Section 4.1 - Exponential Functions

Recall: Linear $f(x) = mx + b$ slow

Quadratic $f(x) = ax^2 + bx + c$ fast

Def: An exponential function is a function in which the variable is in the exponent and the base is a positive constant.

$$f(x) = a^x, \text{ for } a > 0$$

Ex: $f(x) = 10^x$, $g(x) = 750(1.05)^x$, $h(x) = 3^{6x}$,

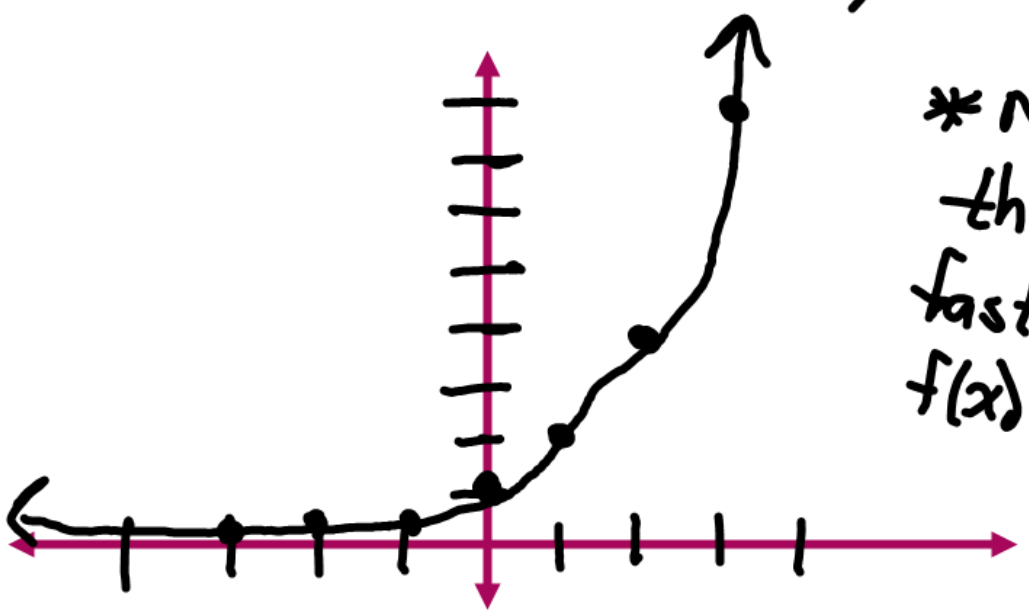
$k(x) = 2^{1-x^2}$

Example #1

Graph $f(x) = 2^x$

x	$f(x)$
-3	$2^{-3} = 1/2^3 = 1/8$
-2	$2^{-2} = 1/2^2 = 1/4$
-1	$1/2$
0	1
1	2
2	4
3	8
4	16

$a^0 = 1$



* Notice that this grows faster than $f(x) = x^2$.

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

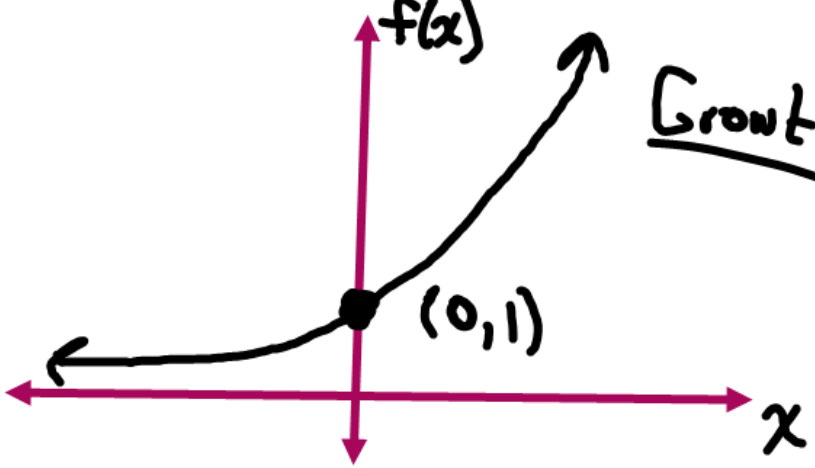
$2^{-1000} = \frac{1}{2^{1000}} \neq 0$

. no x can produce 0.

$f(x) = a^x$

* If $a > 1$, we have exponential growth.
* If $0 < a < 1$, we have exponential decay.

