Moral Hazard and Customer Loyalty Programs†

Leonardo J. Basso*, Matthew T. Clements** and Thomas W. Ross***

August 14, 2007

Abstract

Frequent flier plans (FFPs) -- operated by almost all of the world’s major airlines -- have become the most famous, and probably the most valuable, customer loyalty programs in the world. In addition, plans created on the FFP model are now offered by sellers in a number of other industries such as hotels, car rental agencies and credit cards. In this paper we present a theory of FFPs that models them as efforts to take advantage of the agency relationship between employers -- who pay for airline tickets -- and employees -- who book those tickets. In this view, FFP benefits constitute “bribes”, inducing employees to book flights on airlines with higher prices. The model considers two airlines competing in a differentiated product environment and shows that a single airline offering an FFP has a large advantage. However, when both airlines operate plans, it is very possible that, while raising prices, competition (now via FFP benefits) will be intensified so much that the airlines end up worse off than had they not created the plans. Thus, in contrast to switching cost treatments of FFPs, we may observe prices and profits moving in opposite directions.

† The authors thank Jim Dana for contributions that significantly improved this paper. We are also grateful to the Social Sciences and Humanities Research Council of Canada and the Phelps Centre for the Study of Government and Business in the Sauder School of Business at the University of British Columbia for financial support; to Mark Armstrong, David Gillen, Mara Lederman, Anming Zhang and seminar participants at Lingnan University, the Hong Kong University of Science and Technology, Peking University, the University of British Columbia, Cornell University, Vanderbilt University, Carleton University, Simon Fraser University and the Pan American Conference on Transport Engineering for helpful discussions; and to Jennifer Ng for excellent research assistance.

* Department of Civil Engineering, Universidad de Chile
** Department of Economics, University of Texas at Austin
*** Sauder School of Business, University of British Columbia
1. Introduction

There can be little doubt that airline frequent flier plans (FFPs) are one of the most significant developments in the history of relationship marketing and customer loyalty plans. While varying somewhat one to another, these plans in general reward loyal (and frequent) customers with free flights, upgrades, products and other services. Given their current size in terms of number of participating airlines, plan members and miles collected and redeemed, it may be hard to believe that the oldest plans are barely 25 years old. For example, the Economist magazine recently estimated that the total stock of unredeemed frequent-flier miles is now worth more (at $700 billion US) than all the dollar bills in circulation around the world.¹

While these plans have attracted a great deal of attention in the transportation and marketing literatures, there has been relatively little formal modeling of FFPs by economists. The purpose of this paper is to analyze an often recognized, but never (to our knowledge) modeled aspect of FFPs -- namely that they exploit an agency relationship between employers who pay for business travel and employees who book the travel and collect the benefits of the FFP.² We also comment upon the use of loyalty programs more generally: we observe programs built on the FFP model for hotels, rental car agencies, and some other products and services, and it is not obvious why these programs are present in some industries but not others.

An employee selecting an airline (or hotel, or car rental agency) will not necessarily have the right incentives to find the lowest possible price when the employer

¹ “In Terminal decline?”, Economist, January 6, 2005.  
² Economists are certainly among those who have recognized this property of FFPs. See, e.g. Tretheway [1989] and the Los Angeles Times "Viewpoints" piece by Klemperer and Png [1986]. It was also discussed by Levine [1987, p. 452].
is paying the bill. With respect to business travel on airlines there are two reasons to be concerned. First, there is the normal agency relationship between the employer who pays most (possibly all) the costs of the ticket and the employee who selects the airline to be flown. We show here how this third-party payer problem can, by itself, increase equilibrium prices above the levels we would observe were employees responsible for paying for their own tickets. In addition there are the added temptations for employees stemming from frequent flier plans. Even a relatively small enticement in the form of a frequent flier benefit may be enough to induce morally hazardous behavior, resulting in even higher airline prices.

In this paper, we show how these programs can give an advantage to a lone airline adopter, but that imitation by competitors can dissipate the benefits such that the airlines are no better with them than they were without them; and that they might even be worse off. Employee/passengers who collect the FFP points clearly benefit from the programs while employers will pay higher prices for employee travel.

The next section of the paper provides some basic background on frequent flier plans and presents an overview of the model. Section III presents the formal analysis while Section IV provides our concluding thoughts. While we will present the analysis with respect to FFPs, the reader should keep in mind that our results will also apply to similar loyalty programs in other industries.
II. Background and overview of the model

Since American Airlines launched the first major FFP, its AAdvantage program, in 1981, most major airlines have adopted their own FFPs. In the typical FFP, rewards are based upon the number of “miles” travelers have accumulated by flying with the sponsoring airline. Rewards are most often free flights, but other benefits (e.g. flight class upgrades) are available in many programs as well. As suggested by the Economist article cited above, these programs are now huge, both in terms of the number of members and their accumulated miles. There are a number of purposes FFPs can and do serve that helps to explain their current popularity with airlines.

(i) Price Discrimination: FFPs function as second degree price discrimination by serving as a sort of quantity discount scheme. To the extent that some buyers will not earn enough miles to collect rewards, through the FFP, the airline effectively charges them a higher price than it charges its larger volume purchasers.

(ii) Switching Costs: A common concern with FFPs is that they create switching costs as travelers are inclined to stick with one airline to accumulate the largest balances possible so as to reach the necessary threshold levels. This aspect of FFPs, recognized by Klemperer [1987, p. 376 and 1995, pp. 517-518], has been formally modeled by Banerjee and Summers [1987], Kim et al. [2001] and others. In Banerjee and Summers’ model, the switching costs created by FFPs can soften competition so drastically that the

---

3 See Shaw [2004, p. 235].
4 The same Economist article estimated that by the end of 2004, 14 trillion frequent flier miles had been accumulated worldwide. There are at least eight airlines—All Nippon Airways, American, Continental, Delta, JAL, Northwest, United, and US Air—that each have a worldwide membership in the tens of millions.
5 For example, important work on loyalty programs and switching costs with obvious applications to FFPs includes Caminal and Claici [2007] and Caminal and Matutes [1990]. These papers illustrate cases in which switching costs can make markets more competitive, leading to lower prices and profits. We will discuss these papers further below.
monopoly price is supportable as a non-cooperative equilibrium. Kim et al. contrast the competitive effects of more and less costly award programs. In some cases, firms may choose to adopt more costly award programs in order to reduce the intensity of price competition.\textsuperscript{6}

(iii) Other Barriers to Entry: In addition to creating switching costs, FFPs can add to barriers to entry in other ways. Tretheway [1989] and Levine [1987] recognize that airlines with more extensive route networks may be at a competitive advantage when it comes to offering FFPs since they have many more destinations to offer plan members, raising the value of their points without necessarily raising the costs of providing those benefits. Borenstein [1996] shows how an airline could use its dominance in a particular hub together with its FFP to deter entry by smaller, though possibly more efficient firms.\textsuperscript{7}

(iv) Customer Tracking and Database Marketing: FFPs provide airlines with valuable information about their customers which may be useful for them and for others (to whom they may sell access to the data) in future marketing activities.\textsuperscript{8}

\textsuperscript{6} Carlsson and Lofgren [2004] estimate the switching costs between domestic airlines in Sweden and found that SAS's frequent flier plan contributed to those switching costs. By contrast, the results of Hartmann and Viard [2006] suggest that switching costs induced by the loyalty program of a particular golf course may not be significant for most customers.

