

# The general equilibrium incidence of environmental taxes<sup>☆</sup>

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## Abstract

We study the distributional effects of a pollution tax in general equilibrium, with general forms of substitution where pollution might be a relative complement or substitute for labor or for capital in production. We find closed form solutions for pollution, output prices, and factor prices. Various special cases help clarify the impact of differential factor intensities, substitution effects, and output effects. Intuitively, the pollution tax might place disproportionate burdens on capital if the polluting sector is capital intensive, or if labor is a better substitute for pollution than is capital; however, conditions are found where these intuitive results do not hold. We show exact conditions for the wage to rise relative to the capital return. Plausible values are then assigned to all the parameters, and we find that variations over the possible range of factor intensities have less impact than variations over the possible range of elasticities.

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Policy makers need to know the distributional effects of environmental taxes. Previous studies that find environmental taxes to be regressive have focused on the uses side of income; that is, how low-income consumers use a relatively high fraction of their income to buy gasoline, electricity, and other products that involve burning fossil fuel. Yet these studies ignore the sources side of income. Environmental policies can have important effects on firms' demands for capital and labor inputs, which can impact the returns to owners of capital and labor in a general equilibrium setting.

The literature in public economics contains much work on general equilibrium tax incidence, but the literature on environmental taxation has focused mostly on efficiency effects. As reviewed below, neither literature yet has studied the general equilibrium incidence of a pollution tax in a model with general forms of substitution. Environmental tax incidence has been studied only in partial equilibrium models, in computational general equilibrium (CGE) models, or in analytical general equilibrium models with limited forms of substitution. This paper provides a theoretical general equilibrium model of the incidence of an environmental tax that allows for differential factor intensities and fully general forms of substitution among inputs of labor, capital, and pollution. We show incidence on the sources side as well as the uses side.

Many empirical studies provide partial equilibrium analyses of the incidence of an environmental tax. For example, [Robison \(1985\)](#) examines the distribution of the costs of pollution abatement from 1973 to 1977 and finds regressive burdens equal to 1.09% of the income of the lowest income class and only 0.22% of income for the highest income class. Using CGE models, [Mayeres \(2000\)](#) and [Metcalf \(1999\)](#) look at various ways to return the revenue from an environmental tax, showing that these distributional effects can more than offset the incidence of the environmental tax itself. [Morgenstern et al. \(2002\)](#) discuss four CGE studies that examine various distributional effects of carbon policy, but none derive analytical results and none show effects on factor prices.<sup>1</sup>

Previous theoretical work on environmental tax incidence by [Rapanos \(1992, 1995\)](#) models pollution in one sector as a negative externality that affects production in the other sector. The model is somewhat restrictive in two respects. First, the externality has a specific effect on production in the other sector, which affects incidence. Second, Rapanos assumes that pollution bears a fixed relation to output (or to capital input) of the polluting sector, so a tax on pollution has the same incidence as a tax on output (or on capital input). In contrast, this paper models pollution as a variable input to the dirty sector's production function. In response to any price change, the producer can change the mix of labor, capital, and pollution. In particular, pollution can be a relative complement or substitute for labor or capital, so that a pollution tax can change the relative demands for those other two factors and affect their relative returns.

[Bovenberg and Goulder \(1997\)](#) examine the efficiency costs of a revenue-neutral environmental tax swap and also solve for the change in the wage rate. Their analytical model considers variable pollution, but the production function has a single elasticity of substitution among the three inputs (capital, labor, and pollution). This formulation does not allow for relative complementarity of inputs in production, a possibility that drives significant results below.<sup>2</sup> [Chua \(2003\)](#) presents a model where pollution is a scalar multiple of output, but it can be lowered by paying an abatement

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<sup>1</sup> Also, [West and Williams \(2004\)](#) use micro data to model demand equations and empirically estimate the distribution of burdens of environmental policy. [Parry \(2004\)](#) examines the distribution of the scarcity rents created by grandfathered emissions permits. In a model with unemployment, [Wagner \(2005\)](#) shows that an emissions tax can help labor to the extent that it stimulates employment in the abatement sector.

<sup>2</sup> [DeMooij and Bovenberg \(1998\)](#) allow for complementarity of inputs, and they derive the change in the wage rate, but their model is primarily used to examine the efficiency of revenue-neutral tax swaps. To the extent that they examine incidence, their results are somewhat limited by the fact that capital either has an exogenous price or is supplied inelastically in the polluting industry.

sector that also uses labor and capital. Because the effect on factor prices depends on use of factors in the abatement sector, this model effectively makes some restrictions on the ways that firms can substitute out of pollution and into other factors such as labor and capital.<sup>3</sup>

Our model does not fix pollution as a scalar multiple of output, nor of capital, nor does it posit a third sector for abatement. Rather, pollution is modeled as an input along with capital and labor. Minimal restrictions are placed on that production function, so that the model is free to consider that any pair of inputs may be complements or substitutes. We then solve the model in the style of Harberger (1962) to find closed form solutions for the general equilibrium responses to a change in the tax on pollution, in the presence of other taxes. The model allows for analyses of a wide variety of policies.

Some of the general results are complex and ambiguous, so special cases are used to provide intuition. In the case where the two sectors have equal capital/labor ratios, for example, an increase in the pollution tax unambiguously raises the price of the dirty good relative to the clean good. We then show specific conditions for the pollution tax to raise or lower the equilibrium wage/rental ratio. Most of these results are intuitive, but some are surprising. One might think that the pollution tax raises the relative return of the factor that is the better substitute for pollution, but a surprising result is that the opposite holds if labor and capital are highly complementary. Then the better substitute for pollution bears proportionally *more* of the burden of the pollution tax.

Another special case allows for differential factor intensities but abstracts from differential substitutability for pollution. Normally the pollution tax then lowers the relative return of the factor that is intensively used in the dirty sector, but a second surprising result is that the opposite holds if the dirty sector can substitute among its inputs more easily than consumers can substitute between outputs. Then the factor that is intensively used in the dirty sector bears proportionately *less* of the burden of the pollution tax. A final unusual result is that although the tax withdraws resources from the private economy, one of the factors could actually gain in real terms. We provide explanations for these counterintuitive results.

The next section presents the model and uses it to derive a system of equations. Then the second section offers a general solution and simplifies it in several cases to interpret the results. While the main contributions here are the propositions about incidence, the third section proceeds to insert plausible values for parameters and to calculate examples of the incidence of environmental policy. It shows that varying the factor intensities over their plausible range has less impact on incidence than varying substitution elasticities over their plausible range. The fourth section thus concludes that it is important next to estimate substitution elasticities. This concluding section also notes caveats. Indeed, our model could be extended in many of the same ways that the original Harberger (1962) model was extended over the following decades.<sup>4</sup>

## 1. Model

The simple model developed here is used to solve for all changes in prices and quantities that result from an exogenous change in the pollution tax. No government revenue neutrality is imposed, however, so an increase in one tax need not be offset by a decrease in another tax.

<sup>3</sup> McAusland (2003) develops a theoretical model to examine the role of inequality in endogenous environmental policy choice, and Aidt (1998) explores how heterogeneous agents may influence environmental policy through political processes. While both models are concerned with inequality, neither is strictly an examination of the incidence of environmental policy. Likewise, Bovenberg, Goulder and Gurney (2005) consider the efficiency costs of environmental taxation under a distributional constraint.

<sup>4</sup> See McLure (1975) and Fullerton and Metcalf (2002) for summaries of these extensions.

Instead, as in Harberger (1962) and others, the government is assumed to use the increased revenue to purchase the two private goods in the same proportion as do households. Thus, the transfer from the private sector to the public sector has no effect on relative demands or on prices. We consider a competitive two-sector economy using two factors of production, capital and labor. Both factors are mobile and can be used by either sector. A third variable input is pollution,  $Z$ , necessary to produce one of the outputs. The constant returns to scale production functions are:

$$\begin{aligned} X &= X(K_X, L_X) \\ Y &= Y(K_Y, L_Y, Z), \end{aligned}$$

where  $X$  is the “clean” good,  $Y$  is the “dirty” good,  $K_X$  and  $K_Y$  are capital used in each sector, and  $L_X$  and  $L_Y$  are labor used in each sector.<sup>5</sup> The resource constraints are:

$$\begin{aligned} K_X + K_Y &= \bar{K}, \\ L_X + L_Y &= \bar{L}, \end{aligned}$$

where  $\bar{K}$  and  $\bar{L}$  are the fixed total amounts of capital and labor in the economy. Totally differentiating these two constraints yields:

$$\hat{K}_X \lambda_{KX} + \hat{K}_Y \lambda_{KY} = 0, \tag{1}$$

$$\hat{L}_X \lambda_{LX} + \hat{L}_Y \lambda_{LY} = 0. \tag{2}$$

where a hat denotes a proportional change ( $\hat{K}_X \equiv dK_X/K_X$ ) and  $\lambda_{ij}$  denotes sector  $j$ 's share of factor  $i$  (e.g.  $\lambda_{KX} \equiv K_X/\bar{K}$ ). Notice that  $Z$  has no equivalent resource constraint and is simply a choice of the dirty sector.<sup>6</sup> To ensure finite use of pollution in the initial equilibrium, we start with a pre-existing positive tax on pollution.<sup>7</sup>

Producers of  $X$  can substitute between factors in response to changes in the gross-of-tax factor prices  $p_L$  and  $p_K$ , according to an elasticity of substitution in production  $\sigma_X$ . The definition of  $\sigma_X$  is differentiated and rearranged to obtain the firm's response,  $\hat{K}_X - \hat{L}_X = \sigma_X (\hat{p}_L - \hat{p}_K)$ , where  $\sigma_X$  is defined to be positive. The firm's cost of capital can be written as  $p_K = r(1 + \tau_K)$ , where  $r$  is the net return to capital and  $\tau_K$  is the *ad valorem* rate of tax on capital. Similarly,  $p_L = w(1 + \tau_L)$ , where  $w$  is the net wage and  $\tau_L$  is the labor tax.<sup>8</sup> Here, the only tax change is in the pollution tax, so  $\hat{p}_L = \hat{w}$  and  $\hat{p}_K = \hat{r}$ . Substituting these into the  $\sigma_X$  expression yields:

$$\hat{K}_X - \hat{L}_X = \sigma_X (\hat{w} - \hat{r}). \tag{3}$$

The choice of inputs in sector  $Y$  is more complicated, since it has three inputs. First, note that firms face no market price for pollution except for a tax, so  $p_Z = \tau_Z$  (and  $\hat{p}_Z = \hat{\tau}_Z$ , where  $\hat{\tau}_Z = d\tau_Z/\tau_Z$ ).

