IRREGULAR WAVE SETUP AND RUNUP ON COBBLE BEACHES AND REVETMENTS

by

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ABSTRACT

A time-averaged probabilistic model is developed to predict irregular wave runup statistics on permeable slopes such as cobble beaches and revetments. The cross-shore variations of the mean and standard deviation of the free surface elevation and horizontal fluid velocities above and inside a porous layer are predicted using the time-averaged continuity, momentum and energy equations. The mean and standard deviation of the shoreline elevation measured by a runup wire are estimated from the predicted mean and standard deviation of the free surface elevation. The wave runup height above the mean water level including wave setup is assumed to be given by the Rayleigh distribution. The wave reflection coefficient is estimated from the wave energy flux remaining at the still water shoreline. This computationally efficient model is shown to be in fair agreement with 57 tests conducted on 1/5 and 1/2 permeable slopes situated inside surf zones on impermeable gentle slopes.

The numerical model is shown to predict the cross-shore variations of the mean and standard deviation of the measured free surface elevation and horizontal velocity fairly accurately when the breaker ratio parameter $\gamma$ is calibrated for some of the 1/2 slope tests. The numerical model predicts the mean and standard deviation of the measured shoreline oscillations reasonably well except that the mean is overpredicted for some of the 1/2 slope tests. Nevertheless, the numerical model predicts the significant and 2% runup heights within the error of about 20%. This accuracy is similar to the accuracy of an available empirical formula based on the known wave conditions at the toe of the permeable slope. The advantage of this numerical model is that it can predict the irregular wave transformation on a beach of arbitrary profile. Furthermore, the numerical model may be applied to gentler permeable slopes for which the effect of wave setup is not negligible. This numerical model is computationally efficient and easy to use because no numerical difficulty has been experienced in the region of very small water depth.
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CHAPTER 1  INTRODUCTION

The prediction of irregular wave runup is necessary in determining the crest height of a coastal structure and the landward limit of wave action on a beach. A large number of studies were performed to understand the swash dynamics and predict wave runup for given slope and offshore wave characteristics as reviewed by Kobayashi (1999). The prediction of wave runup was initially based on experiments and empirical formulas because of the complexity involved in wave breaking and runup. Time-dependent numerical models for shallow-water waves were developed to predict regular wave runup (Kobayashi et al. 1987), irregular wave runup on a rough impermeable slope (Kobayashi et al. 1990), and irregular wave runup on a permeable slope (Wurjanto and Kobayashi, 1993). These models are one-dimensional in the cross-shore direction and do not predict the vertical variations of fluid velocities. Vertically two-dimensional models were also developed to predict plunging waves on an impermeable slope (van der Meer et al. 1992) and regular wave interaction with a steep porous structure (Liu et al. 1999). These numerical models predict the detailed temporal and
spatial variations of the free surface elevation and fluid velocities which are needed to understand the complicated hydrodynamics.

For practical applications, the time-dependent models for wave runup have not been applied routinely perhaps because these models require significant computational efforts and experience to run computer programs and obtain quantities of practical importance. On the other hand, empirical formulas for irregular wave runup on coastal structures have been improved to account for various factors (van der Meer and Janssen 1995; van Gent 2001) but are not versatile enough to deal with various combinations of different beaches and structures. These empirical formulas require the input of the representative height and period of incident waves at the toe of the structure which is normally located inside the surf zone during a severe storm. Consequently, a wave model will be necessary to predict the wave transformation from offshore to the toe of the structure.

Irregular wave breaking and wave setup on an impermeable beach of arbitrary profile are generally predicted using numerical models such as that of Battjes and Stive (1985) based on time-averaged momentum and energy equations. Their time-averaged model predicts only the mean and standard deviation of the free surface elevation but is widely used because of its computational efficiency. In this study, the time-averaged model of Battjes and Stive (1985) is extended landward to a permeable slope such as a revetment and a cobble beach. The extended wave propagation model is combined with a probabilistic wave runup model to predict the runup heights of practical importance such as the significant and 2% runup heights. Furthermore, this time-averaged probabilistic model predicts the cross-shore variations of the mean and standard deviation of the free surface elevation and horizontal fluid velocities above and inside the permeable layer.