\textsuperscript{7} In an interesting recent paper, Lederman [2004] studies the impact of frequent flier programs on airline demand with a technique that allows her to disentangle the impact of FFPs from other advantages enjoyed by dominant airlines.

\textsuperscript{8} Shaw [2004, p. 238] argues that “effective use of the database for database marketing is now the key to airlines obtaining value-for-money from FFP investment.” The Supermarket Report issued by the Competition Commission of the United Kingdom in 2000 considered (in Chapter 7) the effects of related loyalty programs in that marketplace. They did not find that the loyalty programs significantly increased switching costs, but did report that the many of the supermarket chains claimed that the “primary benefit of the loyalty cards was the collection of customer information, which allowed them to target better their promotional campaigns and product offerings…” (paragraph 7.81).
Frequent flier plans and the agency relationship.

Another perspective on FFPs suggests that they may be, at least in part, tools by which airlines take advantage of the agency relationship that exists when the party who books air travel and collects the FFP benefits is not the one who pays for the ticket. In this view, FFPs function as bribes to business travelers, tempting them to select higher priced airfares and possibly even take unnecessary (or less convenient) flights, paid for by their employer, in exchange for a small personal benefit in the form of program miles or points.\(^9\) As will become clear, an important property of these plans is that employers are typically not able to claim the FFP benefits for employee travel – airlines resist attempts by employers to collect, insisting that the benefits have been “earned” by the traveler. Moreover, it has been suggested that, in competitive labor markets, some employers may have found that letting employees keep miles (particularly when competing employers do) is an important tool for retaining and attracting employees.\(^10\)

While there has been very little in the way of formal modeling of this effect, the potential for abuse has been well-recognized. Dean [1988] and Arnesen et al. [1997] consider the ethical implications of FFPs and find them to be suspect. Using survey data,

---

\(^9\) While it is clear that business travel represents a substantial fraction of all air travel, it is difficult to get precise measures of this share. As customers do not have to reveal the purpose of their trip when they book their ticket, this information is typically only collected through survey research. The (U.S.) Travel Industry Association’s Domestic Market Report [2005] reports that 28% of domestic trips by air in 2004 were for business/convention purposes and another 9% were for combined business and pleasure. A number of surveys have been done for specific airports as well. For example, a survey done for the Vancouver International Airport revealed that 30% of trips by air through that airport were for business purposes. (YVR Skytalk, April 2006, p. 3). Pels et al. [2001] report a share of 39% for the San Francisco airport.

\(^10\) For example, in recent years both the American and Canadian federal governments changed policies which had prohibited employees from retaining FFP benefits. The American policy had dated from 1994. The reasons given for allowing government employees to keep the benefits in the U.S. included a desire to offer competitive employment opportunities in the government, according to the General Accounting Office (GAO) [2001]. This same GAO report cites a survey by the National Business Travel Association in April 2001 that indicated that 98 per cent of employers surveyed did not try to capture FFP benefits from their employees’ travel. This is not to say that some purchasers have not been able to negotiate special deals with airlines in which FFP “miles” went to the employer. This is simply not the normal arrangement.
Stephenson and Fox [1992] find that a significant fraction of corporate travel managers reported that FFPs had resulted in companies having to pay higher fares than necessary (consistent with the expected effect from the agency relationship we consider here) and also paying for unnecessary air travel. Based on this survey, Stephenson and Fox estimated that FFPs cost the companies an additional 7-9% of their travel costs annually and projecting this led them to estimate a total effect across the U.S. of $6.3 billion of excess costs annually. Nako [1992] and Proussaloglou and Koppleman [1999] find econometric evidence of the effects of FFPs on buyers' willingness to pay and on prices actually paid.

The model

Our purpose here is to model explicitly the principal-agent problem characteristic of FFPs, so that we may understand better their effect on competition and prices in oligopoly markets. Our model will allow us to determine how different features of the marketplace and FFPs themselves -- such as the degree to which employees’ travel is paid by their employers and the costs of providing FFP benefits -- will influence the relationship between FFPs and market prices.

The existing work closest to what we do here is the paper by Cairns and Galbraith [1990], who incorporate elements of the agency problem into their switching cost model (their employee pays an exogenous fraction of the cost of the ticket). However, their

---

11 Tretheway [1989, p. 197] cites sources suggesting that 13-20% of business travel under FFPs is unnecessary. In a widely-cited interview in U.S. News and World Report (April 22, 1985), Judith Dettinger of American Express reported that “A recent survey showed that 25 percent of the frequent travelers polled admitted to taking trips that were totally unnecessary in order to rack up miles.”

12 The data in Proussaloglou and Koppelman [1999] came from a stated preference survey of travelers, with the "mock" choices made treated as raw data in the econometric work. They report, for example, that high frequency business travelers were willing to pay a premium of $72 to travel with the carrier in whose program they most actively accumulate mileage points.
purposes are quite different from ours. While we wish to explore the full equilibrium implications of FFPs in the agency environment, their focus is more on the implications of different airline network sizes for the conditions under which an incumbent can use an FFP to deter entry.\textsuperscript{13}

In our model we treat the FFP as a simple cash transfer to travelers.\textsuperscript{14} In effect, then, each airline sets two prices—one (P) is the price to be paid by the ultimate purchaser of the ticket, and the second (F) is the FFP side-payment from the airline to the traveler. Since FFP benefits are typically not in the form of cash, F should be thought of as the present discounted value of the monetary equivalent of those benefits. So, for example, a free flight that can be claimed at any time during the year would correspond to a higher F than a free flight that has restrictions. We model the cost to the airline of delivering a benefit of F to the traveler as a linear function $\gamma F$, where $\gamma$—which represents the cost to the airline of delivering $1 of benefit to consumers—may be greater than, equal to, or less than 1. If the airline has a great deal of excess capacity it can apply to deliver benefits (e.g. via free flights), it may perceive that $\gamma < 1$, while if the plans are costly to set up and maintain it may be the case that $\gamma > 1$.\textsuperscript{15}

We assume that all travelers for whom these prices apply are employees (we will also use the term “workers”) traveling on company business and that they are all

\textsuperscript{13} Their model can effectively be interpreted as a model of switching costs. Their Proposition 3 is the one point of significant overlap between our sets of results. It corresponds closely with our Proposition 2 on the equilibrium with two identical airlines both with FFPs.