$a > 1$

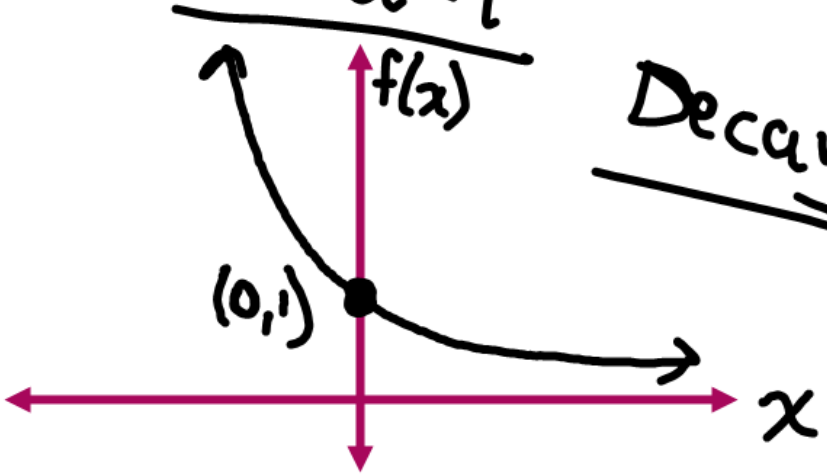


D: $(-\infty, \infty)$ R: $(0, \infty)$

y-int: $a^0 = 1 \rightarrow (0,1)$

$y = 0$ is a H.A.