<sup>5</sup> As is typical in environmental models, the second production function includes pollution as an input. This is simply a rearrangement of a production function where both  $Y$  and  $Z$  are functions of  $K_Y$  and  $L_Y$ .

<sup>6</sup> For example, one could say that pollution  $Z$  arises from the incomplete transformation of an unmodeled input that has a flat supply curve, and that the associated pollution has no cost other than  $\tau_Z$ .

<sup>7</sup> This problem could also be solved by introducing a private cost of pollution, separate from the tax (see Fullerton and Metcalf, 2001). Here, we merely assume that the initial tax is positive and hence examine a *change* in the pollution tax rate rather than the *introduction* of a pollution tax.

<sup>8</sup> Differentiating the price equations yields  $\hat{p}_K = \hat{r} + \hat{\tau}_K$  and  $\hat{p}_L = \hat{w} + \hat{\tau}_L$ , where  $\hat{\tau}_K = d\tau_K/(1 + \tau_K)$  and  $\hat{\tau}_L = d\tau_L/(1 + \tau_L)$ . The model could be used to vary these tax rates and thus to analyze their incidence. Alternatively, it could be used with a revenue-neutrality constraint, so that an increase in the pollution tax is offset by a decrease in some other tax. In this paper, however, we hold these taxes constant ( $\hat{\tau}_K = \hat{\tau}_L = 0$ ).

This tax per unit of pollution is a specific tax rather than an *ad valorem* tax. We then follow Mieszkowski (1972) in modeling the choices among three inputs. To do this, define  $e_{ij}$  as the Allen elasticity of substitution between inputs  $i$  and  $j$  (Allen, 1938). This elasticity is positive when the two inputs are substitutes and is negative when they are complements. Note that  $e_{ij} = e_{ji}$ , that  $e_{ii} \leq 0$ , and that at most one of the three cross-price elasticities can be negative.<sup>9</sup> Also, define  $\theta_{YK} \equiv r(1 + \tau_K)K_Y/p_Y Y$  as the share of sales revenue from  $Y$  that is paid to capital (and similarly define  $\theta_{YL}$ ,  $\theta_{XK}$ , and  $\theta_{XL}$ ). Note that  $\theta_{YZ} \equiv \tau_Z Z/p_Y Y$  is the share of revenue of sector  $Y$  that is paid for pollution, through taxes. Also note that  $\theta_{XK} + \theta_{XL} = 1$  and  $\theta_{YK} + \theta_{YL} + \theta_{YZ} = 1$ . Then, as shown in the Appendix:<sup>10</sup>

$$\hat{K}_Y - \hat{Z} = \theta_{YK}(e_{KK} - e_{ZK})\hat{r} + \theta_{YL}(e_{KL} - e_{ZL})\hat{w} + \theta_{YZ}(e_{KZ} - e_{ZZ})\hat{\tau}_Z \tag{4}$$

$$\hat{L}_Y - \hat{Z} = \theta_{YK}(e_{LK} - e_{ZK})\hat{r} + \theta_{YL}(e_{LL} - e_{ZL})\hat{w} + \theta_{YZ}(e_{LZ} - e_{ZZ})\hat{\tau}_Z, \tag{5}$$

Using assumptions of perfect competition and constant returns to scale, where  $p_X$  and  $p_Y$  are output prices, the Appendix also derives the following two equations:

$$\hat{p}_X + \hat{X} = \theta_{XK}(\hat{r} + \hat{K}_X) + \theta_{XL}(\hat{w} + \hat{L}_X), \tag{6}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YK}(\hat{r} + \hat{K}_Y) + \theta_{YL}(\hat{w} + \hat{L}_Y) + \theta_{YZ}(\hat{Z} + \hat{\tau}_Z), \tag{7}$$

Totally differentiate each sector’s production function and substitute in the conditions from the perfect competition assumption shown in the Appendix to yield:

$$\hat{X} = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X. \tag{8}$$

$$\hat{Y} = \theta_{YK}\hat{K}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}. \tag{9}$$

Finally, consumer preferences for the two goods can be modeled using  $\sigma_u$ , the elasticity of substitution between goods  $X$  and  $Y$  in utility. Differentiate the definition of  $\sigma_u$  to get the equation for consumer demand response to a change in prices:<sup>11</sup>

$$\hat{X} - \hat{Y} = \sigma_u(\hat{p}_Y - \hat{p}_X). \tag{10}$$

This formulation does not preclude disutility from pollution. Rather, it just assumes that pollution (or environmental quality) is separable in utility. For an alternative, Carbone and Smith (2006) model the non-separability of air quality and leisure.

<sup>9</sup> Stability conditions in Allen (1938) require that  $e_{ii}$  are strictly negative. Yet we can solve for the Leontief case where all  $e_{ij} = 0$  below, so we only require  $e_{ii} \leq 0$ .

<sup>10</sup> In a sense, we could avoid saying that pollution is an “input” to the production function  $Y = Y(K_Y, L_Y, Z)$  and instead just specify Eqs. (4) and (5). In that case, the Allen elasticities of substitution are a direct way to model the choices of firms. A higher tax on pollution leads firms to pollute less ( $e_{ZZ} < 0$ ), holding output constant, and it may raise or lower use of labor or capital in ways that depend on the signs and magnitudes of  $e_{LZ}$  and  $e_{KZ}$ . The only necessary point is that these reactions affect the incidence of the tax.

<sup>11</sup> Our model also includes output taxes  $\tau_X$  and  $\tau_Y$ . Thus, in general,  $\hat{X} - \hat{Y} = \sigma_u(\hat{p}_Y + \hat{\tau}_Y - \hat{p}_X - \hat{\tau}_X)$ , but this paper holds these tax rates constant ( $\hat{\tau}_X = \hat{\tau}_Y = \hat{\tau}_K = \hat{\tau}_L = 0$ ). These other taxes are still in the model, however, since firms and consumers in the initial equilibrium are responding to cum-tax prices. Since the levels of those tax rates do not appear in solutions below, it means that the effects of  $\hat{\tau}_Z > 0$  do not depend on the level of other existing tax rates.

Eqs. (1)–(10) are ten equations in eleven unknowns ( $\hat{K}_X, \hat{K}_Y, \hat{L}_X, \hat{L}_Y, \hat{w}, \hat{r}, \hat{p}_X, \hat{X}, \hat{p}_Y, \hat{Y}, \hat{Z}$ ). Good  $X$  is chosen as numeraire, so  $\hat{p}_X=0$ .<sup>12</sup> This system of ten equations then provides solutions to all ten unknown endogenous changes as functions of parameters and of an exogenous positive change in the pollution tax ( $\hat{\tau}_Z>0$ ). If the pollution tax is reduced, then all results hold with the opposite sign.

Our primary purpose is to solve for incidence results, i.e., the effects on output prices and factor prices. Thus, to make the solution more manageable, we omit equations for the proportional changes in quantities  $\hat{K}_X, \hat{K}_Y, \hat{L}_X, \hat{L}_Y, \hat{X}$ , and  $\hat{Y}$ . We keep the proportional change in pollution,  $\hat{Z}$ , since that result is of interest as well, though it is a quantity and not a price.

**2. Results and interpretations**

The Appendix shows how to use Eqs. (1)–(10) above to find the following general solutions for an increase in  $\tau_Z$ , the pollution tax rate:<sup>13</sup>

$$\hat{p}_Y = \frac{(\theta_{YL}\theta_{XK}-\theta_{YK}\theta_{XL})\theta_{YZ}}{D} [A(e_{ZZ}-e_{KZ})-B(e_{ZZ}-e_{LZ}) + (\gamma_K-\gamma_L)\sigma_u]\hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \tag{11a}$$

$$\hat{w} = \frac{\theta_{XK}\theta_{YZ}}{D} [A(e_{ZZ}-e_{KZ})-B(e_{ZZ}-e_{LZ}) + (\gamma_K-\gamma_L)\sigma_u]\hat{\tau}_Z, \tag{11b}$$

$$\hat{r} = \frac{\theta_{XL}\theta_{YZ}}{D} [A(e_{KZ}-e_{ZZ})-B(e_{LZ}-e_{ZZ})-(\gamma_K-\gamma_L)\sigma_u]\hat{\tau}_Z, \tag{11c}$$

$$\begin{aligned} \hat{Z} = &-\frac{1}{C} [\theta_{YK}(\beta_K(e_{KK}-e_{ZK}) + \beta_L(e_{LK}-e_{ZK}) + \sigma_u)\hat{r} + \theta_{YL}(\beta_K(e_{KL}-e_{ZL}) \\ &+ \beta_L(e_{LL}-e_{ZL}) + \sigma_u)\hat{w} + \theta_{YZ}(\beta_K(e_{KZ}-e_{ZZ}) \\ &+ \beta_L(e_{LZ}-e_{ZZ}) + \sigma_u)\hat{\tau}_Z] \end{aligned} \tag{11d}$$

where  $\gamma_K \equiv \frac{\lambda_{KY}}{\lambda_{KX}} = \frac{K_Y}{K_X}$  and  $\gamma_L \equiv \frac{\lambda_{LY}}{\lambda_{LX}} = \frac{L_Y}{L_X}$ . Also, for convenience, this solution combines notation into definitions where  $\beta_K \equiv \theta_{XK}\gamma_K + \theta_{YK}$ ,  $\beta_L \equiv \theta_{XL}\gamma_L + \theta_{YL}$ ,  $A \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ})$ ,  $B \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ})$ , and  $C \equiv \beta_K + \beta_L + \theta_{YZ}$ . It is readily apparent that  $A>0$ ,  $B>0$ , and  $C>0$ . The denominator is  $D \equiv C\sigma_X + A[\theta_{XK}\theta_{YL}(e_{KL}-e_{LZ}) - \theta_{XL}\theta_{YK}(e_{KK}-e_{KZ})] - B[\theta_{XK}\theta_{YL}(e_{LL}-e_{LZ}) - \theta_{XL}\theta_{YK}(e_{KL}-e_{KZ})] - (\gamma_K-\gamma_L)\sigma_u(\theta_{XK}\theta_{YL}-\theta_{XL}\theta_{YK})$ .