\textsuperscript{14} In fact, our model can also be interpreted as a model of customer service more broadly. What we require is that the firm spends money providing benefits to travelers that do not flow back to their employers. These could be wider seats, more legroom, friendlier staff or other such benefits. What is attractive about FFPs as vehicles for the delivery of benefits to travelers is that they are closer to cash gifts given the various ways they can be used by recipients.

\textsuperscript{15} Note that, because of the way we have defined F—the present discounted value of the monetary equivalent of the benefits—airlines cannot decrease $\gamma$ by, for example, putting restrictions on the use of the rewards so that travelers use them only when there is excess capacity: such a strategy would make the rewards less valuable for the customer.
members of the FFPs. Workers’ airfares are largely paid for them by their employers, but
we do assume that workers bear some (possibly very small or even zero) share, \( \theta \), of the
price of the ticket, where \( 0 \leq \theta \leq 1 \).\(^{16}\) Certainly airlines do serve leisure travelers;
however, we assume that the airlines’ much vaunted ability to segregate their markets via
various means is sufficient to allow us to focus on employee travel as independent from
the leisure travel market. In our discussion section below we will consider some of the
implications of introducing leisure travelers into the market we study here.

There are two airlines in the model, and the flights they offer are differentiated
such that workers have preferences for one airline over another. We model this as a
duopoly with the two firms at either end of a Hotelling line with unit length and –without
further loss of generality– unit density. Given equal prices and FFP benefits, workers
would prefer to travel with the “closer” airline, perhaps because of the kinds of services it
offers. The employer does not perceive this differentiation and therefore would like the
employee to book travel at the lowest price possible. But the employer does not observe
the possible prices and can only approve (or not) travel on the employee’s selected
airline. Hence, workers are free to pick either airline they wish as long as the price
charged is below their employer's reservation price, \( V^<\infty \).

The full game studied has two stages. In the first, airlines decide whether or not to
have FFPs. In the second they simultaneously pick their prices \( (P_i, i = 1,2) \) and, if
relevant, the levels of their FFP benefits \( (F_i, i = 1,2) \). We look for subgame perfect

\(^{16}\) What we want to consider is that there is some cost to employees, however small, of paying higher
prices for tickets. It could be that they bear some of this cost --- as it leaves less money in their expense
account budgets for other items perhaps. Or it could be that this cost represents the expected costs to them
of being audited and punished for careless use of their employer’s resources. We will assume that this
fraction is constant and the same for all workers. A natural extension of this model would involve adding
an earlier stage in which firms choose \( \theta \).
equilibria, employing backward induction techniques to solve the game. Hence, we begin by analyzing the second-stage subgame and use those results to determine whether one airline, both or neither will choose to have a FFP, and (when there are FFPs) what they do to the level of prices paid in the market and to airline profits.

An alternative structure for the timing in this game might have us allow airlines to choose their levels of FFP benefits before they choose their levels of prices, however this implies a level of commitment to the $F_i$ that we do not believe exists. Airlines can easily alter the value of their awards by making award redemption easier or harder, for example by allocating more or fewer seats for award travel. In contrast we do believe there is significant commitment value to a firm’s choice at stage one not to have an FFP.

Without taking the steps to create the program at this first stage, the airline will not have any ability to offer a positive $F_i$. Thus, we can think of the decision to create an FFP at stage one as a decision to remain flexible with respect to $F_i$ in the second stage, rather than committing it to be zero.\(^\text{17}\)

If a worker located at $z$ buys from airline 1, her utility can be represented by:

$$ W - \theta P_1 - zt + F_1 $$

while if she buys from airline 2 it will be:

$$ W - \theta P_2 - (1 - z)t + F_2 $$

where $(P_i, F_i)$ are the price and level of FFP benefit offered by airline $i$, $z$ is the “distance” (in real or product space) of the worker from airline 1, and $t$ is the cost per unit of product-space distance for workers flying with an airline located some distance from the

\(^{17}\) In an appendix available upon request from the authors, we show that in the context of this specific Hotelling-style model we would get results equivalent to those presented here were we to allow airlines to commit to their levels of $F_i$ before playing the price-subgame. We do not expect that this equivalence would survive were we to change the model to allow, for example, downward sloping demand curves from individual employers.
worker’s own location. We assume that the worker must choose an airline as a condition of remaining employed, i.e. we effectively assume that $W \to \infty$.\footnote{To be clear, if the employer rejects travel because it is too expensive ($P > V$), the worker will not lose her job. She would lose it only by refusing to travel.}

One more piece of notation is useful, related to a critical level of $\theta$ and an assumption about its magnitude.

**Assumption**: Let $\theta = t / V$. We assume that $\theta < 1$, or equivalently $t < V$.

This assumption guarantees that the two firms would actually compete absent FFPs, at least for the case in which $\theta=1$, i.e. that it would not be an equilibrium for them to charge the reservation value and take exactly half the market each.

Finally, airlines are assumed to have constant marginal costs of providing seats, which without any further loss of generality we set to zero. Therefore, the only airline costs in our model are those related to the frequent flier benefits they pay to workers. If marginal costs are positive, the prices and frequent flier benefits we solve for below can be viewed in terms of the markup of price over marginal cost (unless price is equal to the reservation price, in which case the marginal cost is not a consideration in pricing).

### III. Analysis of the Model

We first note that, whenever $P_1 \leq V$ or $P_2 \leq V$, the market will be fully covered. If $P_1, P_2 > V$, then the market is shut down: the market can never be partially covered. However, the case $P_i > V$ and $P_j \leq V$ cannot arise in equilibrium: firm $i$ would have no
demand, so it would have an incentive to lower its price. Therefore, we are sure that we
are in the case \( P_1, P_2 \leq V \) and the market is always fully covered.

### III.1 0-FFP Case

We begin by examining the subgame in which neither airline has elected to offer
an FFP. Since the market is fully covered, the demands are given by:

\[
D_1(P_1, P_2) = z(P_1, P_2)
\]

\[
D_2(P_1, P_2) = (1 - z(P_1, P_2))
\]

where the location of the worker indifferent between the two airlines is given by:

\[
z(P_1, P_2) = \frac{1}{2} + \theta \frac{P_2 - P_1}{2t}.
\]

First-order conditions on profit maximization lead in a direct manner to

\[
P_1 = P_2 = \bar{P} = \frac{t}{\theta} \quad \text{and} \quad \pi_1 = \pi_2 = \bar{\pi} = \frac{t}{2\theta}
\]

While this is just a step on our path toward examining the effects of FFPs, the results here
are of independent interest as well. They illustrate the moral hazard in a third-party payer
situation with product differentiation. Note that, if \( \theta = 1 \), we are back in a usual Hotelling
model where the third party payer problem does not arise and both prices will equal \( t \), the
transport cost parameter. This is a standard result from this kind of Hotelling structure.
When \( \theta \) falls below 1, however, prices rise above normal Hotelling levels. This is as a
result of the moral hazard present in the model created by the fact that a third party (the
employer) is paying at least part of the cost of the worker’s ticket. This makes the
worker’s demand less elastic and pushes up equilibrium prices – even in the absence of
FFPs.
However, there will be a limit to how high prices can rise. For $\theta$ sufficiently low, $\bar{P} = V$, the employers’ reservation price. Recalling that $\theta = t/V$ this tells us that we get two sets of equilibrium prices and profits as summarized in the following proposition.$^{19}$

**Proposition 1**: In the 0-FFP subgame:

If $\theta > \overline{\theta}$, then $P_1 = P_2 = \bar{P} = \frac{t}{\theta}$ and $\pi_1 = \pi_2 = \bar{\pi} = \frac{t}{2\theta}$

If $\theta \leq \overline{\theta}$, then $P_1 = P_2 = \bar{P} = V$ and $\pi_1 = \pi_2 = \bar{\pi} = \frac{V}{2}$

Again, prices and profits both rise as $\theta$ falls below 1, until price hits the reservation value. Thus, the moral hazard is good for the airlines but costly to the employers.

