

- 1.) A new regional online newspaper with a small market niche estimates that its annual profit from advertising is modeled by

$$P(x) = -0.5x^3 + 8x^2 + 2x + 160$$

thousand dollars when its circulation is x **thousand subscribers**, for $0 \leq x \leq 100$. Currently there are 5,000 subscribers and the number of subscribers is increasing at the rate of 2,000 per year.

- What is the current profit?
- How many **subscribers** will there be in two years?
- Write down a formula $x(t)$ for the number of subscribers as a function of time in years.
- Is profit increasing, decreasing, or constant when there are 9000 subscribers? Explain how you know this.
- Write down a formula for the profit, P , as a function of time, t . You do not need to simplify it.
- At how many years will the profit be a maximum? (Round to the nearest year). Give your reasoning.

Solution:

- $P(5) = -.5 \cdot 5^3 + 8 \cdot 5^2 + 2 \cdot 5 + 160$
= 307.5 thousand dollars.
- $2 \cdot 2 + 5 = 9$ thousand subscribers
- $x(t) = 2t + 5$ thousand subscribers
- Profit is *increasing*. A cubic has at most two turning points. Using a calculator we can see that the profit function is increasing when $x = 9$. (Or they can test points at lower and higher values of x .)
- $P(x(t)) = -0.5(2t + 5)^3 + 8(2t + 5)^2 +$
 $2(2t + 5) + 160$ thousand dollars.
- Using calculator (either by inspection, or using Max function) there is a max when x is approximately 10.8 thousands subscribers. Therefore,

$$2t + 5 = 10.8$$

$$t = \frac{10.8 - 5}{2}$$

$$t = 3 \text{ years, approximately}$$

Scoring Rubric:

- 1 point
- 1 point
- 1 point
- 1 point
- 1 point
- 2 points:
reasoning (1point)
the year (1 point)

2.) The following chart shows the average weight of a baby girl at various time periods in months after being born.

| | | | | | | | | | | | | | | | | | |
|---------------|-----|-----|------|-------|------|------|-------|-------|------|----|------|------|-------|-------|------|-------|------|
| Time (mo.) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Weight (lbs.) | 7.4 | 9.6 | 11.8 | 13.45 | 14.8 | 15.9 | 16.75 | 17.65 | 18.3 | 19 | 19.5 | 20.2 | 20.75 | 22.25 | 23.8 | 25.15 | 26.7 |

- a.) Input the above data in your calculator. According to your **table** what is the average rate of change of weight in the first six months, and in the next six months (i.e. months 0 to 6 and months 7 to 12)? Round to three decimal places and give units.
- b.) Graph the data points using your graphing calculator. From linear, quadratic, exponential, cubic and logarithmic, fit a model to the given data. Why did you choose that model? Provide the formula for $w(t)$ the weight in pounds as a function of t months. Round all coefficients to three decimal places and give units.
- c.) What does your **rounded** model predict for the weight of a one-year baby girl? Use three decimal places.

Solution:

a.) $\text{AROC} = \frac{16.75 - 7.4}{6} = 1.558 \text{ lbs/mo.}$
 $\text{AROC} = \frac{20.75 - 17.65}{6} = 0.517 \text{ lbs/mo.}$

b.) Cubic function; changes concavity once.
 $w(t) = 0.009t^3 - 0.232t^2 + 2.652t + 7.323 \text{ lbs.}$

c.) $w(12) = 21.291 \text{ lbs}$

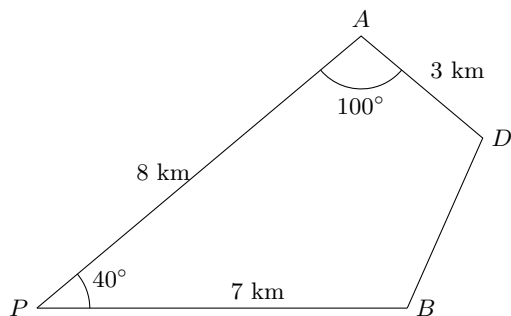
Scoring Rubric:

a.) 3 points:
 for each correct answer (1 point)
 for decimals and units (1 points)

b.) 3 points:
 cubic (1 point)
 reason (1 point)
 correct function

c.) 3 points:
 correct answer (2 points)
 for decimals and units (1 point)

- 3.) Hikers Andrew and Brett take separate trails from their starting point P to get to their destination D . They walk at an angle of 40° apart from each other as shown, and stop to eat their lunch at positions A and B respectively.



- a.) How far apart are lunch positions A and B ?
 b.) How far does Brett still have to walk from B to reach destination D ?

Round all answers to three decimal places and give units.

Solution:

- a.) Let a denotes the distance from A to B .
 By cosine rule we have:

$$a^2 = 8^2 + 7^2 - 2 \cdot 8 \cdot 7 \cos(40^\circ)$$

$$a = 5.216 \text{ km}$$

- b.) Let x denote the measure of the angle PAB .
 By sine rule we have:

$$\frac{\sin x}{7} = \frac{\sin(40^\circ)}{a}$$

$$\sin x = \frac{7 \cdot \sin(40^\circ)}{5.216}$$

$$\sin x = 0.863$$

$$x = 59.655^\circ.$$

Let y denote the measure of the angle BAD ,
 and b the distance from B to D .

$$y = 100^\circ - x = 100 - 59.655 = 40.345^\circ.$$

Using again cosine rule we get:

$$b^2 = (5.216)^2 + 3^2 - 2 \cdot 5.216 \cdot 3 \cos(40.345^\circ)$$

$$b = \mathbf{3.515 \text{ km.}}$$

Scoring Rubric:

- a.) 3 points:
 Cosine rule (1 point)
 Correct information in the cosine rule (1 point)
 Correct answer with decimals and units (1 point)
- b.) 6 points:
 Using the sine rule (1 point)
 Finding $\sin x$ (1 point)
 Correct angle x (1 point)
 Finding y (1 point)
 Cosine rule (1 point)
 Correct answer with decimals and units (1 point)