Calculus Field Day **Practice** **AB Session 1 (No Graphing Calculator)**

School:_____

Team Members: _____

_____

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Part I. (30 minutes; No graphing calculator is permitted.)

Each team has 30 minutes to answer three questions.

Each team submits one set of answers at the end of the thirty minutes. You must cross out (without penalty) whatever you do not wish to be considered.

You must show steps and Reasoning. Partial credit will be given. Answers should be kept in exact form, e.g., 2e or ln(3). Remember to include units: feet/second, meters, etc. You do not have to simplify final numerical answers.

No calculator is allowed for these problems.

Name: _____

School: _____

1.) Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfies $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

a.) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

b.) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$, for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than the exact value $f(1.1)$? Explain your reasoning.

c.) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

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2.) Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

a.) Find $f'(x)$ and $f''(x)$.

b.) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

c.) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

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3.) Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \leq t \leq 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table below.

t (minutes)	0	1	2	3	4	5	6
C(t) (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

a.) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

b.) Is there a time t, $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.

c.) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,\mathrm{dt}$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,\mathrm{dt}$ in the context of the problem.

d.) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.