**III.2 2-FFP Case**

The subgame in which both airlines offer FFPs is also quite straightforward, given the symmetry of the situation. As indicated above, the FFPs are modeled here as simple cash transfers (“bribes”) between the airlines and the workers. Consistent with the way FFPs are managed, the employers are not permitted to capture these benefits. Airline $i$ now offers workers the pair $(P_i, F_i)$. While $F_i$ is the amount of benefit the worker will receive, the cost to the airline of providing this benefit is $\gamma F_i$, where $\gamma$ may be less than, equal to, or greater than 1.

The demands are now given by:

---

$^{19}$ The proof of Proposition 1 is straightforward and is omitted here, but is available upon request from the authors.
\[ D_1(P_1, P_2, F_1, F_2) = z(P_1, P_2, F_1, F_2) \]
\[ D_2(P_1, P_2, F_1, F_2) = (1 - z(P_1, P_2, F_1, F_2)) \]
and hence the profits are
\[ \pi^1 = z(P_1, P_2, F_1, F_2)(P_1 - \gamma F_1) \]
\[ \pi^2 = M(1 - z(P_1, P_2, F_1, F_2))(P_2 - \gamma F_2) \]
but now the indifferent consumer is located at:
\[ z(P_1, P_2, F_1, F_2) = \frac{1}{2} + \frac{\theta (P_2 - P_1) + F_1 - F_2}{2t}. \]
The linearity of the problem (in terms of consumers’ utility and airlines’ costs) we have here simplifies its solution considerably. When \( \gamma > 1/\theta \) (or equivalently \( \gamma > 1 \)), if firm \( i \) has an FFP, the Nash equilibrium must involve \( F_i = 0 \) and hence, equilibrium prices and profits will be as in the 0-FFP case (Proposition 1). While the airline(s) may formally have an FFP, they will choose to set the level of benefits to zero because they are too costly. The intuition here is straightforward. Airlines attract new business by making their offering more attractive. To put a dollar benefit into a buyer’s pocket using the FFP will require an increase in \( F_i \) of $1 which will cost the airline \( \gamma \). To put a dollar into the buyer’s pocket by lowering price, the price will have to fall by an amount \( 1/\theta \) (since the buyer only benefits by \( \theta \) when price falls by $1). Therefore, if \( \gamma > 1/\theta \), it is less costly to the airline to use lower prices to attract business then to use the FFP and it will choose to set \( F_i = 0 \).

On the other hand, when it is less costly to use the FFP to attract business than lower prices, firms will use FFPs as much as possible, raising prices up to the employer’s reservation value. That is, when \( \gamma < 1/\theta \) (or equivalently, \( \gamma < 1 \)), if firm \( i \) has an FFP,
the Nash equilibrium must involve $P_i = V_i$. Thus, we do not need to worry about the case in which $\gamma \theta > 1$. The outcome is known from Proposition 1. When $\gamma \theta < 1$, we know that both firms will set their prices to $V_i$ and hence we only need to worry about their choice of $F_i$. Taking the first order conditions, $\partial \pi_i / \partial F_i = 0$, and solving for the equilibrium we obtain:

$$F_i^* = \frac{V - t \gamma}{\gamma}$$

Comparative statics reveal that:

$$\frac{dF_i^*}{dV} = \frac{1}{\gamma}, \quad \frac{dF_i^*}{dt} = -1, \quad \frac{dF_i^*}{d\gamma} = -\frac{V}{\gamma^2}, \quad \frac{dF_i^*}{d\theta} = 0$$

These results tell us that the level of frequent flier plan benefits ($F$) is:

(i) increasing in the reservation price ($V$);
(ii) decreasing in the degree of differentiation ($t$);
(iii) decreasing in the cost of the FFP program ($\gamma$); and
(iv) independent of the degree to which workers pay for their flights ($\theta$).

The intuition behind observation (iv) is that, since $F$ is a direct transfer to workers, the proportion of price paid by the workers does not affect airlines’ ability to use $F$ to capture workers. As we shall see below, price does not depend directly on $\theta$ either, although the range of $\theta$ influences whether price is equal to the reservation price. Note that, if $\gamma \geq 1/\theta$, then $F_1 = F_2 = F = 0$ and $\pi_1 = \pi_2 = \pi^f = \frac{V}{2}$; the FFPs are, again,

---

$^{20}$ To see these results, consider that the airline will choose its $F_i$ and $P_i$ to maximize profit given by $P_i - \gamma F_i$, subject to a constraint that the cost to employee, $0P_i - F_i$, be kept to some fixed level and border conditions $0 \leq P_i \leq V$ and $F_i \geq 0$. Substituting the employee constraint into the objective function and maximizing we see quickly that $P_i$ will go to the maximum value allowed if $\gamma \theta < 1$ and that $F_i$ will go to zero if $\gamma \theta > 1$. A more detailed treatment is contained in an earlier version of this paper available from the authors upon request.
(endogenously) inactive and we are back into the 0-FFP case, just as when $\gamma \theta > 1$. If $\gamma < 1/\theta$, then $F_i^* = \frac{V - t \gamma}{\gamma}$, which leads to $\pi_1 = \pi_2 = \pi^f = \frac{t \gamma}{2}$.