While the interpretation of this general solution is limited by its complexity, some basic effects can be identified. For example, the last term in (11b) or (11c) is  $(\gamma_K-\gamma_L)\sigma_u$ , which Mieszkowski (1967) calls the “output effect”: a tax on emissions is a tax only in the dirty sector and therefore reduces output (in a way that depends on consumer demand via  $\sigma_u$ ). Less output means less demand for all inputs, but particularly the input used intensively in that sector. The term  $(\gamma_K-\gamma_L)$

<sup>12</sup> Harberger (1962) chose  $w$  as numeraire and interpreted the expression for  $dr$  as the change in the return to capital relative to labor. In this model, we provide expressions for both  $\hat{w}$  and  $\hat{r}$ . These results can be compared to those of Harberger by considering the value of  $\hat{r}-\hat{w}$ .

<sup>13</sup> If solely in terms of exogenous parameters, the expression for  $\hat{Z}$  would be long. The three preceding equations can be substituted into the fourth to get that closed-form solution, as done in special cases below.

is positive when the dirty sector is capital-intensive. Then, assuming  $D > 0$ , the output effect places relatively more burden on capital. Whether this intuitive results hold depends on the sign of the denominator  $D$ , however, and this complicated expression can have either sign in this general solution.

Furthermore, the first two terms in Eqs. (11b) or (11c) represent “substitution effects”. As pollution become more costly, the dirty sector seeks to adjust its demand for all three inputs. How it does so is determined by the Allen elasticities of substitution, which figure prominently in those first two terms. The constants  $A$  and  $B$  also come into play, weighing the impact of the elasticities on the incidence results. These constants can be signed, but their magnitudes are complicated functions of the factor share parameters, making an interpretation difficult from these general solutions alone.

Thus, while several effects are at work, their combination and interactions are quite difficult to analyze in these equations. We therefore isolate each effect by assuming away the other effects in a series of special cases. Although  $A > 0$ ,  $B > 0$ , and  $C > 0$ , the denominator  $D$  cannot be signed, and so nothing definitive can be said yet about the effect of the pollution tax on the output price (11a), factor prices (11b) and (11c), or even on the amount of pollution (11d). In fact, an increase in the pollution tax might increase pollution.<sup>14</sup> Thus the following special cases are also useful to seek definitive results.<sup>15</sup>

Before proceeding, consider implications for who bears the burden of this tax. The interpretation of  $\hat{w} = \hat{r} = 0$  is not that factors bear no burden, but that burdens are proportional to their shares of national income. Labor is unaffected by the tax on the sources side if the real wage does not change,  $\hat{w} - \hat{p} = 0$ , where the overall price index is  $p \equiv \varphi p_X + (1 - \varphi)p_Y$  and where  $\varphi$  is the share of total expenditure on  $X$ . Of course,  $p_X$  is numeraire, but  $p_Y$  may rise. And this discussion presumes that both factors spend similarly on the two goods; on the uses side, if  $p_Y$  rises, the tax places more burden on anybody who spends more than the average fraction of income on the polluting good.

### 2.1. Case 1: Equal factor intensities

First, consider the case where both industries have the same factor intensities, that is, both are equally capital (and labor) intensive. This amounts to setting  $\gamma_L$  and  $\gamma_K$  equal to each other. Let their common value be  $\gamma$ , and note that this condition implies that  $L_Y/L_X = K_Y/K_X$ . In this case, the solution to the system of equations is:

$$\hat{p}_Y = \theta_{YZ} \hat{\tau}_Z \tag{12a}$$

$$\hat{w} = \frac{-\theta_{XK} \theta_{YZ} \gamma (e_{KZ} - e_{LZ})}{D_1} \hat{\tau}_Z \tag{12b}$$

<sup>14</sup> De Mooij and Bovenberg (1998) obtain a similar perverse result. In our model, it is possible with certain extreme parameter values. For a case that satisfies all restrictions from Allen (1938), suppose  $\lambda_{KX} = 0.2$ ,  $\lambda_{LX} = 0.1$ ,  $\theta_{XK} = 0.9$ ,  $\theta_{YK} = 0.72$ ,  $\theta_{YZ} = 0.1$ ,  $e_{KL} = 2$ ,  $e_{KZ} = -1$ ,  $e_{LZ} = 5$ ,  $e_{ZZ} = -1.8$ , and  $\sigma_u = \sigma_X = 1$ . The increase in  $p_Z$  has a direct effect that reduces pollution (since  $e_{ZZ} = -1.8$ ) but a larger indirect effect that raises pollution. The 10% higher  $p_Z$  decreases demand for capital ( $e_{KZ} = -1$ ), which decreases the rate of return ( $\hat{r} = -.0025$ ). It also increases demand for labor ( $e_{LZ} = 5$ ). This labor is hard to get from the other sector  $X$ , which is small and capital-intensive, so  $w$  rises steeply ( $\hat{w} = .0223$ ). Both of these factor price changes have positive feedback effects on pollution, since the fall in  $r$  raises  $Z$  ( $e_{ZK} = -1$ ), and the rise in  $w$  raises  $Z$  ( $e_{ZL} = 5$ ). The net result is 0.197% more emissions. This result depends on the fixed supply of labor and capital, however; if either factor supply were endogenous,  $\hat{Z} > 0$  would be less likely.

<sup>15</sup> A very special case of this model reduces exactly to the model in Harberger (1962), even for his analysis of a partial tax on capital. In our model, when capital and pollution are perfect complements in production of  $Y$ , then a tax on pollution is a partial factor tax on capital (as in Rapanos, 1992).

$$\hat{r} = \frac{\theta_{XL}\theta_{YZ}\gamma(e_{KZ}-e_{LZ})}{D_1} \hat{\tau}_Z \tag{12c}$$

$$\hat{Z} = \frac{-\sigma_u\theta_{YZ}-\theta_{YZ}(\beta_K(e_{KZ}-e_{ZZ}) + \beta_L(e_{LZ}-e_{ZZ}))}{\gamma + 1} \hat{\tau}_Z$$

$$- \frac{[\theta_{XL}\theta_{YK}(\beta_K(e_{KZ}-e_{KK}) + \beta_L(e_{KZ}-e_{KL}) + \beta_K(e_{KL}-e_{LZ}) + \beta_L(e_{LL}-e_{LZ}))]\gamma\theta_{YZ}(e_{LZ}-e_{KZ})}{D_1(\gamma + 1)} \hat{\tau}_Z \tag{12d}$$

where  $D_1 = (\sigma_X - \theta_{XL}\theta_{YK}\gamma e_{KK} - \theta_{XK}\theta_{YL}\gamma e_{LL}) + \gamma(\theta_{XL}\theta_{YK} + \theta_{XK}\theta_{YL})e_{KL}$ .

One of the most striking observations from this solution is how the long general expression for  $\hat{p}$  in (11a) reduces to such a simple expression in (12a). In this special case, then, we can provide a definite sign:

**Proposition 1A.** *In Case 1,  $\hat{p}_Y > 0$ .*

**Proof.** Since  $\hat{\tau}_Z > 0$ , Eq. (12a) implies  $\hat{p}_Y > 0$ . □

Furthermore, the increase in the pollution tax affects this relative price only through the share that pollution constitutes of output,  $\theta_{YZ}$ . The fact that Eq. (11a) of the general solution is more complicated implies that the capital and labor intensities of production also effect  $p_Y$  in the general case. Here, those effects have been assumed away, and the uses side of the incidence of an environmental tax is clear: consumers of the dirty good bear a cost of the tax increase.

To interpret the incidence on the sources side, or the effect of a change in the pollution tax on returns to inputs, we must know something about the sign of the denominator  $D_1$ . From the expression for  $D_1$  above, note that  $D_1 > 0$  if and only if  $e_{KL} > \frac{-\sigma_X + \theta_{XL}\theta_{YK}\gamma e_{KK} + \theta_{XK}\theta_{YL}\gamma e_{LL}}{\gamma(\theta_{XL}\theta_{YK} + \theta_{XK}\theta_{YL})}$ . Call this last inequality “Condition 1”. The expression to the right of the inequality sign is strictly negative, so  $e_{KL} > 0$  is sufficient but not necessary for Condition 1. Remember that  $e_{ij}$  is positive whenever inputs  $i$  and  $j$  are substitutes, and negative for complements. To make the denominator  $D_1 > 0$ , it is not necessary that capital and labor are substitutes in production of  $Y$  ( $e_{KL} > 0$ ), but only that they are not too complementary (Condition 1).