$0 < a < 1$

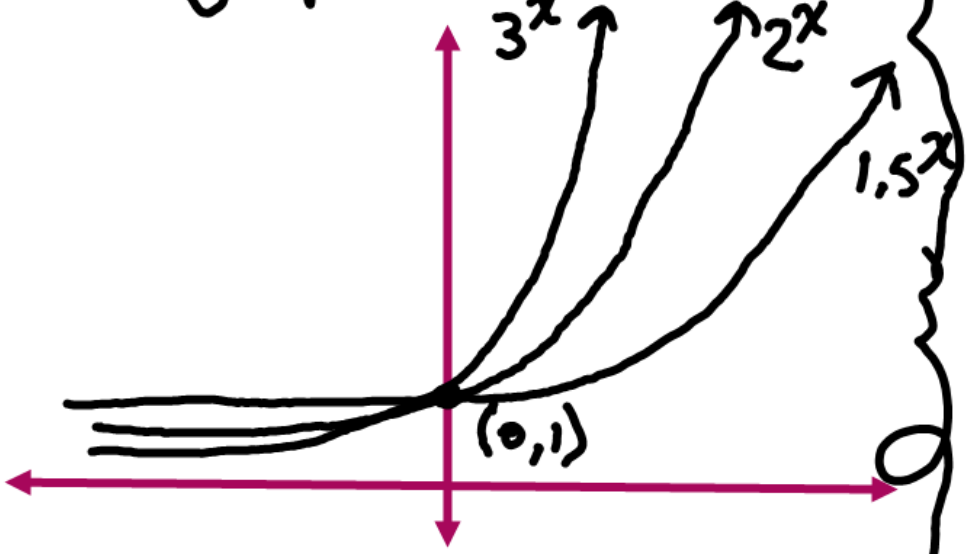


D: $(-\infty, \infty)$ R: $(0, \infty)$

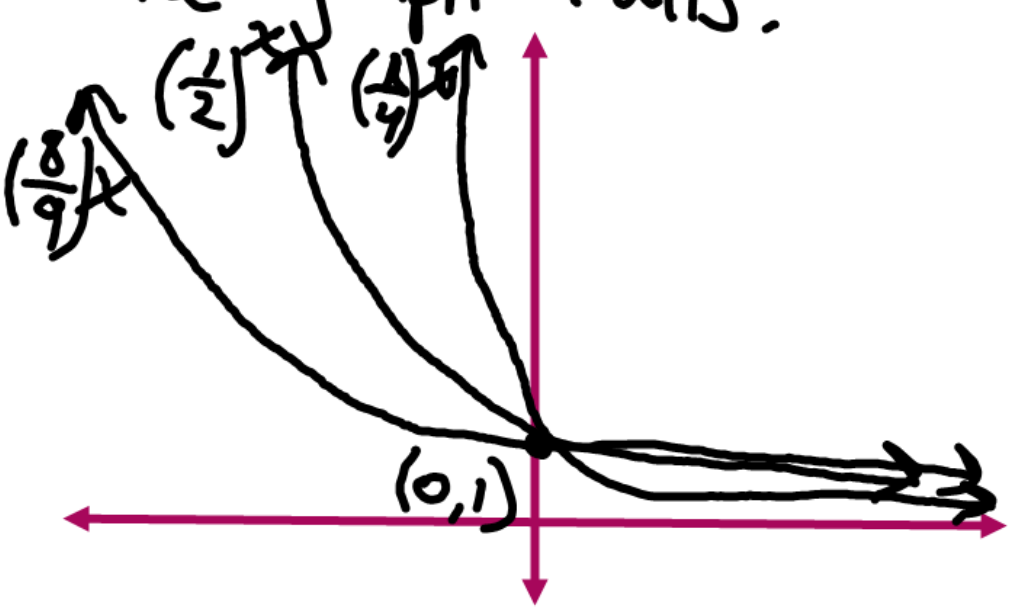
y-int: $(0,1)$

$y = 0$ is a H.A.

The larger the base "a", the more steeply the graph rises.



The smaller the base "a", the more steeply the graph falls.



Start #29

$f(x) = a^x$

$f(3) = 27$

$\sqrt[3]{27} = \sqrt[3]{a^3}$

$27^{1/3}$

$3 = a$

$f(x) = 3^x$

Recall:

1. $a^x \cdot a^y = a^{x+y}$
2. $(a^x)^y = a^{xy}$
3. $a^x / a^y = a^{x-y}$
4. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Ex: $3^{x+1} = 3^x \cdot 3^1 = 3(3)^x$

$5^{3x} = (5^3)^x = 125^x$

$3^{x-1} = 3^x \cdot 3^{-1} = 3^x \cdot \frac{1}{3}$

$= \boxed{\frac{1}{3}(3)^x}$

$3^{1-x} = 3^1 \cdot 3^{-x} = 3 \cdot \frac{1}{3^x}$

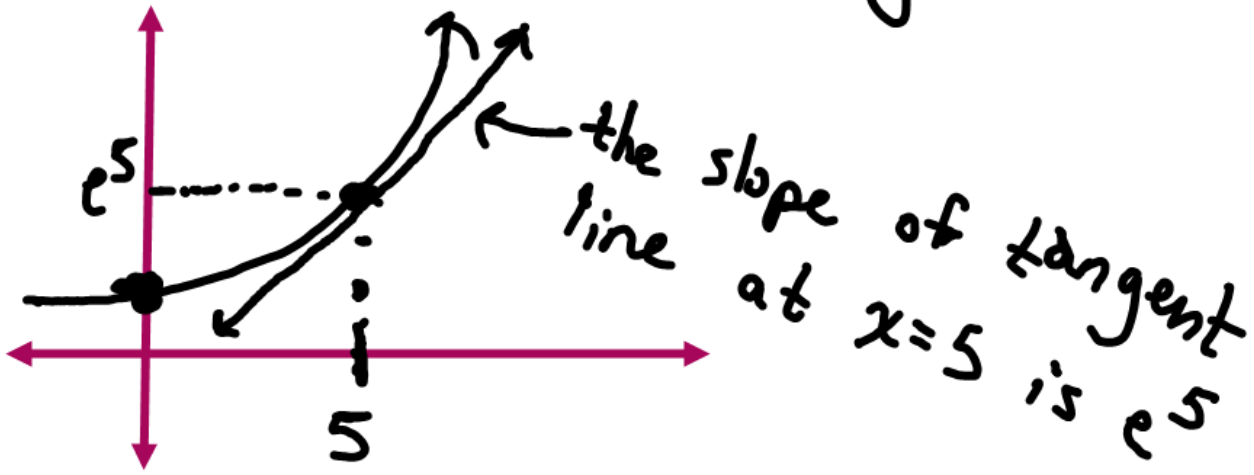
$= \boxed{3 \cdot \left(\frac{1}{3}\right)^x}$

* The number e: one of most useful, naturally-occurring bases

$\pi = 3.14159\ldots$

$e \approx 2.718281828459\ldots$

↳ Interesting fact about $f(x) = e^x$: The value of the function is always equal to the rate of change at that x-value.



Example 2

Section 4.1 – Exponential Functions

The assets (in billions of dollars) for the multinational insurance company American International Group Inc. (AIG) can be approximated by the function

$$A(x) = 1756e^{-.091x}$$

where $x = 7$ corresponds to the year 2007.

a. What are the approximate assets of AIG in 2007 (prior to the great recession)? What about in 2015?
b. What was the first full year when AIG fell below $600 billion?

a.) $x = 7$ $\quad A(7) = 1756e^{-.091(7)} = \$928.71$ billion

$x = 15$ $\quad A(15) = 1756e^{-.091(15)} = \$448.45$ billion

b.) 2011 → 645.35 billion

2012 → 589.22 billion

2012