Finally, in the case when $\gamma \theta = 1$, the profits would be exactly as in $\gamma \theta < 1$ but we would have a multiplicity of equilibria; many $(P, F)$ pairs would sustain those profits. Restricting our attention to the Nash equilibrium with higher prices in the case $\gamma \theta = 1$, we can summarize the 2-FFP case in the following proposition:

**Proposition 2**: In the 2-FFP subgame:

When $\gamma \leq \min\{\frac{1}{\theta}, \frac{1}{\theta}\}$:

- $P_1 = P_2 = P^f = V$, $F_1 = F_2 = F = \frac{V}{\gamma} - t$, and $\pi_1 = \pi_2 = \pi^f = \frac{t \gamma}{2}$

When $\gamma > \min\{\frac{1}{\theta}, \frac{1}{\theta}\}$:

- If $\theta < \theta$, $P_1 = P_2 = P^f = V$, $F_1 = F_2 = F = 0$, and $\pi_1 = \pi_2 = \pi^f = \frac{V}{2}$

- If $\theta > \theta$, $P_1 = P_2 = P^f = \frac{t}{\theta}$, $F_1 = F_2 = F = 0$, and $\pi_1 = \pi_2 = \pi^f = \frac{t}{2 \theta}$

Graphically, Proposition 2 leads to the following distribution of profits:
Notice that when $\gamma=1$, the profits of the airlines are equal to the profits that would be observed in a typical Hotelling equilibrium in which there was no moral hazard (i.e. $\theta=1$ and no FFP). That is, all the extra profits that could have been made from exploitation of the third-party payer feature (as in the 0-FFP case) are dissipated. Moreover, when $\gamma<1$ profits are even lower than in the typical Hotelling model. Not only are the extra profits stemming from the third-party payer feature dissipated, but some of the profits due to differentiation are lost as well. In the extreme case in which $\gamma=0$, in fact, profits are zero, just as if there were no differentiation at all (i.e. $t=0$). Most importantly, lower profits are realized despite the fact that prices are now higher than with no FFPS: when employees do not bear the entire cost of a flight ($\theta<1$), competition—in terms of prices—is softer than in the typical Hotelling model, yet profits are lower. The agency problem leads to higher
prices, even in the absence of FFPs. Since FFP benefits are a direct transfer to workers, the addition of FFPs as an instrument intensifies the competition between airlines. The result is, from the airlines’ perspective, more like typical price competition, since the extra revenues from higher prices are given away through the FFPs. When both airlines offer FFPs, both the airlines and the employers are worse off (prices are higher but profits are lower); only the employees benefit.

At this point, it seems important to contrast our results with the range of views of FFPs based upon their role in creating switching costs. On one hand, the Banerjee and Summers [1987] model suggests that FFP-induced switching costs will reduce the intensity of competition to point of supporting monopoly outcomes with higher airline prices and profits. On the other hand, a recent paper by Caminal and Claici [2007] shows that loyalty-rewarding pricing schemes can be important “business-stealing” strategies that can lead to lower prices and profits for the sponsoring firms. However, our results differ from both views, since we illustrate the possibility that prices and profits will move in opposite directions with FFPs: prices may go up and profits down.

The effectiveness of this added competitive instrument depends on its cost – and airlines are better off when it is less efficient. We see here that airlines’ profits increase with $\gamma$: the more expensive the FFP program, the higher are airline profits. Hence, when

---

21 See also Caminal and Matutes [1990] for a more general earlier discussion of how the switching costs created by some loyalty programs can enhance competition.

22 There are parallels here with results in the literature on price discrimination in oligopoly. For example, Armstrong [forthcoming, section 3.3] uses a model with a similar Hotelling structure to show that duopolists’ profits may fall if they choose to adopt strategies of price discrimination. See also, Armstrong and Vickers [2001]. While the price discrimination literature has illustrated cases in which profits are lower with price discrimination, those results have tended to rely on either some sort of asymmetry in best-responses or two-stop shopping, neither of which are features of our model. We are grateful to Mark Armstrong for his assistance on this point.

23 With a method that distinguishes FFP from other hub dominance factors, Lederman (forthcoming and 2006) provides empirical evidence that enhancements to FFPs leads to higher prices and that this effect is more pronounced in the higher fare classes which she argues are more likely to be represent business travelers.
airlines have competing FFP programs, they would prefer them to be expensive; any small increment in the cost of the FFPs would imply that the firm would choose smaller bribes in equilibrium, softening the competition (in terms of profits, not prices) between them. If $\gamma$ is high enough, bribes can be driven to zero. But in the best case, the airlines would achieve the same profits they would get if the FFPs were not feasible: comparing $\pi^f$ to $\pi$, it is easy to see that the latter weakly dominates the former. They are equal only when $\gamma \geq \min\{\frac{1}{\theta}, \frac{1}{\theta}\}$. We formalize this in the following proposition.

**Proposition 3:** Equilibrium profits in the 0-FFP case weakly dominate equilibrium profits in the 2-FFP case, that is $\bar{\pi} \geq \pi^f$. $\pi^f$ is equal to $\pi$ if and only if $\gamma \geq \min\{\frac{1}{\theta}, \frac{1}{\theta}\}$ (and, in this case, the level of FFP benefits will be set equal to zero).

In particular, if workers do not pay the full amount of their ticket ($\theta < 1$), the absence of FFPs will be always more profitable if $\gamma \leq 1$.

The last aspect of the proposition, that $\bar{\pi} > \pi^f$ if $\gamma \leq 1$, implies that, even if FFPs are inexpensive for airlines ($\gamma < 1$), they cannot be used effectively as ‘money pumps’ because competition precludes it.

### III.3 1-FFP Case

---

24 More precisely, they would prefer that their rival’s FFP is expensive. Here, however, both FFPs have the same cost parameter and in this case airlines are better off when both have more costly programs. Kim et al [2001] also found that “more costly” FFPs allowed airlines to achieve higher profits but this was by reducing the intensity of price competition. In our case the effect on profits is through the degree to which firms use their FFPs.
When only one firm offers an FFP, we assume, without loss of generality, that
firm 1 is the one that offers an FFP. Since the market is still fully covered, but only firm 1
has the additional pricing instrument, the demands are given by:

\[
D_1(P_1, P_2, F_1) = z(P_1, P_2, F_1)
\]

\[
D_2(P_1, P_2, F_1) = (1 - z(P_1, P_2, F_1))
\]

and profits are

\[
\pi^1 = z(P_1, P_2, F_1)(P_1 - \gamma F_1)
\]

\[
\pi^2 = (1 - z(P_1, P_2, F_1))P_2
\]

where the indifferent consumer is

\[
z(P_1, P_2, F_1) = \frac{1}{2} + \frac{\theta(P_2 - P_1) + F_1}{2t}.
\]

Again, due to the linearity in the problem we are studying, we do not need to worry about
the case in which \(\gamma \theta > 1\). The outcome is known from Proposition 1. When \(\gamma \theta < 1\) we
know that \(P_1 = V\) and we have now a game with two decision variables, in which the
second-order conditions do hold. First order conditions lead to:

\[
P_2^* = \frac{3t\gamma - V(1 - \gamma \theta)}{3\gamma \theta}, \quad F_1^* = \frac{V(2 + \gamma \theta) - 3t\gamma}{3\gamma}, \quad z^* = \frac{V(1 - \gamma \theta) + 3t\gamma}{6t\gamma}
\]

Now that we have an asymmetric solution, the possibility of corner solutions is
considerably greater. Here, there are three possible corners that may be hit:

1. \(P_2^* < 0\) and \(z^* > 1\) if \(\gamma < g_1(\theta) = \frac{V}{3t + V\theta}\)
2. $P_2^* > V$ if $\gamma > g_2(\theta) = \frac{V}{3t-2V\theta}$, for $\theta \in [0, 3\theta/2]$.