In the following, the time-averaged probabilistic model is presented first. The laboratory experiments using 1/5 and 1/2 permeable slopes are described second. The
developed model is compared with 57 tests in the two experiments and used to examine the permeability effects on the wave motion on the slope and the sensitivity of the model to the roller effect, the breaker ratio parameter and the bottom friction factor. Finally, the findings of this study are summarized.

It is noted that the results in this report will be presented concisely by de los Santos, Kobayashi, Meigs and Losada (2005) and Kobayashi, de los Santos and Kearney (2005).
CHAPTER 2  NUMERICAL MODEL

The numerical model based on the time-averaged continuity, momentum and energy equations is presented first in this chapter. The effects of bottom friction and the presence of a roller on the cross-shore momentum and energy equations are included. The formula for the energy dissipation rate due to wave breaking developed by Battjes & Stive (1985) is modified to include the effect of a relative steep slope to allow situations where the energy dissipation is more concentrated locally. Furthermore, an extra energy dissipation term is included to account for the flow resistance inside the porous layer. Finally, a probabilistic model for irregular wave runup is developed using the computed mean and standard deviation of the free surface elevation $\eta$ on the permeable slope.

2.1 TIME-AVERAGED WAVE MODEL

The problem examined here is depicted in Fig. 2-1 where alongshore uniformity and normally incident waves are assumed. The cross-shore coordinate $x$ is positive onshore. The vertical coordinate $z$ is positive upward with $z = 0$ at the still water level (SWL). The upper
and lower boundaries of the permeable stone layer are located at \( z = z_b \) and \( z_p \), respectively, where the lower boundary is assumed to be impermeable. The beach in front of the permeable slope is assumed to be impermeable and \( z_b = z_p \) on the beach. The instantaneous water depth and free surface elevation are denoted by \( h \) and \( \eta \), respectively, and \( h = (\eta - z_b) \). The horizontal fluid velocity \( u \) is the depth-averaged velocity. The still water depth \( d_i \) at the toe of the permeable slope is located inside the surf zone on the beach in the experiments in this study.

![Fig. 2-1: Time averaged model for wave propagation on permeable slope.](image)

The time-averaged continuity, momentum and energy equations used here are those given by Kobayashi et al. (2005) for the prediction of irregular breaking wave transmission over a submerged porous breakwater together with the roller effect discussed by Kobayashi et al. (2005) for sand suspension. These equations are summarized in the following. The time-averaged momentum and energy equations are expressed as

\[
\frac{dS_{xx}}{dx} = -\rho g \bar{h} \frac{d\bar{\eta}}{dx} \tau_b + \frac{dF}{dx} = -D_b - D_f - D_r
\]  

(1)

where \( S_{xx} \) = cross-shore radiation stress; \( \rho \) = fluid density; \( g \) = gravitational acceleration; \( \bar{h} \) = mean water depth with the overbar denoting time averaging; \( \bar{\eta} \) = wave setup or setdown; \( \tau_b \) = time-averaged bottom shear stress; \( F \) = wave energy flux per unit width; and \( D_b, D_f \)
and $D_f$ = time-averaged energy dissipation rate per unit horizontal area due to wave breaking, bottom friction, and porous flow resistance, respectively.

Linear wave theory for onshore progressive waves is used to estimate $S_{ss}$ and $F$ where the root-mean-square wave height $H_{rms}$ is defined as $H_{rms} = \sqrt{8} \sigma_{\eta}$ with $\sigma_{\eta}$ = standard deviation of $\eta$ (Battjes and Stive 1985).

$$S_{ss} = \rho g \sigma_{\eta}^2 (2n - 0.5) + \rho C_p q_r ; \quad F = \rho g C_p \sigma_{\eta}^2$$ (2)

where $n = C_g / C_p$ with $C_g$ and $C_p$ = group velocity and phase velocity in the mean water depth $\bar{h}$ corresponding to the spectral peak period $T_p$ of incident waves, and $q_r$ is the volume flux due to the roller on the steep front of a breaking wave.

