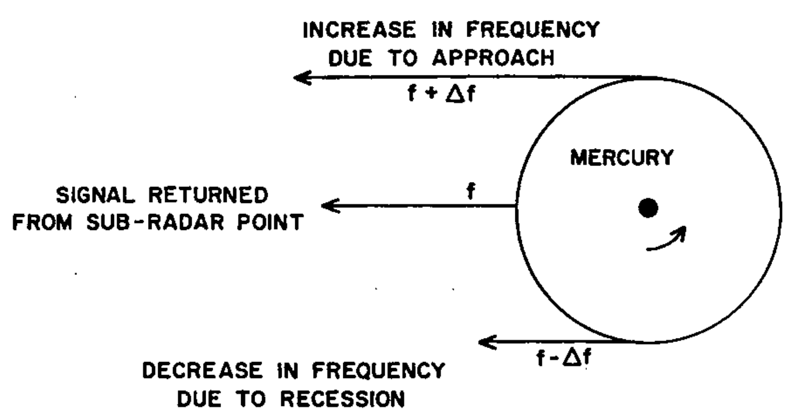
Mercury's Rotational Period

Determining the basic properties of the planets (size, mass, motion) is an important first step for the observational astronomer. This information can be used to build a more complete understanding of the solar system. Sometimes direct observation suffices, but in other cases it provides misleading information. The rotational period of Mars was successfully determined in this manner. However, similar attempts for Mercury made by Bessel, Schiaparelli, and others produced conflicting results. Bessel deduced a period of 24 hours, but long-term observations by Schiaparelli (and apparently confirmed by others) placed its period at 88 days. This is the same as Mercury's period of revolution, and was often described as being in "captive rotation" with one face always towards the sun.

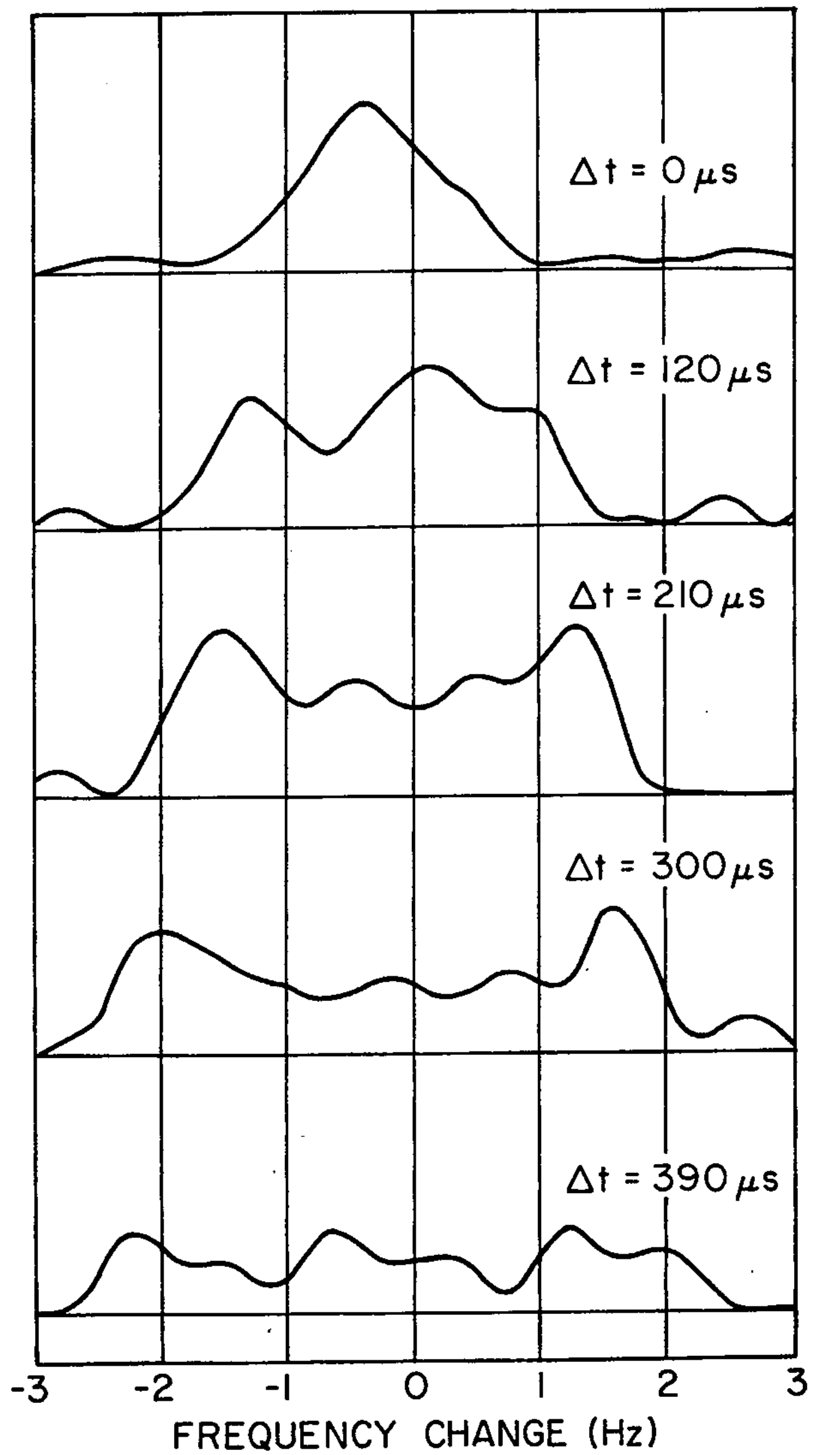
The first attempts were made to use spectroscopy about 1900 for the purpose of determining rotation periods of planets. It was first applied by Keeler to the rings of Saturn, and attempts were made by Slipher and others to determine Mercury's rotational period. The slit of a spectrograph was laid parallel to the equator of the planet. The lines from the receding edge were red-shifted while those from the approaching edge were blue shifted, obeying the classic Doppler formula. The results obtained by this method indicated that Mercury took several days to rotate, but precise measurements could not be made.