3. $F_1^* < 0$ if $\gamma > g_3(\theta) = \frac{2V}{3t-\gamma\theta}$, for $\theta \in [0, 3\theta]$.

It is straightforward to establish the properties of $g_1$, $g_2$ and $g_3$ in order to define the four regions in Figure 2. For the purpose of illustration here, we make the additional assumption that $V/t < 3/2$, which simply establishes that the intercepts in the figure are all less than 1.25

Figure 2: Regions in the 1-FFP Case

---

25 It is easy to verify that: (i) the intercept of $g_3$ is the largest and the three intercepts are below $1/\theta$. If $V/t > 3$, then the three intercepts would be above 1, if $3 > V/t > 3/2$ then only $g_3$’s intercept would be above 1, an if $V/t < 3/2$ the three intercepts would be below one (note that when $V$ is big, $\theta$ is small). (ii) $g_2$ lies below $g_3$ and $\gamma = 1/\theta$ in the interval $[\theta, \theta]$ and (iii) $g_2$ and $g_3$ intersect exactly when $\theta = \theta$ and, at that point, they achieve a value of $\gamma$ that is exactly equal to $1/\theta$. Assuming arbitrarily that $V/t < 3/2$ and, hence, that the intercepts are below one, the curves look like those in Figure 2.
Interior solutions are found in region B and correspond to $P_1 = V, 0 < P_2 < V$, and $F_1 > 0$.

In region A, $P_2 = 0$ and $z = 1$. Firm 2 is then effectively out of the market. In the limit, when $\gamma \to g_1(\theta)$ from above and $P_2 \to 0$ and $z \to 1$, $F_1 > 0$. This would be the equilibrium in region A. However, if firm 2 can decide whether to enter or not, it would not, since it would not expect to make positive profits. Firm 1 would then set $F = 0$ because it would get all the demand and earn $\pi_1 = V$. For the full game, the only thing that matters is that in this region, firm 1 does not have actual competition, so it makes the largest profits and firm 2 makes nothing.

The boundary between regions C and D is actually not relevant, because when moving from interior solutions (region B) to corner solutions, the frontier between B and C, $g_2(\theta)$, is hit first. Above $g_2$, firm 2 finds it optimal to choose $P_2 = V$. But firm 1, despite charging $V$, may still find it optimal to choose $F_1 > 0$ because that allows it to attract more demand. That value of $F_1$ is obtained by optimizing with respect to $F_1$ given $P_1 = P_2 = V$ for both firms. We then obtain $F_1 = \frac{V - 1}{2\gamma}$. Note that $F_1 > 0$ as long as $\gamma < 1/\theta$. Otherwise, the FFP is too expensive, and firm 1 does not want to use its FFP. It sets $F = 0$ and both firms charge prices of $V$.

If $\gamma \theta = 1$, the profits would be the same as for $\gamma \theta < 1$ but we would have multiplicity of equilibria. Restricting our attention to the Nash equilibrium with higher prices when $\gamma \theta = 1$, and recalling that when $\gamma \theta > 1$ the outcome is as in the 0-FFP case, we can summarize the 1-FFP case in the following proposition:
**Proposition 4:** In the 1-FFP subgame:

When $\gamma < g_1(\theta)$:

\[
P_1 = V, \quad P_2 = 0 \quad \text{and} \quad F_1 = 0, \quad \pi_1 = V \quad \text{and} \quad \pi_2 = 0
\]

When $g_1(\theta) \leq \gamma < \min\{g_2(\theta), 1/\theta\}$:

\[
P_1 = V, \quad P_2 = \frac{3t\gamma - V(1 - \gamma \theta)}{2\gamma \theta} \quad \text{and} \quad F_1 = \frac{V(2 - \gamma \theta) - 3t\gamma}{3\gamma}
\]

\[
\pi_1 = \frac{(V(1 - \gamma \theta) + 3t\gamma)^2}{18t\gamma} \quad \text{and} \quad \pi_2 = \frac{(3t\gamma - V(1 - \gamma \theta))^2}{18t\gamma^2 \theta}
\]

When $g_2(\theta) \leq \gamma < 1/\theta$, for $\theta \in [0, \theta]$:

\[
P_1 = V, \quad P_2 = V \quad \text{and} \quad F_1 = \frac{V - t\gamma}{2\gamma}
\]

\[
\pi_1 = \frac{(V + t\gamma)^2}{8t\gamma} \quad \text{and} \quad \pi_2 = \frac{V(3t\gamma - V)}{4t\gamma}
\]

When $\gamma > \min\{\frac{1}{\theta}, \frac{1}{\theta}\}$:

If $\theta < \theta$, $P_1 = P_2 = P^f = V$, $F_1 = F_2 = F = 0$, and $\pi_1 = \pi_2 = \pi^f = \frac{V}{2}$

If $\theta > \theta$, $P_1 = P_2 = P^f = t/\theta$, $F_1 = F_2 = F = 0$, and $\pi_1 = \pi_2 = \pi^f = \frac{t}{2\theta}$

Graphically, Proposition 4 leads to the following distribution of profits:
III.4 Equilibria of the Full Game

We can now determine the equilibrium of the full game. To do this we simply compare the profits earned by each airline in the 0-FFP, 1-FFP and 2-FFP scenarios outlined above and ask whether firms, anticipating these levels of profits, would elect to create FFPs. This analysis is straightforward but tedious and is found in Appendix A. Here we simply summarize the results.26

26 For each \((\theta, \gamma)\) point at stage 1 we have a simple two-by-two matrix game, in which the strategies available to each airline are to create a FFP or not to do so. The Nash equilibrium choice at this stage will determine the course of the full game.
**Proposition 5:** In the full game, in which airlines choose whether to offer FFPs in the first period and compete in prices and FFP benefits in the second period:

(i) If $\gamma$ is sufficiently low (region H), both airlines offer FFPs, but profits are lower than they would be if neither airline offered an FFP (in effect, a prisoner’s dilemma game).

(ii) If $\gamma$ is intermediate (region I), one airline offers an FFP, as this is more profitable given that its competitor does not offer an FFP. It is not profitable for the second airline to offer an FFP also (this is essentially a game of “chicken”\textsuperscript{27}).

\textsuperscript{27} The simple two-player game of chicken has two equilibria. Each involves the two players (who have identical sets of strategic options) picking different strategies (here: adopt/not an FFP) and having different preferred equilibria (here each airline prefers the equilibrium in which it has the FFP and the other does not). It is very similar to the more famous “battle of the sexes” game (in which the multiple equilibria
(iii) If $\gamma$ is sufficiently high (region J), there are effectively no FFPs. Zero, one, or two FFPs are all equilibria, but in all cases FFP benefits are set to zero.\footnote{28}

Thus we see that in the region (H) which includes the less costly FFPs and the more heavily subsidized travel for workers, we are most likely to find both airlines creating FFPs but, as in the prisoner’s dilemma, suffering lower profits than if neither of them had done so. As the FFP becomes more costly, it is less effective as a competitive tool and we enter region I, where only one firm elects to have a FFP and then into J where neither airline adopts an FFP.

