free to discuss things, but **don't be looking at someone else's paper when you are writing your solution**. It's too easy to freeload that way and not learn anything. See the class website for more guidelines.

Looking at solutions to homeworks and quizzes from previous offerings of ECE6279 is expressly forbidden. Look, here I am expressing how forbidden it is. Forbidden! Forbidden!!!

A 30% penalty will be assessed on late homeworks (even homeworks turned in later the day it is due, at my discretion); I will distribute solutions to students in the different sections (on-campus or distance learning) shortly after class on the associated "late option" due date, so I will not accept solutions after that. If you cannot make a class, try to make arrangements to get your homework to me ahead of time.

1 Required Problems

1. Consider a linear four-element array with elements at $x = -3, -1, 0$ and 2. (Note there isn't an element at $x = 1$ and $x = -2$.)

   (a) Find the co-array, along with the coarray values. Give your answer by plotting the co-array values along the vertical axis, with the postition in the coarray indicated by the horizontal axis. (To do problems like this, I find it convenient to draw out the array with little marks, draw out its mirror image, and then manually convolve the two, building up marks in the answer as I go along. Your mileage may vary.)

   (b) Is the array redundant, i.e., are there any non-zero lags with a co-array value greater than 1?

2. Here, we'll consider the coarray of a two-dimensional array. Suppose the array has five elements, at (0,0), (1,0), (4,0), (0,1), and (0,4).

   (a) What is the *coarray value* of the central lag (0,0)? Explain how you could find this without doing very much work, i.e. without having to do part (b).

   (b) Sketch a top-down view of the coarray. (We won't be able to use a vertical axis to show the coarray values like we did on the homework, so just use dots or small crosses to indicate the points of the coarray. You do not need to indicate the coarray values.)

   (c) Is this array redundant?

3. Consider a linear 7-element array with elements at $x = -4, -3, -1, 0, 1, 3, 4$. (Note there aren't elements at $x = \pm 2$.) Find three compatible subarrays of three elements each that you could use to do subaperture averaging.

4. Consider a function of a complex-valued vector defined by $f(\mathbf{z}) = \mathbf{z}^H\mathbf{A}\mathbf{z} + 2\text{Re}\{\mathbf{b}^H\mathbf{z}\}$, where $\mathbf{A}$ is Hermitian. Find the critical point of $f(\mathbf{z})$. (There is not enough information in this problem to tell whether this critical point is a minimum, a maximum, or a saddle point, which is why I'm using the word "critical point").