The roller effect has been represented by its area or energy (Svendsen 1984) but the roller volume flux is used here because the roller effect is the most apparent in the increase of undertow current. The term $\rho C_p q_r$ in $S_{ss}$ is the roller momentum flux due to the roller propagating with the speed of $C_p$ and causes the landward shift of $\bar{\eta}$ in the breaker zone (Kobayashi et al. 2005). The dissipated wave energy is converted to the roller energy which is assumed to be governed by (Stive and DeVriend 1994)

$$\frac{d}{dx} (\rho C_p^2 q_r) = D_f - \rho g \beta_q q_r$$ (3)

where the roller dissipation rate, $\rho g \beta_q q_r$, is assumed to equal the rate of work to maintain the roller on the wave-front slope $\beta_q$ of order 0.1 (Deigaard 1993).

The bottom shear stress $\tau_b$ and the corresponding dissipation rate $D_f$ are expressed using the formulas based on the quadratic drag force based on the horizontal velocity $u$. The mean and standard deviation of $u$ are denoted by $\bar{u}$ and $\sigma_u$, respectively. The Gaussian
distribution of $u$ and the equivalency of the time and probabilistic averaging are assumed to express $\tau_b$ and $D_f$ in terms of $\bar{u}$ and $\sigma_u$

$$\tau_b = \frac{1}{2} \rho f_b \sigma_u^2 G_2 (u_*) ; \quad D_f = \frac{1}{2} \rho f_b \sigma_u^3 G_3 (u_*) ; \quad u_* = \frac{\bar{u}}{\sigma_u}$$

(4)

where $f_b$ = bottom friction factor which is taken as $f_b = 0$ on the beach and $f_b = 0.01$ on the stone slope (Kobayashi et al. 2005). The analytical functions $G_2 (r)$ and $G_3 (r)$ for the arbitrary variable $r$ are given by Kobayashi et al. (2005) and can be approximated as $G_2 \approx 1.64 r$ and $G_3 = (1.6 + 2.6 r^2)$ for $|r| < 1$.

The standard deviation $\sigma_u$ is estimated using the relationship between $\sigma_u$ and $\sigma_n$ based on linear shallow-water wave theory (Kobayashi et al. 1998)

$$\sigma_u = \sigma_n \left( g h \right)^{0.5} ; \quad \sigma_* = \sigma_n / h$$

(5)

The mean $\bar{u}$ is estimated using the time-averaged, vertically-integrated continuity equation $(\sigma_u \sigma_n + \bar{u} h + \bar{v} h_p + q_r) = 0$ with the condition of no net landward water flux. In this equation, $\sigma_u \sigma_n$ is the onshore flux due to linear shallow-water waves (Kobayashi et al. 1998), $\bar{u} h$ is the offshore flux due to the return current $\bar{u}$, $q_r$ is the volume flux due to the roller of a breaking wave, and $\bar{v} h_p$ is the water flux inside the permeable layer of vertical height $h_p$ due to the time-averaged horizontal discharge velocity $\bar{v}$. Substitution of Eq. (5) into the continuity equation yields

$$\bar{u} = - \left[ \sigma_u^2 \sqrt{g h + \bar{v} h_p / h + q_r / h} \right] ; \quad h_p = z_p - z_p$$