The size of these regions clearly depends on the parameter values in the model, the ratio of $V/t$ being most critical. As can be seen in Figure 4, as $V/t$ increases, both the border between H and I, and the border between I and J rise, while $\theta$ approaches 0. This expands the region (in $\gamma$-$\theta$ space) in which the 2-FFP prisoner’s dilemma holds. If $V/t$ is sufficiently high, the full length of the line $\gamma = 1$ will be in region H, implying that even if the FFP is costless redistribution it may lead to a prisoners’ dilemma and lower profits for both airlines (and in that case, area I would be tall but thin).

It is interesting to contrast these results with those that we would obtain were we to remove the first stage of the game and simply let the two firms chose their prices and frequent-flier benefits simultaneously, permitting them to set $F_i = 0$ if they wished. In our model, the first stage does not allow a firm to commit to offering an FFP since it can set $F_i = 0$ in the second stage, but it does permit firms to commit not to offer an FFP, and there is value to that commitment. Without the ability to commit, the game would be

\footnote{28 If there were any small set-up cost for a FFP, only the 0-FFP equilibrium would survive.}
exactly as in our subgame with 2 FFPs. Hence, the entire region I in Figure 4 in which the
two-stage game leads to an asymmetric game of chicken outcome with one FFP will be
added to the range of two-FFP prisoner dilemmas, as in Figure 1. In this region, the
action of one airline to commit not to employ an FFP softens competition and benefits
both airlines.

IV. Discussion and Conclusions

Frequent flier plans, now almost universal among the world’s major airlines, may
serve many purposes, including price discrimination and establishing switching costs.
Similar kinds of loyalty plans exist in other industries, most notably the hotel business.
In this paper we have explored one aspect of FFPs that had not been formally modeled by
economists before as far as we know. Our focus has been on the role that FFPs can play
in helping airlines take advantage of the agency relationship between employers who pay
for tickets and employees who select the airline. Since the employee is the one to receive
the FFP benefit, the plan provides a mechanism through which airlines can “bribe”
employees to select them over possibly lower-priced rivals. Our results indicate that,
consistent with survey reports in the transportation literature, FFPs can lead to higher
prices to employers and lower profits for airlines.29 Significantly, this contrasts with the
“switching cost” approach in which the FFPs can, depending on the model, raise both
prices and airline profits or lower both. We also showed that more costly FFPs may

29 On the view that FFPs may be more trouble than they are worth (perhaps suggesting they may be a
prisoner's dilemma) see, for example Hu et al [1988] and especially Kearney [1990]. A recent N.Y Times
article by David Leonhardt [2006] also makes the point that FFPs may no longer be accomplishing for the
airlines what they were designed to accomplish. (Interestingly, the same article notes the kind of
employee-employer moral hazard problem modeled here.)
actually lead to higher profits than less costly plans, as they provide less efficient ways for firms to compete. The inefficiency of the FFPs mitigates the degree to which FFPs can intensify competition.

It is interesting to consider why the moral hazard associated with purchases for business purposes has led to these programs for air travel, hotels and car rentals, but not to any great extent for all the other inputs that employees order and their employers pay for. We offer two thoughts on this point. For most of the significant purchases a firm makes there will be specific employees tasked with doing the purchasing – a purchasing department, for example. When purchasing is concentrated, while bribery is still possible, the monitoring of employees is less costly. In the case of employee travel, however, purchases are made on a more distributed basis: that is, individual employees often arrange their own travel. This may well be efficient because the right choice of flight or hotel will often depend on personal characteristics of the traveler such as what time of day he or she is available to fly. With so many different people making purchasing decisions, monitoring their behavior is more costly. It may also be the case that loyalty programs of this type may be less costly for airlines, which can provide benefits out of excess capacity at a lower cost (i.e. airlines may have a lower $\gamma$) than firms in other industries.

We would also note that there are other behaviors that sellers employ that might be interpreted as, at least in part, attempts to induce employee-agents into making purchasing decisions that are not in the interest of the employer-principal paying for the purchase. For example, manufacturer money-back rebates on office-equipment have this potential if the rebate flows back to the employee, not necessarily ending up with the
employer who reimbursed the employee for the purchase. Another example, returning us to the airline industry, involves the posting of very high price “full fare economy” tickets priced just below the price of business class travel and very much higher than other regularly available economy fares. Employees may use these fares to circumvent employer policies against paying for business class travel – the employee can travel business class and claim the only slightly lower “economy” price for reimbursement.30

There is much additional work that can be done in this area. We suggest a few directions here. First, we could endogenize the labor contract between the worker and the firm to allow these parties to take into account the FFP benefits to be earned by the worker. If employers are able to lower wages by an amount equal to the expected FFP benefits they will gain from the more intense competition between airlines produced upon the introduction of these plans. The potential benefits to the employer are magnified if the FFP benefits are not taxed, though in this case we have another harmed party -- taxpayers. In this paper we have chosen to assume the labor contract is not affected by the presence of FFP benefits, in part because we have seen no evidence that such adjustments are made in contracts. This is perhaps with good reason: given the general amount of uncertainty regarding the amount of traveling any given worker will be asked to do (and the fact that the employer typically controls to some extent the amount of flying the worker will do) it may not be easy for employers to credibly offer a compensation package with certainly lower wages in return for uncertain FFP benefits over time.31

30 It has been suggested to us that this is also a concern with respect to travel funded by government agencies that support academic research. We are grateful to John Chant for suggesting this point.
31 This is a topic of ongoing research however.
Second, on the assumption that airlines can discriminate in price between those flying on business (at the expense of their employer) and leisure travelers (people paying for their own seats), we have included only workers in our model. It would clearly be valuable to consider a model in which this market segmentation is not so perfect so that the market studied included some leisure travelers as well. Preliminary work in this direction suggests that the qualitative nature of the results is largely the same, such as the existence of game of chicken and prisoner’s-dilemma equilibria. One complication is that, when both airlines offer FFPs, prices and FFP benefits can be decreasing in $\theta$ (rather than being independent of $\theta$ as in the all-worker case). The presence of leisure travelers (i.e. consumers for whom $\theta = 1$) gives airlines some incentive to lower price below $V$, and $F$ is adjusted accordingly. This incentive is increasing in $\theta$ (and decreasing in the relative proportion of workers). These results clearly hold in some region of the parameter space; however, the concavity issues that arise in the all-worker case become much more severe in the model that includes leisure travelers. Since the model with leisure travelers is far less tractable and does not appear to offer any crucial insights that the all-worker model lacks, we have excluded leisure travelers from our analysis.