With that condition, we can interpret the effect of a change in the pollution tax rate on the returns to factors of production.

**Proposition 1B.** *In Case 1, under Condition 1,  $\hat{w} > 0$  and  $\hat{r} < 0$  if and only if  $e_{LZ} > e_{KZ}$ .*

**Proof.** Since  $D_1 > 0$  under Condition 1, Eqs. (12b) and (12c) imply this result. □

A special case of 1B is where labor and pollution are substitutes in sector  $Y$ , while capital and pollution are complements (i.e.,  $e_{LZ} > 0$  and  $e_{KZ} < 0$ ). In this case, the condition  $e_{LZ} > e_{KZ}$  of Proposition 1B holds, and  $\tau_Z$  raises the wage.<sup>16</sup> It is not necessary, however, for these terms to have opposite signs. Even if both capital and labor are substitutes for pollution, the relative price of labor still rises from an increase in the pollution tax as long as labor is a better substitute for pollution than is capital.

<sup>16</sup> Note that the change in the wage rate always has the opposite sign as the change in the rental rate. This follows directly from the choice of  $X$  as numeraire and the zero-profit Eq. (6). This relationship need not hold for other choices of numeraire. Only the relative change in the returns to capital and labor is of interest here, and this value is independent of the choice of numeraire.

It is also of interest and quite counterintuitive to note when the above proposition does not hold. If the value of  $D_I$  is negative, then the results are exactly the opposite.

**Proposition 1C.** *In Case 1, but where Condition 1 does not hold,  $\hat{w} > 0$  and  $\hat{r} < 0$  if and only if  $e_{LZ} < e_{KZ}$ .*

**Proof.**  $D_1 < 0$ , and Eqs. (12b) and (12c).  $\square$

Normally with  $e_{LZ} < e_{KZ}$ , we would say that capital is a better substitute for pollution, so the pollution tax would tend to increase demand for capital and hence to increase  $r$ . This effect is more than offset, however, when  $e_{KL}$  is sufficiently negative (Condition 1 does not hold). Capital and labor are complementary inputs, and the increased demand for capital also leads to an increased demand for labor. With a sufficiently high degree of complementarity, this effect dominates, and  $w$  increases relative to  $r$ . While we would not necessarily expect this case to be common, it demonstrates a perverse possibility: even when both sectors have equal factor intensities, the better substitute for pollution can bear *more* of the burden of a tax on pollution.

The top half of Fig. 1 summarizes Case 1:  $\hat{p}_Y$  is always positive, but the signs of  $\hat{w}$  and  $\hat{r}$  depend on whether Condition 1 holds, and on whether  $e_{LZ} < e_{KZ}$ . On the knife's edge where  $e_{LZ} = e_{KZ}$  is a special case of Case 1 that we label Case 2.

### 2.2. Case 2: Equal factor intensities and $e_{KZ} = e_{LZ}$

In addition to equal factor intensities ( $\gamma_K = \gamma_L = \gamma$ ), suppose also that capital and labor are equally good substitutes for pollution ( $e_{LZ} = e_{KZ}$ ). In this special case, we have:

**Proposition 2A.** *In Case 2,  $\hat{w} = 0$ ,  $\hat{r} = 0$ ,  $\hat{p}_Y = \theta_{YZ} \hat{\tau}_Z$ , and*

$$\hat{Z} = \frac{-\sigma_u - (\beta_K + \beta_L)(e_{LZ} - e_{ZZ})}{\gamma + 1} \theta_{YZ} \hat{\tau}_Z. \tag{13}$$

**Proof.** Eqs. ((12a) (12b) (12c) (12d)), substituting in  $e_{KZ} = e_{LZ}$ .  $\square$

For factor prices, as in Case 1, equal factor intensities eliminate the output effect of Mieszkowski (1967). The additional assumption of equal substitution elasticities in Case 2 eliminates his substitution effect. Without either of these effects, the change in tax has no effect on the relative prices of capital and labor. Thus, neither factor bears a disproportionate burden. Next, the effect on the price of the dirty good is the same as in Case 1. More interesting in this case is that we can finally sign the effect on pollution.

**Proposition 2B.** *In Case 2,  $\hat{Z} < 0$ .*

**Proof.** In Eq. (13), note that  $e_{ZZ} \leq 0$ ,  $e_{LZ} \geq 0$ , and that all of the other parameters are positive.<sup>17</sup>  $\square$

The purpose of this example is to reduce the general model to the simple model where  $Y$  has only two inputs, and where  $\tau_Z$  is said to have two effects that both reduce pollution.<sup>18</sup> The “substitution effect” is the second term in the numerator of (13), where  $\tau_Z$  increases the relative

<sup>17</sup> In general,  $e_{LZ}$  could be positive or negative, but here where  $e_{LZ} = e_{KZ}$ , the fact that  $e_{ZZ} \leq 0$  implies that  $e_{LZ} = e_{KZ} \geq 0$ .

<sup>18</sup> The substitution and output effects here refer to effects on pollution (whereas elsewhere we use these terms to mean Mieszkowski's (1967) effects of taxes on factor prices). To reduce the model to only two inputs, we could add the assumptions  $e_{KL} = \sigma_X = 0$ ; then  $L$  and  $K$  are used in fixed proportions and can be considered a single composite input. Sector  $X$  uses only this clean input, with no ability to substitute. Sector  $Y$  has effectively only two inputs between which it can substitute: one is pollution and the other is this clean input. The results with these assumptions are identical to the results in Case 2, however, so we do not need to say whether  $e_{KL} = \sigma_X = 0$ .

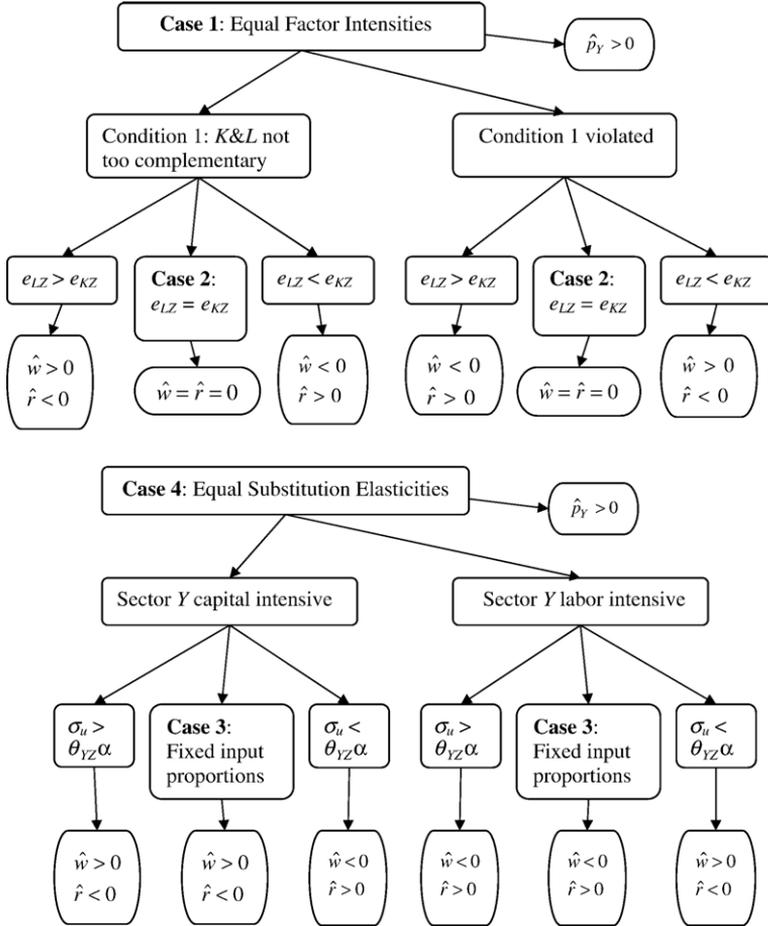


Fig. 1. A schematic representation of special cases and results (in the general case,  $\hat{p}_Y \leq 0$ ,  $\hat{w} \leq 0$ , and  $\hat{r} \leq 0$ ).

price of pollution, which reduces pollution per unit output. The “output effect” is the first term in (13), where  $\tau_Z$  increases the price of output, which reduces total demand. Thus, pollution is definitely reduced by the tax on pollution. Somewhat surprisingly, this intuitive result does *not* hold in general (see Footnote 14).

2.3. Case 3: Fixed input proportions ( $e_{ij}=0$ )

By eliminating factor intensity differences, the special cases above concentrate on the signs of input demand elasticities. Now we eliminate the differential effects of input demand elasticities in order to concentrate on relative factor intensities ( $\gamma_K - \gamma_L$ ). If this value is positive, then industry Y is capital-intensive.

We start with the simplest way to eliminate differential elasticities by assuming that *all*  $e_{ij}=0$  (but this is a special case of Case 4 below, a less restrictive way to eliminate differences). We now expect no substitution effect, but only the output effect arising from the implicit tax on Y

associated with an increase in the tax on pollution. This absence of a substitution effect is precisely what materializes in the solution:<sup>19</sup>

$$\hat{p}_Y = \frac{C\sigma_X\theta_{YZ}}{D_3}\hat{\tau}_Z \tag{14a}$$

$$\hat{w} = \frac{(\gamma_K-\gamma_L)\theta_{XK}\theta_{YZ}\sigma_u}{D_3}\hat{\tau}_Z \tag{14b}$$

$$\hat{r} = \frac{-(\gamma_K-\gamma_L)\theta_{XL}\theta_{YZ}\sigma_u}{D_3}\hat{\tau}_Z \tag{14c}$$

where  $D_3 = C\sigma_X - (\gamma_K - \gamma_L)\sigma_u(\theta_{XK}\theta_{YL} - \theta_{XL}\theta_{YK})$ . It can be shown that  $(\gamma_K - \gamma_L)$  and  $(\theta_{XK}\theta_{YL} - \theta_{XL}\theta_{YK})$  always have opposite sign, so  $D_3 > 0$ . Thus, just as in cases above, the price of the dirty good relative to the clean good increases unambiguously in response to an increase in the tax on pollution ( $\hat{p}_Y > 0$ ).