(6)

where $h_p = 0$ on the impermeable beach.
The energy dissipation rate $D_r$ in Eq. (1) is estimated using the formula by Wurjanto and Kobayashi (1993) based on the discharge velocity $v$ whose probability distribution is assumed to be Gaussian

$$D_r = \rho h_p \left[ \alpha \sigma_v^2 (1 + v^2) + \beta \sigma_v^2 G_3(v_r) \right] ; \quad v_r = \frac{\bar{v}}{\sigma_v} \tag{7}$$

where $\sigma_v$ = standard deviation of $v$; $G_3$ = same function as in Eq. (4) except for $r = v$; and $\alpha$ and $\beta$ = laminar and turbulent flow resistance coefficients. Kobayashi et al. (2005) modified the formulas of $\alpha$ and $\beta$ by van Gent (1995) for irregular waves in the form

$$\alpha = \alpha_o \left(\frac{1-n_p}{n_p}\right)^2 \frac{\nu}{D_{s50}^2} ; \quad \beta = \left(\beta_1 + \beta_2 \frac{\sigma_v}{\nu}\right) ; \quad \beta_1 = \frac{\beta_o (1-n_p)}{n_p^2 D_{s50}} ; \quad \beta_2 = \frac{7.5 \beta_o (1-n_p)}{\sqrt{2} n_p^2 T_p} \tag{8}$$

where $\alpha_o$ and $\beta_o$ = empirical parameters calibrated as $\alpha_o = 1,000$ and $\beta_o = 5$; $n_p$ = porosity of the stone; $D_{s50}$ = nominal stone diameter defined as $D_{s50} = \left(\frac{M_{s50}}{\rho_s}\right)^{1/3}$ with $M_{s50}$ = median stone mass and $\rho_s$ = stone density; $\nu$ = kinematic viscosity of water ($\nu = 0.01 \text{cm}^2/\text{s}$); and $T_p$ = spectral peak period. The mean $\bar{v}$ and standard deviation $\sigma_v$ are estimated assuming the local force balance between the horizontal gradient of hydrostatic pressure and the flow resistance inside the permeable layer

$$(\alpha + 1.64 \beta \sigma_v) \bar{v} = -g \frac{d \bar{\eta}}{dx} ; \quad \alpha \sigma_v + 1.9 \beta \sigma_v^2 = g k_p \bar{h} \sigma_v \tag{9}$$

where $k_p$ = linear wave number based on $\bar{h}$ and $T_p$. Eq. (9) can be solved analytically to obtain $\sigma_v$ and $\bar{v}$ for known $k_p \bar{h} \sigma_v$ and $d \bar{\eta}/dx$.

The energy dissipation rate $D_\beta$ due to wave breaking in Eq. (1) is estimated using the formula by Battjes and Stive (1985) which is modified by Kobayashi et al. (2005) as
\[ D_b = \frac{\rho g a Q H_b^2}{4T_p} \quad ; \quad a = \frac{T_p S_b}{b \left( \frac{g}{h} \right)^{0.5}} \geq 1 \quad ; \]

\[ \frac{Q-1}{\ln Q} = \left( \frac{H_{nm}}{H_m} \right)^2 \quad ; \quad H_m = 0.88 \frac{\gamma k_p \bar{h}}{k_p \tanh \left( \frac{\gamma k_p \bar{h}}{0.88} \right)} \]

where \( a \) = empirical coefficient; \( Q \) = fraction of breaking waves with \( Q = 0 \) for no wave breaking and \( Q = 1 \) when all waves break; \( H_b \) = wave height used to estimate \( D_b \); \( S_b \) = local bottom slope defined as \( S_b = \frac{dz_b}{dx} \); \( b \) = slope adjustment factor calibrated as \( b = 3 \); \( H_m \) = local depth-limited wave height with \( H_m = \gamma \bar{h} \) in shallow water; and \( \gamma \) = breaker ratio parameter. The coefficient \( a \) is the ratio between the wavelength and the horizontal length scale \( (b \bar{h}/S_b) \) imposed by the small depth \( \bar{h} \) and the bottom slope \( S_b \) near the shoreline when \( a > 1 \).

The increase of \( D_b \) due to the slope effect results in the increase of \( q_r \) in Eq. (3). To offset this increase, use is made of \( \beta_r = (0.1 + S_b) \geq 0.1 \), which implies that the wave-front slope increases on the upward slope (Kobayashi et al. 2005).

