Apertures, Part II

ECE 6279: Spatial Array Processing
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Lecture 5

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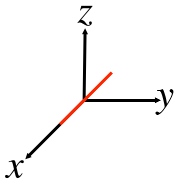
Where We Are in J&D

- **Lecture material drawn from:**
  - –Sec. 3.3, 3.3.1-3.3.2
- **Might be a good idea to read through Sec. 3.2**
  - –Reviews sampling theory

Last Time: Filled Linear Aperture

$$b(x) = \begin{cases} 1, & |x| \le D/2 \\ 0, & \text{otherwise} \end{cases}$$

$$w(\vec{x}) = b(x)\delta(y)\delta(z)$$

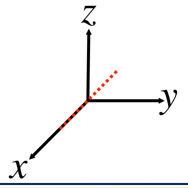


Regularly Sampled Linear Aperture

- **Odd number of sensors *M*, w/spacing *d***
- **Total length *D=Md***

$$b(x) = \sum_{m=-(M-1)/2}^{(M-1)/2} \delta(x - md)$$

$$w(\vec{x}) = b(x)\delta(y)\delta(z)$$



Aperture Smoothing Function (1)

$$W(k_x) = \int_{-\infty}^{\infty} b(x)\exp(\oplus jk_x x)\,dx$$

$$= \int_{-\infty}^{\infty} \sum_{m=-(M-1)/2}^{(M-1)/2} \delta(x - md)\exp(jk_x x)\,dx$$

$$= \sum_{m=-(M-1)/2}^{(M-1)/2} \exp(jk_x md) \times \left[\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)\right]$$

$$\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)$$

Aperture Smoothing Function (2)

$$\sum_{m=\underbrace{-(M-1)/2}_{a}}^{(M-1)/2} \exp(jk_x md) \times \left[\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)\right]$$

- **Terms from first set:**

$$\boxed{\left\{a + \frac{1}{2}\right\}} \cancel{\left\{(a-1) + \frac{1}{2}\right\}} \cdots \cancel{\left\{-(a-1) + \frac{1}{2}\right\}} \cancel{\left\{-a + \frac{1}{2}\right\}}$$

- **Subtract terms from second set:**

$$\cancel{\left\{a - \frac{1}{2}\right\}} \cancel{\left\{(a-1) - \frac{1}{2}\right\}} \cdots \cancel{\left\{-(a-1) - \frac{1}{2}\right\}} \boxed{\left\{-a - \frac{1}{2}\right\}}$$

Our Old Friend, the *Sind*

$$W(k_x) = \frac{\exp\left(\frac{jk_x Md}{2}\right) - \exp\left(-\frac{jk_x Md}{2}\right)}{\exp\left(\frac{jk_x d}{2}\right) - \exp\left(-\frac{jk_x d}{2}\right)}$$

$$= \frac{\sin\left(\frac{k_x Md}{2}\right)}{\sin\left(\frac{k_x d}{2}\right)}$$

$$W(0) \equiv M$$

In Polar Wavenumber Coordinates

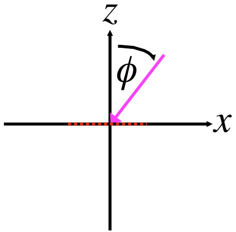
$$\frac{\sin\left(\frac{k_x Md}{2}\right)}{\sin\left(\frac{k_x d}{2}\right)} = \underbrace{\frac{\sin\left(\frac{k_r Md \sin\phi\cos\theta}{2}\right)}{\sin\left(\frac{k_r d \sin\phi\cos\theta}{2}\right)}}_{W(k_r,\phi,\theta)}$$

$$k_x = -k_r \sin\phi\cos\theta$$

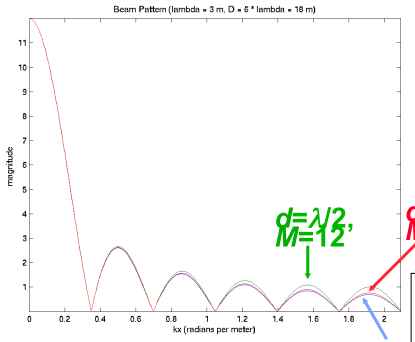
Limiting to the X-Z Plane

- **Suppose** $\theta = 0$

$$W\left(\frac{2\pi}{\lambda},\phi,0\right)=\underbrace{\frac{\sin\left(\frac{Md\pi}{\lambda}\sin\phi\right)}{\sin\left(\frac{d\pi}{\lambda}\sin\phi\right)}}_{W(\phi)}$$