More interesting is the effect on the relative return to labor and capital. The sign of each of those two changes is based only on the relative factor intensities of the two industries,  $\gamma_K - \gamma_L$ . We can write this conclusion as the following:

**Proposition 3.** *In Case 3, if Y is capital-intensive, then  $\hat{w} > 0$  and  $\hat{r} < 0$ .*

**Proof.** Eqs. (14b) and (14c), since  $D_3 > 0$ , and Y being capital-intensive means  $\gamma_K - \gamma_L > 0$ . □

The interpretation is solely in terms of the output effect for sector Y. Because of fixed input proportions ( $e_{ij} = 0$ ), a pollution tax increase is equivalent to a tax on output Y and leads to decreased output of that good. Therefore, sector Y demands less labor and less capital. If Y is capital-intensive, then the fall in demand for capital exceeds the fall in demand for labor, and hence  $r$  falls relative to  $w$ . This simple case helps establish the presumption for the more surprising result of the next section.

#### 2.4. Case 4: Equal elasticities of factor demand

To abstract from differential input demand elasticities, it is not necessary to suppose that all are zero. A less restrictive way to do this is to suppose that all of the own-price Allen elasticities  $e_{ii}$  are equal to  $a_1 \leq 0$  and that all of the cross-price elasticities,  $e_{ij}$  for  $i \neq j$ , are equal to  $a_2 \geq 0$ .<sup>20</sup> Furthermore, define  $\alpha \equiv (a_2 - a_1) \geq 0$ . (Then Case 3 is the special case where  $\alpha = a_2 = a_1 = 0$ .) The more general solution then is:

$$\hat{p}_Y = \frac{\theta_{YZ}}{D_4} \{ \theta_{YZ}\alpha(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK}) + C\sigma_X + \alpha(A\theta_{XL}\theta_{YK} + B\theta_{YL}\theta_{XK}) \} \hat{\tau}_Z \tag{15a}$$

$$\hat{w} = \frac{(\gamma_K-\gamma_L)\theta_{XK}\theta_{YZ}(\sigma_u - \theta_{YZ}\alpha)}{D_4}\hat{\tau}_Z \tag{15b}$$

$$\hat{r} = \frac{-(\gamma_K-\gamma_L)\theta_{XL}\theta_{YZ}(\sigma_u - \theta_{YZ}\alpha)}{D_4}\hat{\tau}_Z \tag{15c}$$

where  $D_4 = C\sigma_X + \alpha(A\theta_{XL}\theta_{YK} + B\theta_{YL}\theta_{XK}) - (\gamma_K - \gamma_L)\sigma_u(\theta_{XK}\theta_{YL} - \theta_{XL}\theta_{YK})$ . In this case, the denominator  $D_4$  is definitely positive. The second term is nonnegative, since  $\alpha \geq 0$ ,  $A > 0$ , and  $B > 0$ . And the remaining terms are the same as in  $D_3 > 0$ . Thus  $D_4 > 0$ .

<sup>19</sup> The expression for  $\hat{Z}$  can be evaluated using the other three expressions, but it is not included here because it does not prove illuminating.

<sup>20</sup> Our Appendix discusses restrictions demonstrated by Allen (1938). Since  $e_{ii}$  cannot be positive, Case 4 assumes that the matrix of  $e_{ij}$  has  $a_1 \leq 0$  down the diagonal and  $a_2 \geq 0$  everywhere else.

As in the other special cases, the sign of the change in the price of the dirty good relative to the price of the clean good is unambiguous.

**Proposition 4A.** *In Case 4,  $\hat{p}_Y > 0$ .*

**Proof.** Since  $(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})$  has the same sign as  $(\gamma_K - \gamma_L)$ , and  $D_4 > 0$ , the coefficient in Eq. (15a) is positive.  $\square$

We can now interpret the changes in factor prices in terms of an output effect and substitution effect in the polluting industry. The sign of the change in factor prices in (15b) and (15c) depends the signs of  $(\gamma_K - \gamma_L)$  and  $(\sigma_u - \theta_{YZ}\alpha)$ . The former is positive when  $Y$  is capital-intensive. In the latter,  $\alpha$  is a measure of the overall ability of firms in sector  $Y$  to substitute among inputs. It is equal to  $e_{ij} - e_{ii}$ , so a larger  $\alpha$  means easier substitution away from the more costly input ( $e_{ii} < 0$ ) and into the other inputs ( $e_{ij} > 0$ ). Also,  $\sigma_u$  represents the ability of consumers to substitute between  $X$  and  $Y$ . Thus, in combination, the expression  $(\sigma_u - \theta_{YZ}\alpha)$  represents whether it is relatively easier for consumers to substitute between goods  $X$  and  $Y$  than for producers of  $Y$  to substitute among their three inputs  $K$ ,  $L$ , and  $Z$ . This interpretation leads to the following proposition about an increase in the pollution tax  $\tau_Z$ :

**Proposition 4B.** *In Case 4, when sector  $Y$  is capital intensive, then  $\hat{w} > 0$  and  $\hat{r} < 0$  whenever  $\sigma_u > \theta_{YZ}\alpha$ . When sector  $Y$  is labor intensive, then  $\hat{w} < 0$  and  $\hat{r} > 0$  whenever  $\sigma_u > \theta_{YZ}\alpha$ .*

**Proof.** Eqs. (15b) and (15c), since  $D_4 > 0$ .  $\square$

To explain,  $\hat{\tau}_Z > 0$  induces a substitution effect for producers of  $Y$  that increases demand for  $K$  and could be expected to increase  $r$ . In addition, however, the output effect raises the price of  $Y$  and reduces production. When  $Y$  is capital intensive, this output effect reduces overall demand for capital and would tend to decrease  $r$ . These two effects work in opposite directions. If  $\sigma_u > \theta_{YZ}\alpha$ , then the output effect dominates the substitution effect and less capital is demanded by sector  $Y$ . When sector  $Y$  is capital-intensive, its reduced demand for capital outweighs sector  $X$ 's increased use, and the economy-wide  $r/w$  falls. Hence the result in Proposition 4B.

This proposition includes both the intuitive result above and the reverse counterintuitive result: abstracting from different cross-price elasticities, capital intensity in the dirty industry can lead to a disproportionately high burden of the pollution tax on labor rather than on capital.<sup>21</sup> Again, capital intensity of  $Y$  has two opposite effects. On the one hand, it reduces  $r$  through the output effect, since consumers demand less of the capital-intensive good. However,  $\sigma_u < \theta_{YZ}\alpha$  means that the effect is relatively small. The larger substitution effect means that firms are trying to substitute out of  $Z$  and into both  $K$  and  $L$ . The firms in  $Y$  want to increase both factors in proportion to their own use, which is capital-intensive, but they must get that capital from  $X$ , which is labor intensive. They can only get that extra capital by bidding up its price.

The bottom half of Fig. 1 summarizes all of these results for Case 4, as well as its special case where all  $e_{ij} = 0$  (Case 3).

### 2.5. Case 5: All equal cross-price substitution elasticities

A final special case can help with intuition and relate our model to other models in the literature. Here, we impose no constraints on factor intensities but suppose that  $\sigma_u$  and all cross-price

<sup>21</sup> This result is reminiscent of Harberger's (1962) result that the partial tax on capital can hurt labor.

substitution elasticities have the same value ( $\sigma_u = e_{KL} = e_{KZ} = e_{LZ} = c$ , some positive constant).<sup>22</sup> We then have the following proposition:

**Proposition 5A.** *In Case 5, regardless of factor intensities, then  $\hat{w} = 0$ ,  $\hat{r} = 0$ ,  $\hat{p}_Y = \theta_{YZ}\hat{\tau}_Z$ , and  $\hat{Z} = -c\hat{\tau}_Z$ .*

**Proof.** Substitute  $c$  into all of Eqs. (11a) (11b) (11c) (11d).  $\square$

It is interesting that these results are similar to those of Case 2 even though Case 2 employs different assumptions (equal factor intensities and  $e_{KZ} = e_{LZ}$ ). Instead, this case with equal substitution elasticities is almost Cobb–Douglas, but a smaller elasticity  $c < 1$  implies that the tax  $\tau_Z$  has less effect on pollution. It can be shown that Cobb–Douglas production in the  $Y$  sector means  $e_{KL} = e_{KZ} = e_{LZ} = 1$  and  $e_{ii} = (\theta_{Yi} - 1)/\theta_{Yi}$  for each input  $i$ . Cobb–Douglas utility means  $\sigma_u = 1$ . Then, with these assumptions, we have:

**Proposition 5B.** *If utility and production of  $Y$  are Cobb–Douglas, then  $\hat{w} = 0$ ,  $\hat{r} = 0$ ,  $\hat{p}_Y = \theta_{YZ}\hat{\tau}_Z$ , and  $\hat{Z} = -\hat{\tau}_Z$ .*

**Proof.** Substitute  $c = 1$  into Proposition 5A.  $\square$

This Cobb–Douglas case is worth stating explicitly because of its clear intuition. Consumers spend a constant fraction of income on  $Y$ , and the firms in  $Y$  spend a constant fraction of sales revenue to pay for pollution. Thus, total spending  $\tau_Z Z$  is constant. Then any increase in the pollution tax implies the same percent fall in pollution and no effect on any other factor of production. In fact, the price  $p_Y$  rises by the same percentage that the quantity  $Y$  falls.

