ECE4893A/CS4803MPG: Multicore and GPU Programming for Video Games

Arbitrary Rotations

Prof. Hsien-Hsin Sean Lee
School of Electrical and Computer Engineering
Georgia Institute of Technology

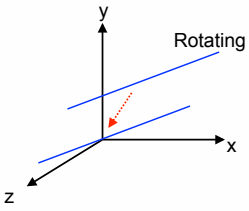
Georgia Institute of Technology

Rotation along an arbitrary axis

1. Transform the axis to cross origin (0,0,0)
2. Transform the axis around X onto the XZ plane
3. Transform the axis around Y onto the Z axis
4. Perform rotation (angle=θ)
5. Inverse transform of 3.
6. Inverse transform of 2.
7. Inverse transform of 1.

Georgia Institute of Technology

Step 1: Translate to (0, 0, 0)



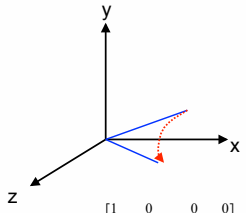
$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -Tx & -Ty & -Tz & 1 \end{bmatrix}$$

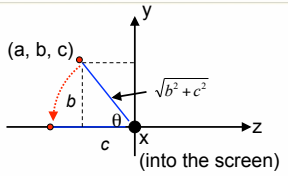
$$T_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Tx & Ty & Tz & 1 \end{bmatrix}$$

- For easier viewing, I use RHS.

Georgia Institute of Technology

Step 2: Rotate around X, project to XZ





$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

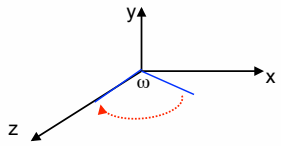
$$\cos\theta = \frac{c}{\sqrt{b^2+c^2}} \quad \text{and} \quad \sin\theta = \frac{b}{\sqrt{b^2+c^2}}$$

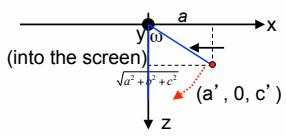
$$\cos\theta = \frac{c}{\sqrt{b^2+c^2}} \quad \text{and} \quad \sin\theta = \frac{b}{\sqrt{b^2+c^2}}$$

- Let's pick up a random point (a, b, c) on the axis to figure out cosθ

Georgia Institute of Technology

Step 3: Transform around Y onto the Z axis





$$T_3 = \begin{bmatrix} \cos\omega & 0 & -\sin\omega & 0 \\ 0 & 1 & 0 & 0 \\ \sin\omega & 0 & \cos\omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_3^{-1} = \begin{bmatrix} \cos\omega & 0 & \sin\omega & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\omega & 0 & \cos\omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos\omega = \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} \text{ and } \sin\omega = \frac{a}{\sqrt{a^2+b^2+c^2}} \qquad \cos\omega = \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} \text{ and } \sin\omega = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

Georgia Institute of Technology

Step 4: Perform Rotation (angle=θ) on Z

$$T_4 = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Georgia Institute of Technology

Combine all matrices with steps 5, 6, 7

$$[x', y', z', 1] = [x, y, z, 1] \cdot T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_3^{-1} \cdot T_2^{-1} \cdot T_1^{-1}$$

Georgia Institute of Technology