Comparison (Wavenumber)



d=λ/2, *M*=12

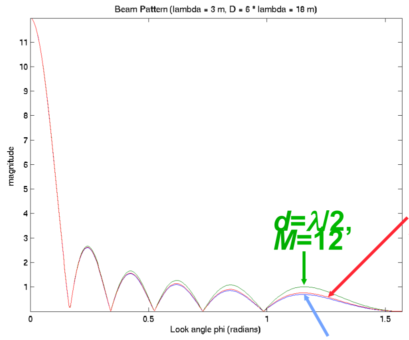
d=λ/4, *M*=24

Filled (sinc)

To study changes in overall shape, all plotted curves are artificially normalized to have gain 12 at angle 0

- 100 MHz wave
- 18 meter long aperture

Comparison (Phi)



d=λ/2, *M*=12

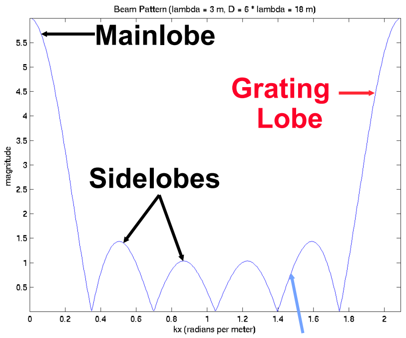
d=λ/4, *M*=24

Filled (sinc)

To study changes in overall shape, all plotted curves are artificially normalized to have gain 12 at angle 0

- 100 MHz wave
- 18 meter long aperture

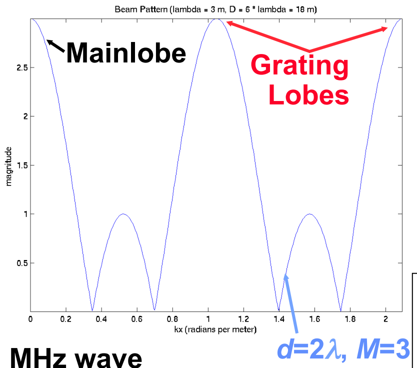
Grating Lobes



d=λ, *M*=6

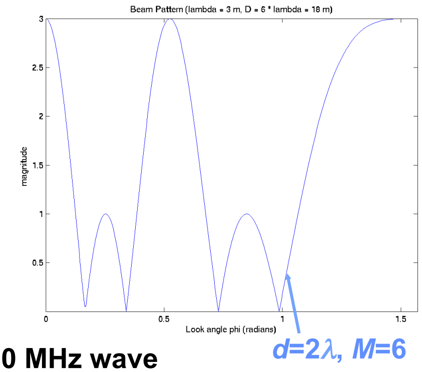
- 100 MHz wave
- 18 meter long aperture

More Grating Lobes



- **100 MHz wave**
- **18 meter long aperture**

Stretching of Grating Lobes



- **100 MHz wave**
- **18 meter long aperture**

Sampling Issues

- **When designing a uniform linear array, generally try to pick $d \leq \lambda/2$**
- **Can sometimes get away $d > \lambda/2$ with if you have side information**
  - Detect a target with a low frequency
    - **Long wavelength**
  - Then track it with a high frequency
    - **Short wavelength**
    - **Use knowledge from detector to disambiguate aliases**
- **Most papers will assume $d \leq \lambda/2$**
  - Usually assume $d = \lambda/2$

2-D & 3-D Apertures

- **Covered in detail by VT-IV, Chapter 4**
- **Can solve "cone of ambiguity" problem**
  - Careful: some 2-D apertures have their own kinds of cones of ambiguity
    - **i.e. circular disk**
- **Sampling issues get trickier to analyze**
  - Theory from ECE6258: Digital Image Processing applies here
    - **Hexagonal sampling, etc.**
  - Discussed in Sec. 3.2 of J&D

Windowing

- **By weighting an aperture by a window, we can**
  - Reduce sidelobe levels…
  - …at the expense of a wider mainlobe

- **All your favorites from ECE4270 apply**
  - Bartlett, Dolph-Chevshev, Hamming, Hann, Kaiser, etc.
  - Just use them in space (ECE6279) instead of time (ECE4270)

Integrating Across Apertures

- **Here's one way aperture smoothing functions show up**
- **Typically integrate across the aperture**

$$z(t) = \int_{-\infty}^{\infty} w(\vec{x}) f(\vec{x},t) d\vec{x}$$

- **"Input" a monochromatic plane wave to the "system"**

$$f(x,t) = \exp\{j(\omega_0 t - \vec{k}^0 \cdot \vec{x})\}$$

$$z(t) = \exp(j\omega_0 t) \underbrace{\int_{-\infty}^{\infty} w(\vec{x}) \exp(-j\vec{k}^0 \cdot \vec{x}) d\vec{x}}_{W(-\vec{k}^0)}$$