### 3. Numerical analysis

To explore the likely size of these effects, we now assign plausible values to parameters. The goal here is not to calculate a point estimate for effects of a pollution tax on pollution and factor prices, as this model is too simple for that purpose. Rather, the goal is to examine numerically the theoretical effects derived above, to see the direction of changes in these outcomes from changes in key parameters. We therefore vary the factor intensities and substitution elasticities. Other parameters are chosen to approximate the current U.S. economy or to match estimates in the available literature. Although many of these parameters have not been estimated in the form required here, other structural models may be similar enough to use their parameter values in our model.

For a definition of the “dirty” sector, we use the top thirteen polluting industries by SIC 2-digit codes from the EPA’s Toxic Release Inventory for 2002.<sup>23</sup> All other industries are deemed “clean” for present purposes. We then use industry-level data on labor and capital employed in the U.S., from Jorgenson and Stiroh (2000).<sup>24</sup> When we add labor and capital across industries within each sector, we find that the 13 most polluting industries represent about 20% of factor income. Therefore, our first “stylized fact” is that the clean sector is 80% of income. Using the same data,

<sup>22</sup> This case is analyzed, for example, by Harberger (1962) in his section VI, part 10.

<sup>23</sup> The top 13 polluters are those with at least 120,000,000 lb of on- and off-site reported releases of all chemicals monitored by the TRI (nearly 650 listed at <http://www.epa.gov/tri/chemical/index.htm>). These industries are metal mining, electric utilities, chemicals, primary metals, fabricated metals, food, paper, plastics, transportation equipment, petroleum, stone/clay/glass, lumber, and electrical equipment. These data are publicly available at <http://www.epa.gov/triexplorer/industry.htm>. Although the dividing line between clean and dirty industries is arbitrary, changes in this line do not yield much change in parameters.

<sup>24</sup> Available online at <http://post.economics.harvard.edu/faculty/jorgenson/data/35klem.html>.

we find that the capital share of factor income is .3985 in the clean sector and .4105 in the dirty sector.<sup>25</sup> Thus the dirty sector is slightly capital-intensive, as consistent with prior findings (e.g. Antweiler et al., 2001, p. 879). The difference is quite small, however, and we wish to avoid the perception that we are trying to calculate incidence with such precision. Appropriate rounding suggests that the capital share in both sectors is about 40%, and so that is the second stylized fact used here (as a starting point, before sensitivity analysis).<sup>26</sup>

In the clean sector, the implication is that  $\theta_{XK}$  is 0.40, and  $\theta_{XL}$  is 0.60 (so  $K/L$  is 2/3). In the dirty sector, however, these data on labor and capital do not help determine the fraction of output attributed to the value of pollution ( $\theta_{YZ}$ ). This parameter is not in available data, since most industries do not pay an explicit price for pollution. For most pollutants in most industries, this value is implicit – a shadow value, or scarcity rent. Therefore, somewhat arbitrarily, we set  $\theta_{YZ}$  to 0.25, and so our third “stylized fact” is that the dirty industry spends 25% of its sales revenue on the pollution input.<sup>27</sup> For the remaining 75% to have the same  $K/L$  ratio as the clean sector,  $\theta_{YK}$  must be 0.30, and  $\theta_{YL}$  must be 0.45. In fact, given various equations of the model, the three “stylized facts” are enough to determine all of the remaining parameters shown in Table 1.<sup>28</sup>

No empirical estimates of substitution parameters are available specifically for our definition of the dirty sector. The clean sector is 80% of factor income, however, so an economy-wide estimate represents a decent approximation of  $\sigma_X$  for the clean sector. Lovell (1973) and Corbo and Meller (1982) estimate the elasticity of substitution between capital and labor for all manufacturing industries. They find an elasticity of unity, which we employ for  $\sigma_X$ .<sup>29</sup> We also use unity for the elasticity of substitution in consumption between the clean and dirty goods,  $\sigma_u$ .<sup>30</sup>

The only further parameters needed are the  $e_{ij}$  input demand elasticities in the dirty sector. The model includes six of these parameters, but only three can be set independently.<sup>31</sup> Therefore, we vary only the three cross-price elasticities, which effectively sets the other parameters. To the best of our knowledge, these cross-price elasticities have never been estimated with inputs defined as

<sup>25</sup> These values are also consistent with other literature. The capital share is similar across studies, though again no study considers clean industries only. Griliches and Mairesse (1998) conclude that the capital share is approximately 0.4, and Blundell and Bond (2000) use GMM to yield an estimate of approximately 0.3, which rises to 0.45 when constant returns to scale is imposed.

<sup>26</sup> Other definitions of the dirty sector might yield more divergent factor intensities. If it includes only utilities and chemicals, for example, then capital is 55% of factor income. Also, however, that dirty sector is only 8% of the economy. In that case we found very small changes in  $w$  and  $r$ , and so those results are not so interesting. A general equilibrium model is not necessary when the taxed sector is small.

<sup>27</sup> This choice for  $\theta_{YZ}$  is arbitrary, and not really a stylized “fact”, but appropriate data are not available. It would be underestimated using data from an emissions permit market such as the one in place for sulfur dioxide from electric utilities, since most pollutants are restricted by mandates rather than permits or taxes. If shareholders own the right to emit a restricted amount of any pollutant, then they earn a scarcity rent that we would characterize as the return  $p_Z Z$ . In available data, this return might appear as part of the normal return to capital of the shareholders. In effect, then, we suppose that existing pollution restrictions and shadow prices are first converted to their equivalent explicit tax rates  $\tau_Z$ . We then evaluate marginal effects of environmental policy by calculating the effects of a small increase in that pollution tax.

<sup>28</sup> We define a unit of any input or output such that all initial prices are one ( $p_X = p_Y = p_K = p_L = p_Z = 1$ ). Then zero profit conditions imply  $X = K_X + L_X$  and  $Y = K_Y + L_Y + Z$ . We consider an economy with total factor income equal to one (which could be in billions or trillions). Then  $K_X + K_Y + L_X + L_Y = 1$ , and factor shares in each industry are enough to determine all  $\lambda$  and  $\theta$  values.

<sup>29</sup> In a more recent paper estimating this parameter, Claro (2003) finds elasticities of approximately 0.8, which is close to one. Babiker et al. (2003) also use  $\sigma_X = 1$  in their computational model.

<sup>30</sup> This is the same initial value used by Fullerton and Metcalf (2001). Little evidence exists on the substitution in utility between goods produced using pollution and those produced otherwise.

<sup>31</sup> The Appendix defines  $a_{ij} = \theta_{Yj} e_{ij}$ , where the parameters must satisfy  $a_{iL} + a_{iK} + a_{iZ} = 0$  for all  $i$ .

Table 1

Data and parameters for the case with equal factor intensities (where  $\theta_{YZ}=.25$ )

$K_Y=0.0800$	$L_Y=0.1200$
$K_X=0.3200$	$L_X=0.4800$
$\lambda_{KY}=0.2000$	$\lambda_{LY}=0.2000$
$\lambda_{KX}=0.8000$	$\lambda_{LX}=0.8000$
$\theta_{YK}=0.3000$	$\theta_{YL}=0.4500$
$\theta_{XK}=0.4000$	$\theta_{XL}=0.6000$

labor, capital, and pollution.<sup>32</sup> We therefore allow them to take alternative values of  $-1$ ,  $-1/2$ ,  $0$ ,  $1/2$ , and  $1$ . As these parameters vary, we consider the implications for the results  $(\hat{p}_Y, \hat{w}, \hat{r}, \hat{Z})$ .

In performing these calculations, we always use a 10% increase in pollution tax. Table 2 allows the Allen elasticities  $e_{KZ}$  and  $e_{LZ}$  to vary, holding constant the factor intensities.<sup>33</sup> Notice in the first column of results that the change in pollution is always negative, but the magnitude of the change varies drastically in a way that depends on the two varied parameters. The smallest change in pollution occurs when both of those parameters are zero ( $\hat{Z}=-.0200$  in row 2), but it is also small whenever the two are of opposite signs (e.g. row 1 or 5). The change in pollution is larger when both are positive, and it is largest in the Cobb–Douglas case where all three cross-price Allen elasticities are equal to one (row 12). As consistent with Proposition 5B, this row shows  $\hat{Z}=-\hat{\tau}_Z$ .

The changes in the wage rate and capital rental rate in the next two columns of Table 2 are always small, no more than about a half of a percent in either direction. The increase in output price is more substantial, always 2.5%. This result is no coincidence. With equal factor intensities, the simple result of Case 1 and Eq. (12a) is  $\hat{p}_Y=\theta_{YZ}\hat{\tau}_Z$ . Thus, the 10% increase in  $\tau_Z$  always raises the output price by 2.5%.

The primary purpose of Table 2 is to illustrate the effects of different cross-price elasticities, as in Case 1 where both sectors have the same factor intensities. Indeed, since  $e_{KL}=1$  satisfies Condition 1, the table reflects Proposition 1B where the pollution tax always imposes more burden on the relative complement to pollution. That is, labor bears more burden ( $\hat{w}<0$ ) in rows such as 3–4 where  $e_{KZ}>e_{LZ}$ , and capital bears more burden ( $\hat{r}<0$ ) in rows like 5–6 where  $e_{KZ}<e_{LZ}$ . When neither is a relative complement ( $e_{KZ}=e_{LZ}$ ), both are burdened equally as in Proposition 2A (rows 2, 7, and 12).