A much more powerful method became possible during the a960s when radar signals were successfully bounced from planetary surfaces. Pettengill, Dyce, and Shapiro produced an accurate rotational period for the planet Mercury using this method. In 1965 they used the 1000-ft radio telescope at Arecibo, Puerto Rico to beam a series of 0.0005-second and 0.0001-second radar pulses at 430 MHz toward the planet. Since the round trip travel time of the pulses was much greater than the pulse length, they could see how the pulses were broadened by reflection from the rotating planet. Frequency shifts also resulted from the relative motions of the planet and the earth's rotation, but these were corrected by using careful timing and computer compensation.

The figure below shows that when a radar signal is reflected from a rotating spherical planet the echo is spread out in time as well as in frequency. The echo first returned is from the sub-radar point. After a small time delay the echo is received from a ring-shaped area centered on this point. That part of the signal returned from the approaching edge will be returned with an increase in frequency ("blue-shifted") and that part returned from the receding edge will be returned with a decrease in frequency ("red-shifted").



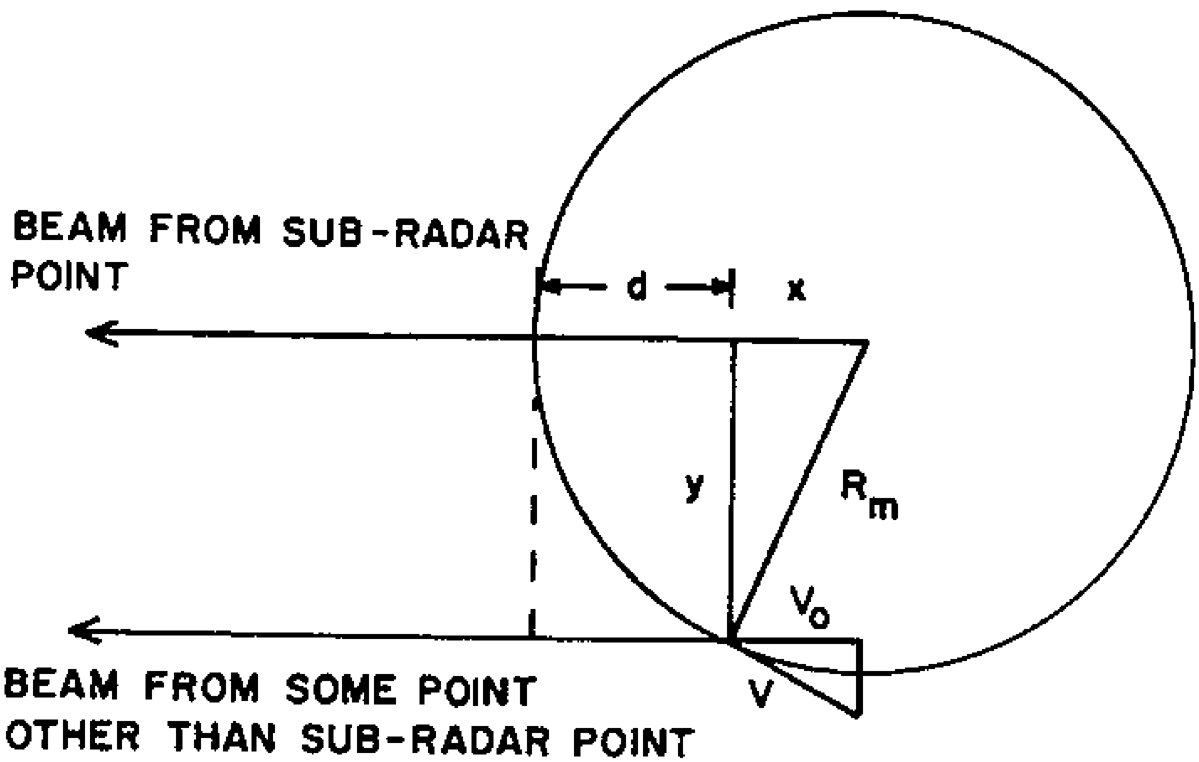
The chart below shows the spectrum of the radar echo for five different time delays (Δt). Note that the longer the time delay, the broader is the return signal in frequency. This broadening is because successive signals return from farther and farther from the sub-radar point. The portion of the planet rotating toward the earth causes the signal to have an increase in frequency (+) and the portion rotating away has a decrease (-). This increase or decrease obeys the Doppler law.



