ECE4893A/CS4803MPG: Multicore and GPU Programming for Video Games

Arbitrary Rotations

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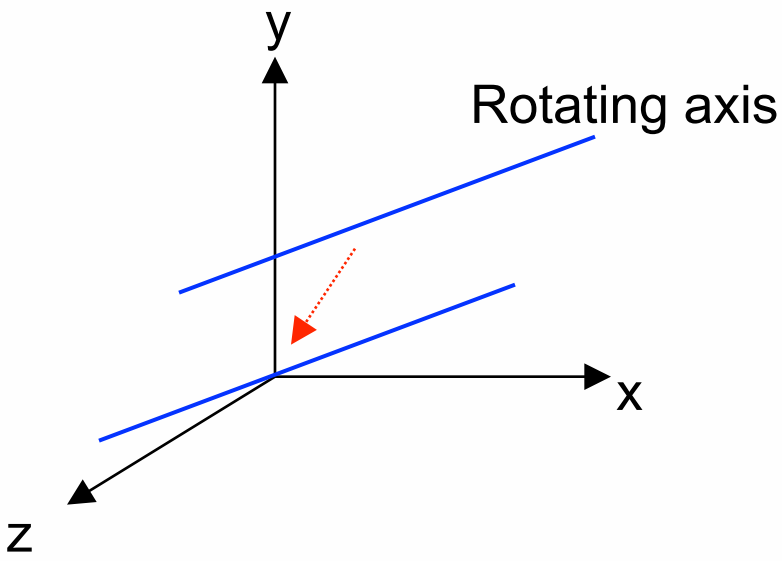
Rotation along an arbitrary axis

1. Transform the axis to cross origin (0,0,0)
2. Transform the axis around X onto the XZ plane
3. Transform the axis around Y onto the Z axis
4. Perform rotation (angle=θ)
5. Inverse transform of 3.
6. Inverse transform of 2.
7. Inverse transform of 1.

Step 1: Translate to (0, 0, 0)

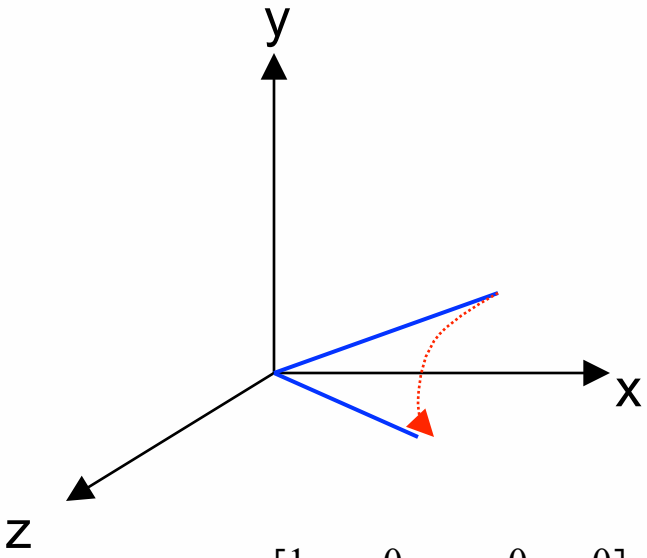


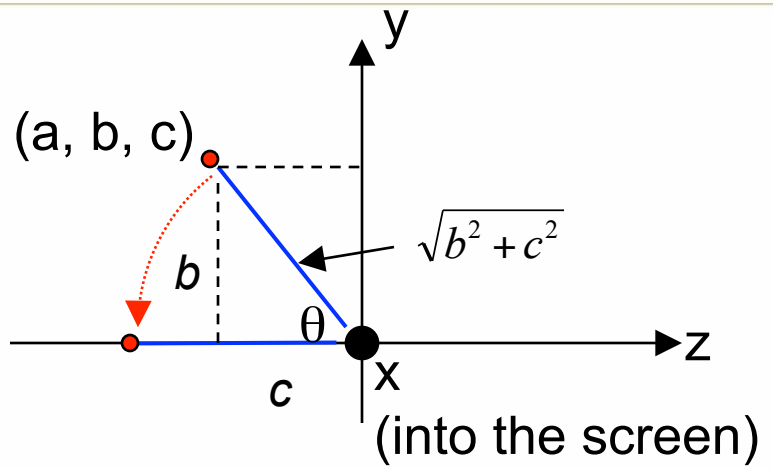
$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -Tx & -Ty & -Tz & 1 \end{bmatrix}$$

$$T_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Tx & Ty & Tz & 1 \end{bmatrix}$$

- For easier viewing, I use RHS.

Step 2: Rotate around X, project to XZ





$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos\theta = \frac{c}{\sqrt{b^2+c^2}} \quad \text{and} \quad \sin\theta = \frac{b}{\sqrt{b^2+c^2}}$$

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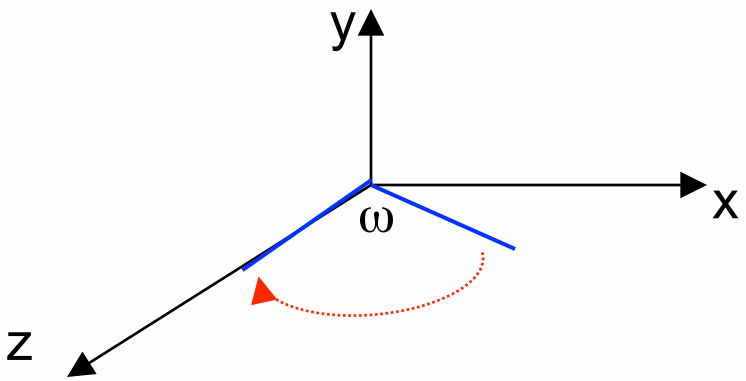
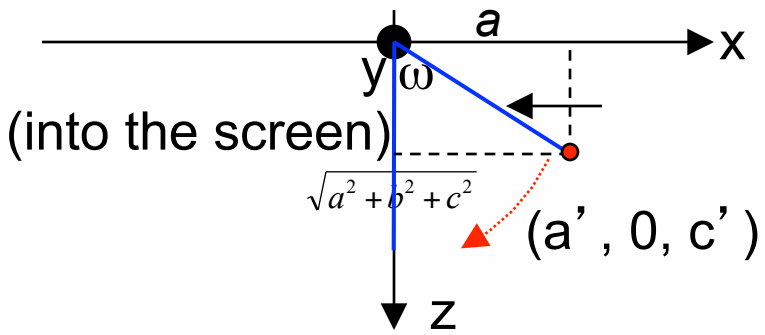
- Let's pick up a random point (a, b, c) on the axis to figure out cosθ

Step 3: Transform around Y onto the Z axis





$$T_3 = \begin{bmatrix} \cos\omega & 0 & -\sin\omega & 0 \\ 0 & 1 & 0 & 0 \\ \sin\omega & 0 & \cos\omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^{-1} = \begin{bmatrix} \cos\omega & 0 & \sin\omega & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\omega & 0 & \cos\omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos\omega = \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} \quad \text{and} \quad \sin\omega = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$\cos\omega = \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} \quad \text{and} \quad \sin\omega = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

Step 4: Perform Rotation (angle=θ) on Z

$$T_4 = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Combine all matrices with steps 5, 6, 7

$$[x', y', z', 1] = [x, y, z, 1] \cdot T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_3^{-1} \cdot T_2^{-1} \cdot T_1^{-1}$$