The requirement of \( 0 \leq Q \leq 1 \) implies \( H_{nm} \leq H_m \) but \( H_{nm} \) becomes larger than \( H_m \) in very shallow water. When \( H_{nm} > H_m \), use is made of \( Q = 1 \) and \( H_b = H_{nm} \) instead of \( H_b = H_m \) for \( H_{nm} \leq H_m \). In addition, \( \sigma_* = \sigma_\eta / \bar{h} \) in Eqs. (5), (6) and (9) becomes too large due to the local use of linear shallow-water wave theory and use is made of \( \sigma_* = (\sigma_* \sigma_q / \bar{h})^{0.5} \) if \( \sigma_* > \sigma_{r*} = \gamma / \sqrt{8} \). The values of \( \gamma \) calibrated by Battjes and Stive (1985) were in the range of \( 0.6 - 0.8 \). In the subsequent comparisons, \( \gamma = 0.6, 0.7 \) and \( 0.8 \) are tried, whereas the other empirical parameters are kept the same as those used by Kobayashi et al. (2005) for the related problem of wave transmission over a submerged porous breakwater.
Eqs. (1) – (10) are solved using a finite difference method with constant nodal spacing \( \Delta x \) of approximately 1 cm for the small-scale experiments in this study. The bottom elevation \( z_b(x) \) and the impermeable boundary \( z_p(x) \) are specified as input. The stone is characterized by its nominal diameter \( D_{n50} \) and porosity \( n_p \). The measured values of \( T_p, \eta \) and \( H_{mn} = \sqrt{8 \sigma_q} \) as well as \( q_r = 0 \) are specified at the seaward boundary \( x = 0 \) outside the surf zone. The landward-marching computation is continued until the computed value of \( \tilde{h} \) or \( \sigma_q \) becomes negative in the region of \( \tilde{h} \) on the order of 0.1 cm. For the option of \( \text{IROLL} = 0 \), the roller volume flux \( q_r = 0 \) and Eq. (3) is not used. The computation time is of the order of one second.

The time-averaged model based on Eqs. (1) – (10) neglects reflected waves. An attempt is made to estimate the degree of wave reflection. The onshore energy flux \( F \) in Eq. (1) decreases landward due to wave breaking, bottom friction and porous flow resistance. The residual energy flux \( F_{swa} \) at the still water shoreline located at \( z_b = 0 \) is assumed to be reflected and propagate seaward. This assumption neglects the fact that the landward-marching computation is made without regard to wave reflection. The root-mean-square wave height \( \langle H_{rms} \rangle \), due to the reflected wave energy flux is crudely estimated as

\[
\langle H_{rms} \rangle = \left[ \frac{8F_{swa}}{(\rho g C_g)} \right]^{0.5}
\]  

(11)

2.2 PROBABILISTIC RUNUP MODEL

A probabilistic model for irregular wave runup is developed using the computed \( \tilde{\eta}(x) \) and \( \sigma_q(x) \) on the permeable slope. A runup wire is used in the subsequent experiments to measure the shoreline oscillations above the slope as shown in Fig. 2-1. The vertical height
\( \delta_r \) of the wire above the average stone surface is known in the following. The wire measures the instantaneous elevation \( \eta_r(t) \) above SWL of the intersection between the wire and the free surface unlike a wave gauge that measures \( \eta(t) \) at given \( x \). Fig. 2-2 depicts an intuitive method used to estimate the mean \( \bar{\eta}_r \) and standard deviation \( \sigma_r \) of \( \eta_r(t) \).

![Diagram showing runup wire and elevations](image)

**Fig. 2-2:** Elevations \( Z_1, Z_2 \) and \( Z_3 \) of intersections of \((\bar{\eta} + \sigma_\eta)\), \( \bar{\eta} \) and \((\bar{\eta} - \sigma_\eta)\) with runup wire where \( \bar{\eta} \) and \( \sigma_\eta \) are the mean and standard deviation of free surface elevation.