Can all of the burden on the sources side be shifted from one factor to the other? For either factor to bear none of the burden, or to gain, its return would have to rise by more than the overall price index  $p\equiv\varphi p_X+(1-\varphi)p_Y$  where  $\varphi=0.75$  is the share of expenditure on  $X$ . When  $\hat{p}_Y=0.025$ , the change in this price index is  $\hat{p}=.00625$ . In Table 2, the largest  $\hat{w}$  is 0.00341 (row 9), just over half of that price increase. In other words, labor can avoid “most” of the burden when the pollution tax induces the firm to use more labor ( $e_{LZ}=1$ ) and less capital ( $e_{KZ}=-.5$ ). The last two columns of Table 2 show that  $\hat{w}-\hat{p}<0$  and  $\hat{r}-\hat{p}<0$  for all parameters considered

<sup>32</sup> Humphrey and Moroney (1975) estimate Allen elasticities using capital, labor, and natural resource products. Bovenberg and Goulder (1997) use estimates of elasticities between labor, capital, energy, and materials. They interpret energy to be a proxy for pollution, strictly valid only if pollution is fixed per unit of energy. DeMooij and Bovenberg (1998) review such estimates and find that  $e_{KL}=0.5$ ,  $e_{KZ}=0.5$ , and  $e_{LZ}=0.3$  best summarize the existing literature. These figures suggest that capital might be a slightly better substitute for energy than is labor, but the difference is not precisely estimated. Their estimates are taken from data on Western European countries.

<sup>33</sup> The table does not contain every permutation of  $e_{KZ}$  and  $e_{LZ}$  between  $-1$  and  $1$ , because not all permutations are possible. Both cannot be negative, since we know that at most one of the three cross-price elasticities is negative. Furthermore, we omit combinations that result in a positive value for any  $e_{ii}$ . Finally, Table 2 always uses  $e_{KL}=1$ .

Table 2

Effects of a 10% increase in the pollution tax, equal factor intensities and varying substitution elasticities (where  $\theta_{YZ}=.25$  and  $\sigma_X=e_{KL}=\sigma_u=1$ )

Row	$e_{KZ}$	$e_{LZ}$	$\hat{Z}$	$\hat{w}$	$\hat{r}$	$\hat{p}_Y$	$\hat{w}-\hat{p}$	$\hat{r}-\hat{p}$
1	1.0	-0.5	-0.02559	-0.00335	0.00503	0.02500	-.00960	-.00122
2	0.0	0.0	-0.02000	0.00000	0.00000	0.02500	-.00625	-.00625
3	0.5	0.0	-0.03573	-0.00112	0.00169	0.02500	-.00737	-.00456
4	1.0	0.0	-0.05094	-0.00221	0.00331	0.02500	-.00846	-.00294
5	-0.5	0.5	-0.02690	0.00230	-0.00345	0.02500	-.00395	-.00970
6	0.0	0.5	-0.04373	0.00113	-0.00169	0.02500	-.00512	-.00794
7	0.5	0.5	-0.06000	0.00000	0.00000	0.02500	-.00625	-.00625
8	1.0	0.5	-0.07574	-0.00109	0.00164	0.02500	-.00734	-.00461
9	-0.5	1.0	-0.04955	0.00341	-0.00511	0.02500	-.00284	-.01136
10	0.0	1.0	-0.06693	0.00223	-0.00335	0.02500	-.00402	-.00960
11	0.5	1.0	-0.08374	0.00110	-0.00165	0.02500	-.00515	-.00790
12	1.0	1.0	-0.10000	0.00000	0.00000	0.02500	-.00625	-.00625

here, so both factors bear some burden. Yet one factor could gain if elasticity differences were greater.<sup>34</sup>

In the next table, we consider the impact of changes in factor intensities (with unchanging elasticities). We cannot just set the cross-price elasticities equal to each other, however, because then Proposition 2A says we get *no* effects on factor prices. Instead, all rows of Table 3 assume  $e_{LZ}=1$  and  $e_{KZ}=-.5$  (as in row 9 of Table 2, with the *largest* effects on factor prices). Then, with those elasticities fixed, we vary the factor intensities. The first column shows  $\gamma_K-\gamma_L$ , which is positive if the dirty sector is capital intensive. We vary this value from -0.25 to 0.25, which effectively changes the “data” of the initial economy. We then calculate new parameters  $\lambda$  and  $\theta$  that are consistent with each other (and with a fixed overall size of each sector  $X$  and  $Y$  and fixed resource quantities  $\bar{K}$  and  $\bar{L}$ ). The second column shows corresponding increases in  $\theta_{YK}$  from 0.15 to 0.44 (and row 6 shows prior results with  $\theta_{YK}=0.30$  and  $\theta_{XK}=0.40$ ).

The first five rows of Table 3 illustrate how labor can bear less than its share of the burden, even though the dirty sector is labor intensive, because labor is a better substitute for pollution. As the polluting sector is changed from labor intensive to capital intensive, the pollution tax burden is shown to shift even more onto capital. The fall in the rental rate enlarges from 0.22% to 0.83%. The change in the wage is always positive, because labor is the better substitute for pollution, but  $\hat{w}$  rises from 0.0018 to 0.0045 as the dirty sector becomes more capital-intensive. In this last row labor avoids “most” of the burden, with  $\hat{w}=0.0045$  relative to  $\hat{p}=0.00625$ , but labor still cannot avoid all of the burden – even in this combination where labor is a better substitute for pollution *and* the dirty sector is very capital intensive.

In general, the factor intensities of the two sectors are better estimated than are the Allen elasticities of substitution. In Table 3, the factor intensities are varied over a range that is much wider than the range of possible estimates, and still the proportional change in the wage rate varies by only 0.0027 (from 0.0018 to 0.0045). In contrast, Table 2 varies the cross-price Allen elasticities only from -0.5 to 1.0, a range that is *less* wide than the range of possible estimates, and  $\hat{w}$  varies by more than twice as much (by 0.0068, from -0.0034 to +0.0034). Thus, for the incidence of the pollution

<sup>34</sup> Suppose  $e_{KZ}=-1$  and  $e_{LZ}=3$ , so that capital and pollution are complements, while labor and pollution are strong substitutes. Then  $\hat{w}-\hat{p}=.00195$ , and labor gains.

Table 3

Effects of a 10% increase in the pollution tax, varying factor intensity (with  $e_{LZ}=1$ ,  $e_{KZ}=-0.5$ ,  $\theta_{YZ}=.25$ , and  $\sigma_X=e_{KL}=\sigma_u=1$ )

Row	$\gamma_K - \gamma_L^a$	$\theta_{YK}$	$\theta_{XK}$	$\hat{w}$	$\hat{r}$	$\hat{p}_Y$	$\hat{Z}$
1	-0.25	0.1515	0.4495	0.0018	-0.0022	0.0258	-0.0750
2	-0.20	0.1818	0.4394	0.0022	-0.0028	0.0257	-0.0699
3	-0.15	0.2118	0.4294	0.0025	-0.0033	0.0256	-0.0648
4	-0.10	0.2416	0.4195	0.0028	-0.0039	0.0255	-0.0597
5	-0.05	0.2710	0.4097	0.0031	-0.0045	0.0253	-0.0546
6 <sup>b</sup>	0.00	0.3000	0.4000	0.0034	-0.0051	0.0250	-0.0496
7	0.05	0.3286	0.3905	0.0037	-0.0057	0.0247	-0.0445
8	0.10	0.3566	0.3811	0.0039	-0.0064	0.0243	-0.0395
9	0.15	0.3841	0.3720	0.0041	-0.0070	0.0238	-0.0346
10	0.20	0.4110	0.3630	0.0044	-0.0076	0.0233	-0.0297
11	0.25	0.4373	0.3542	0.0045	-0.0083	0.0228	-0.0248

<sup>a</sup> Note that  $\gamma_K - \gamma_L = K_Y/K_X - L_Y/L_X$ .

<sup>b</sup> Results in row 6 match those in Table 2, row 9.

tax, we conclude that the impact of factor intensities over the plausible range is less important than the impact of the elasticities of substitution between pollution and capital or labor.

#### 4. Conclusion

Using a simple general equilibrium model of production with pollution, this paper has found the incidence of a pollution tax on the prices of outputs *and* on the returns to inputs. We present the system of equations that can be solved for the incidence of any tax on capital, labor, output, or pollution. A small increase in the pollution tax rate alters the return to labor relative to capital in a way that depends on the substitutability of labor for pollution, the substitutability of capital for pollution, and the relative factor intensities of the two sectors. When both sectors are equally capital-intensive and capital is a better substitute for pollution than is labor, then intuitively we expect the return to capital to rise relative to the wage. If labor and capital are highly complementary, however, then this intuitive result does not hold.

Another surprising result is in the case where both factors are equally substitutable for pollution. In that case, the pollution tax can increase the return to capital even when the polluting sector is more capital-intensive than the other sector, if consumers are less able to substitute among goods than producers of the dirty good are able to substitute among their inputs.

Numerically, it is shown that the elasticities of substitution in production between capital and pollution and between labor and pollution have an important effect on the incidence of a pollution tax. The impact of the uncertainty about substitution elasticities outweighs the impact of the uncertainty about factor intensities.

These results provide evidence that the substitutability of capital, labor, and emissions has very important consequences for environmental policy, and that more work needs to be done in estimating these parameters and analyzing their effects. Not only do these elasticities affect tax incidence, as shown in the main results of this paper, they affect the impact of environmental policy on the environment itself. For alternative parameter values used here, a 10% increase in the pollution tax rate reduces pollution anywhere from 2% to 10%. For extreme parameter values, it can lead to more pollution.

Further research could extend in many directions. First, any of the simplifying assumptions of our model could be relaxed to see how results are affected by alternative assumptions such as: imperfect factor mobility, adjustment costs, imperfect competition, non-constant returns to scale, international trade in goods or factors, tax evasion, or uncertainty. In many cases, the results of such extensions can be predicted from the literature that followed the original article by Harberger (1962). In a model with perfect international capital mobility, for example, the net return to capital is fixed by world capital markets, and so the pollution tax cannot place a burden on capital – in contrast to the results here for effects on the wage and capital return in a closed economy.