Spectrum of radar pulses returning from Mercury made on August 17, 1965
Δt=time delay. (Taken from *Astronomical Journal*, 72, p.351, 1967.)

In principle it ought to be easy to determine the rotational velocity of Mercury's limb, and (by knowing the planet's circumference) to calculate the rotational period. However, the echo weakens towards the edge of the planet and the signal return from the limb is unobtainable. We will use the echo from a ring intermediate between the sub-radar point and the limb to calculate the line-of-sight component of Mercury's rotational velocity and from this find its true rotational velocity.

Examine the figure below. Note that each signal in the chart above is labeled with its time delay in microseconds (μs) (1 μs = 10^{-6} s). It is easy to calculate the distance d that any delayed beam has traveled beyond that sub-radar point by multiplying half the time delay by the speed of the radar wave (the speed of light). This information is needed to obtain the line-of-sight velocity (v_0) in order to get the true rotational velocity (v).



Procedure

1.) Choose one of the time delayed signals in the chart and calculate

$$d = \frac{1}{2} c\Delta t$$

where d is the distance in meters, Δt is the time delay in s, and c = 3 X 10^8 m/s.

2.) In the figure above, the lengths x and y are given by

$$x = R - d$$

$$y = \sqrt{R^2 - x^2}$$

where R is the radius of Mercury (R = 2.42 X 10^6 m). Calculate x and y.

3.) Using the previously selected signal from the chart find v_0, the observed line-of-sight component of the rotational velocity at some point indicated in the figure above. The Doppler equation is generally stated in terms of a change in wavelength ($\Delta\lambda$) relative to the "rest" wavelength (λ), but it can also be stated in terms of frequency (f) and frequency shift (Δf)

$$\frac{\Delta f}{f} = \frac{v_0}{c}$$

where Δf is the shift in frequency, f is the frequency of the transmitted signal (f = 430 X 10^6 Hz), v_0 is the observed velocity, and c is the speed of the radar wave (c = 3 X 10^8m/s).
Examine your selected radar signal from the chart and mark the points to the left and right of center where the signal *begins* to drop down toward the baseline. Read the frequency change at each of these points as accurately as you can. Disregarding algebraic signs, average the results from the two shoulders. The actual Doppler frequency shift, Δf, is half the value as this is a reflected signal and the radar pulse is shifted "going in" and again "coming out." Calculate v_0 in meters per second.

4.) From the line-of-sight component v_0, calculate v, the foreshortened rotational velocity. As seen in the last figure above, the triangle containing x, y, and R is similar to the triangle containing v_0 and v.

$$\frac{v}{v_0} = \frac{R}{y}$$

Calculate v from this equation. The result is the true rotational velocity in meters per second.

5.) Calculate Mercury's rotational period by dividing v into the circumference of Mercury (C = 1.52 X 10^7 m). Compare your result with the accepted value of the rotation period of 58 d 15 h 30 m (5.067 X 10^6 s).