Third, we could explore the implications were airlines to have different costs associated with FFPs. It has been argued that airlines with more extensive networks are at a competitive advantage with respect to delivering FFP benefits to travelers because the travelers have more choices of places to fly. In our model this would translate into allowing one firm to have a $\gamma$ parameter smaller than that of the other firm. Our study of
the 1-FFP subgame hints at what we might find: we suspect that when one airline has an advantage of this kind, it will translate into higher profits.\textsuperscript{32}

Fourth, we have not really explored the welfare consequences of FFPs in this model, in part because the unit nature of demand, and the fact that the market is always fully covered, makes them somewhat less interesting. In this model, when $\gamma = 1$ (so that benefits are costless transfers), as long as everyone is flying with the closest airline, we have full efficiency.\textsuperscript{33} Another direction in which to take this work would then be to allow travelers (or their employers) to have downward-sloping demand curves for flights. Workers could then select the airline, but the employer would determine how many flights they could take at that airline’s price. The inflation in airline prices we have observed here would then have some efficiency consequences as prices above marginal costs create deadweight loss.

Beyond extensions to the theoretical framework employed here, we believe it would be valuable to get more systematic empirical evidence of the extent to which the moral hazard at the core of our model is important. It would also be useful to explore the extent to which the effects we have described here exist in other markets such as hotels and car rentals. We think it is no coincidence that these very large loyalty programs exist in areas in which a large fraction of purchases are work-related and therefore where these moral hazard problems present themselves. We thus have perhaps

\textsuperscript{32} We might wish to push this even further to consider decisions airlines might make, for example on route structure, that could serve to lower the costs to them of delivering FFP benefits (i.e. lower their $\gamma$). It may be profitable, for example, for an airline to add a route to a popular holiday destination to a largely business-centre related network just to give its FFP members a highly desired use for their free flights.

\textsuperscript{33} The efficiency story is a little more complicated if $\gamma > 1$ or if $\gamma < 1$. In the former case, any use of the FFP at all is inefficient in that the costly transfers lower total surplus. In the latter case, FFPs generate welfare themselves (since the benefits are valued more than the associated costs) and in this respect the use of the FFPs creates wealth. Of course, if they lead fliers to travel with the “wrong” airline they would create some offsetting costs.
a more compelling explanation for the presence of FFPs than those previously offered, since we can also explain why similar programs do not exist in many industries.
Appendix A

Equilibrium of the Full Game

The full game has two stages: (1) airlines decide whether or not to offer FFPs; (2) they choose prices and FFP benefits. We want to find the equilibria in each of these regions:

As discussed in Section 3.3, given our assumption that \( \frac{V}{t} < \frac{3}{2} \), the common intercept of \( g_1 \) and \( g_2 \), \( \frac{V}{3t} \), will be below one half.

Using Figures 1 and 3 we can easily see that in region A, the equilibrium is 2-FFP and is a prisoner’s dilemma by virtue of Proposition 4; and that in regions E-G, the
equilibria are 0-FFP, or 1-FFP or 2-FFP, but the F’s are zero. Note that, if there is any small set-up cost for an FFP, the equilibrium would be 0-FFP for regions E-G.

In regions B, C, and D it is evident that \( \pi_1^{1-FFP} > \pi_1^{0-FFP} > \pi_1^{2-FFP} \). The first inequality follows from the fact that \( F_t > 0 \); if the profits for the firm with the FFP are smaller than when it does not have an FFP, the firm can always set \( F \) equal to zero and make the same profits as in the 0-FFP case (as in regions E and G). The second inequality follows from Proposition 4.

Second, from Proposition 4 we know that \( \pi_2^{0-FFP} > \pi_2^{2-FFP} \) in the three regions.

Third, along the curve \( \gamma = g_1(\theta), \) we have that \( \pi_2^{1-FFP} = 0 \)

Now, consider region B. It is easy to see that \( \pi_2^{1-FFP}( \gamma = 1/\theta) = \frac{f}{2\theta} \). Therefore, in region B we know that \( \pi_2^{1-FFP} < \pi_2^{0-FFP} \). But since \( \pi_2^{1-FFP} \) varies from 0 to \( \frac{f}{2\theta} \) as \( \gamma \) varies from \( g_1 \) to \( 1/\theta \), there is a locus inside region B in which \( \pi_2^{1-FFP} = \pi_2^{2-FFP} \). This locus is given by:

\[
\gamma_b(\theta) = \frac{6tV + V^2\theta + \sqrt{V^3\theta(12t + V\theta)}}{18t^2}
\]

which is upward sloping in \( \theta \). It intersects \( \gamma = \frac{1}{\theta} \) exactly at the vertical asymptote of \( g_2 \), which is located at \( \theta = \frac{3t}{2V} \). This is to the right of \( \theta = 1 \) if \( \frac{V}{t} < \frac{3}{2} \) and hence, \( \gamma_b \) intersect \( \theta = 1 \) below \( \gamma = 1 \). Therefore, region B is divided into two sub-regions: an upper region in which \( \pi_2^{1-FFP} > \pi_2^{2-FFP} \), and a lower region in which \( \pi_2^{1-FFP} < \pi_2^{2-FFP} \).
Region C is similarly divided into an upper and a lower region, by the same locus. The locus intersects $g_2$ at $V = \frac{V}{2t}$, which is evidently below $\gamma = 1$ (see figure 2 or 3).

Given the inequalities of profits in the lower and upper regions inside B and C, we know that the equilibrium in the lower regions is that both airlines offer FFPs, and it is a prisoner’s dilemma; in the upper regions, the equilibrium is that one firm offers an FFP and it is a game of chicken.

Finally, consider region D. Since $\pi_1^{\text{VFFP}} > V / 2$ and the maximum joint profit is $MV$, it follows that $\pi_2^{\text{VFFP}} < V / 2 = \pi_2^{\text{0-FPP}}$. Noting that $\pi_2^{\text{VFFP}}$ does not depend on $\theta$ in region D, and that $\pi_2^{\text{VFFP}}(\gamma = V / 3t) = 0$ and $\pi_2^{\text{VFFP}}(\gamma = \theta) = V / 2$, we see that there is a locus inside region D in which $\pi_2^{\text{VFFP}} = \pi_2^{\text{0-FPP}}$. This locus is given by:

$$\gamma_D = \frac{V}{2t}$$

which obviously connect with $\gamma_B$ at $g_2$. The equilibrium of the game is thus as given by Proposition 5 and Figure 4.
References


Hartmann, Wesley and V. Brian Viard, “Do Frequency Reward Programs Create Switching Costs?”, mimeo, Stanford Graduate School of Business, July 31, 2006.


Lederman, Mara, “Are Frequent Flyer Programs a Cause of the ‘Hub Premium’?” mimeo, Rotman School of Management, University of Toronto, January 2007.


Stephenson, Frederick J. and Richard J. Fox, "Corporate Attitudes toward Frequent-Flier Programs", *Transportation Journal*, Fall 1987, 10-22.