Another direction for further research is to calculate the effects of these price changes on different income groups (or regions, or racial groups). With data on the labor and capital income of each group, our results for  $\hat{w}$  and  $\hat{r}$  could be used to calculate the effect of a pollution tax on the sources of income for each group. With additional data on the expenditures, our results for  $\hat{p}_Y$  could be used to calculate the effect on the uses side. Finally, our analytical model could be extended to a computational general equilibrium (CGE) model with more factors, sectors, and groups.

Still, the model in this paper provides the first theoretical analysis of the incidence and distributional effects of environmental policy that allows for fully general forms of substitution among factors and that solves for all general equilibrium effects of the pollution tax. The analytical model is used to derive general propositions that do not depend upon the particular parameter values that must be used in a CGE model. Our model also is used to identify the crucial parameters. In particular, we show how differential substitution between factors can greatly affect the burdens of a pollution tax.

## Appendix A. Deriving Eqs. (4)–(7)

Given a set of input prices  $p_K$ ,  $p_L$ , and  $p_Z$ , and output decision  $Y$ , the solution to the dirty sector's cost-minimization problem consists of three input demand functions:

$$\begin{aligned} K_Y &= K_Y(p_K, p_L, p_Z, Y) \\ L_Y &= L_Y(p_K, p_L, p_Z, Y) \\ Z &= Z(p_K, p_L, p_Z, Y) \end{aligned}$$

Totally differentiating and dividing through by the appropriate input level yields:

$$\begin{aligned} \hat{K}_Y &= a_{KK} \hat{p}_K + a_{KL} \hat{p}_L + a_{KZ} \hat{p}_Z + \hat{Y}, \\ \hat{L}_Y &= a_{LK} \hat{p}_K + a_{LL} \hat{p}_L + a_{LZ} \hat{p}_Z + \hat{Y}, \\ \hat{Z} &= a_{ZK} \hat{p}_K + a_{ZL} \hat{p}_L + a_{ZZ} \hat{p}_Z + \hat{Y}, \end{aligned}$$

where  $a_{ij}$  is the elasticity of demand for input  $i$  with respect to the price of input  $j$ . As Mieszkowski (1972) notes, this  $a_{ij}$  equals  $e_{ij}\theta_{Yj}$ . Note that  $a_{ij}$  does not have to equal  $a_{ji}$  even though  $e_{ij}=e_{ji}$ . Also,  $e_{ii}$  and  $a_{ii}$  must be  $\leq 0$ , and  $a_{iK}+a_{iL}+a_{iZ}=0$  (Allen, 1938). Thus, at most one of these two cross-price elasticities can be negative.<sup>35</sup>

<sup>35</sup> Moreover, given symmetry ( $e_{ij}=e_{ji}$ ) this result means either that all three cross-price elasticities ( $e_{KL}$ ,  $e_{KZ}$ , and  $e_{LZ}$ ) are positive or that one is negative and the other two are positive.

The three input demand functions are not independent, since the production function gives the relationship between the three inputs and output  $Y$ . Hence we can use any two of these functions. Again following Mieszkowski, we subtract the third equation from each of the first two equations so that the system of the two remaining equations contains all of the Allen elasticities of substitution. When we substitute in the expressions for  $a_{ij}$  and the price changes, we get Eqs. (4) and (5) in the text.<sup>36</sup>

Assuming perfect competition, each input to production must be paid a price equal to its marginal revenue product:

$$\begin{aligned} p_X X_K &= r(1 + \tau_K) = p_Y Y_K, \\ p_X X_L &= w(1 + \tau_L) = p_Y Y_L, \\ p_Y Y_Z &= p_Z = \tau_Z. \end{aligned}$$

where  $X_K$ ,  $X_L$ ,  $Y_K$ ,  $Y_L$ , and  $Y_Z$  are the derivatives of the production functions with respect to each input. Perfect competition and constant returns to scale assure that the value of output must equal the sum of factor payments:

$$\begin{aligned} p_X X &= r(1 + \tau_K)K_X + w(1 + \tau_L)L_X, \\ p_Y Y &= r(1 + \tau_K)K_Y + w(1 + \tau_L)L_Y + \tau_Z Z. \end{aligned}$$

Totally differentiate these equations, divide through by sales revenue ( $p_X X$  or  $p_Y Y$ ), and rearrange terms to obtain Eqs. (6) and (7) in the text.

## Appendix B. General solution to the system

First, to eliminate  $\hat{X}$  and  $\hat{Y}$ , subtract Eq. (8) from Eq. (6) and Eq. (9) from Eq. (7):

$$\hat{p}_X = \theta_{XK} \hat{r} + \theta_{XL} \hat{w} \tag{A1}$$

$$\hat{p}_Y = \theta_{YK} \hat{r} + \theta_{YL} \hat{w} + \theta_{YZ} \hat{\tau}_Z. \tag{A2}$$

Eqs. (A1) and (A2) tell us how changes in net-of-tax factor prices are passed on to output prices, according to the factor shares in each industry. Substituting Eqs.(8) and (9) into Eq. (10) yields

$$\theta_{XK} \hat{K}_X + \theta_{XL} \hat{L}_X - \theta_{YK} \hat{K}_Y - \theta_{YL} \hat{L}_Y - \theta_{YZ} \hat{Z} = \sigma_u (\hat{p}_Y - \hat{p}_X). \tag{A3}$$

Solving Eqs. (1) and (2) for  $\hat{K}_Y$  and  $\hat{L}_X$ , respectively, and substituting these expressions in to Eqs. (3) and (A3) gives us

$$-\gamma_K \hat{K}_Y + \gamma_L \hat{L}_X = \sigma_X (\hat{w} - \hat{r}), \tag{A4}$$

$$-(\theta_{XK} \gamma_K + \theta_{YK}) \hat{K}_Y - (\theta_{XL} \gamma_L + \theta_{YL}) \hat{L}_X - \theta_{YZ} \hat{Z} = \sigma_u (\hat{p}_Y - \hat{p}_X). \tag{A5}$$

<sup>36</sup> De Mooij and Bovenberg (1998) derive analogous expressions with a fixed input factor or price.

Finally, to eliminate  $\hat{K}$  and  $\hat{L}$ , we use Eqs. (4) and (5) to solve for them and then substitute the resulting expressions into (A4) and (A5). After some rearrangement, these equations become:

$$\begin{aligned} \sigma_X(\hat{w}-\hat{r}) = & (\gamma_L-\gamma_K) \hat{Z} + \theta_{YK}[\gamma_L(e_{LK}-e_{ZK})-\gamma_K(e_{KK}-e_{ZK})] \hat{r} \\ & + \theta_{YL}[\gamma_L(e_{LL}-e_{ZL})-\gamma_K(e_{KL}-e_{ZL})] \hat{w} \\ & + \theta_{YZ}[\gamma_L(e_{LZ}-e_{ZZ})-\gamma_K(e_{KZ}-e_{ZZ})] \hat{\tau}_Z \end{aligned} \quad (\text{A6})$$

and

$$\begin{aligned} -\sigma_u(\hat{p}_Y-\hat{p}_X) = & C \hat{Z} + \theta_{YK}[\beta_K(e_{KK}-e_{ZK}) + \beta_L(e_{LK}-e_{ZK})] \hat{r} \\ & + \theta_{YL}[\beta_K(e_{KL}-e_{ZL}) + \beta_L(e_{LL}-e_{ZL})] \hat{w} \\ & + \theta_{YZ}[\beta_K(e_{KZ}-e_{ZZ}) + \beta_L(e_{LZ}-e_{ZZ})] \hat{\tau}_Z \end{aligned} \quad (\text{A7})$$

At this point we set good  $X$  as the numeraire, so that  $\hat{p}_X=0$ . Then, plugging Eq. (A2) into (A7) to eliminate  $\hat{p}_Y$  yields, after some rearrangement:

$$\begin{aligned} C \hat{Z} = & -\theta_{YK}[\beta_K(e_{KK}-e_{ZK}) + \beta_L(e_{LK}-e_{ZK}) + \sigma_u] \hat{r} - \theta_{YL}[\beta_K(e_{KL}-e_{ZL}) \\ & + \beta_L(e_{LL}-e_{ZL}) + \sigma_u] \hat{w} - \theta_{YZ}[\beta_K(e_{KZ}-e_{ZZ}) + \beta_L(e_{LZ}-e_{ZZ}) + \sigma_u] \hat{\tau}_Z \end{aligned} \quad (\text{A8})$$

Solve (A6) and (A8) for  $\hat{Z}$ , equate, and rearrange, to get:

$$\begin{aligned} & [\theta_{YK}(-A(e_{KK}-e_{ZK}) + B(e_{LK}-e_{ZK}) + (\gamma_K-\gamma_L)\sigma_u) + C\sigma_X] \hat{r} \\ & + [\theta_{YL}(-A(e_{KL}-e_{ZL}) + B(e_{LL}-e_{ZL}) + (\gamma_K-\gamma_L)\sigma_u) - C\sigma_X] \hat{w} \\ & = \theta_{YZ}[-A(e_{ZZ}-e_{KZ}) + B(e_{ZZ}-e_{LZ}) + (\gamma_L-\gamma_K)\sigma_u] \hat{\tau}_Z \end{aligned} \quad (\text{A9})$$

Now Eqs. (A1) and (A9) are two equations in only two unknowns,  $\hat{r}$  and  $\hat{w}$ . Use (A1) to solve for  $\hat{w}$  in terms of  $\hat{r}$ , substitute into (A9), and simplify, to reach Eq. (11c) in the text. Substitute this back into previous equations to obtain the equations for  $\hat{p}_Y$ ,  $\hat{w}$ , and  $\hat{Z}$  [(11a), (11b), (11d)].

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