The probabilities of \( \eta \) exceeding \((\bar{\eta} + \sigma_\eta)\), \( \bar{\eta} \) and \((\bar{\eta} - \sigma_\eta)\) are assumed to be the same as the probabilities of \( \eta_r \) exceeding \((\bar{\eta}_r + \sigma_r)\), \( \bar{\eta}_r \) and \((\bar{\eta}_r - \sigma_r)\), respectively. The elevations of \( Z_1, Z_2 \) and \( Z_3 \) of the intersections of \((\bar{\eta} + \sigma_\eta)\), \( \bar{\eta} \) and \((\bar{\eta} - \sigma_\eta)\) with the runup wire are obtained using the computed \( \bar{\eta}(x) \) and \( \sigma_\eta(x) \) together with the wire elevation.
\[ z_b(x) + \delta_r \]. The obtained elevations are assumed to correspond to \( Z_1 = (\overline{\eta_r} + \sigma_r), Z_2 = \overline{\eta_r} \) and \( Z_3 = (\overline{\eta_r} - \sigma_r) \). The mean and standard deviation of \( \eta_r(t) \) are estimated as

\[
\overline{\eta_r} = \frac{(Z_1 + Z_2 + Z_3)}{3} \quad ; \quad \sigma_r = \frac{(Z_1 - Z_3)}{2}
\]

(12)

where the use of \( Z_1, Z_2 \) and \( Z_3 \) to estimate \( \overline{\eta_r} \) is slightly more reliable than \( \overline{\eta_r} = Z_2 \) because the elevation \( Z_2 \) is somewhat sensitive to the detailed spatial variation of \( \overline{\eta}(x) \).

The runup height \( R \) is defined as the crest height above SWL of the temporal variation of \( \eta_r \). The time series of \( [\eta_r(t) - \overline{\eta_r}] \) is analyzed using a zero-upcrossing method to identify the crests in the time series. This procedure is the same as that used for the analysis of the wave crests in the time series of \( \eta(t) \) except that the wave crest is defined as the height above the mean water level. The probability distribution of linear wave crests is normally given by the Rayleigh distribution [e.g., Goda (2000)], whereas Tayfun (2004) presented the distribution of nonlinear wave crests in deep water. As a first approximation, the runup height \( (R - \overline{\eta_r}) \) above the mean level \( \overline{\eta_r} \) is given by the Rayleigh distribution

\[
P(R) = \exp \left[ -2 \left( \frac{R - \overline{\eta_r}}{R_{1/3} - \overline{\eta_r}} \right)^2 \right]
\]

(13)

where \( P(R) \) = exceedance probability of the runup height \( R \) above SWL; and \( R_{1/3} \) = significant runup height defined as the average of 1/3 highest values of \( R \). The mean \( \overline{\eta} \), related to wave setup is normally neglected in Eq. (13) for the prediction of irregular wave runup on steep coastal structures [e.g., van der Meer and Janssen (1995)]. However, wave setup on gentler slopes is not negligible as will be shown for the permeable slope experiments in this study.
Finally, it is necessary to express $R_{i,j}$ in terms of $\bar{\eta}_r$ and $\sigma_r$ estimated using Eq. (12).

If the probability distribution of $\eta_r$ is approximately Gaussian, use may be made of

\[
(R_{i,j} - \bar{\eta}_r) = 2\sigma_r \quad \text{(Goda 2000)}.
\]

For the following experiments using 1/5 and 1/2 permeable slopes, $R_{i,j}$ is estimated as

\[
R_{i,j} = \bar{\eta}_r + (2 + \tan \theta)\sigma_r
\]

where $\theta = \text{slope angle from the horizontal}$ and $\tan \theta = 1/5$ and 1/2 in the experiments. The slope correction in Eq. (14) is purely empirical and needs to be verified for other slopes.
CHAPTER 3  EXPERIMENTS

Thirty tests under incident irregular waves were conducted to analyze the cross-shore wave evolution and horizontal velocity in the surf zone on a gently sloping impermeable beach and a 1/5 slope with a porous cobble layer. This chapter will review the experimental setup for the 1/5 slope experiment and summarize the experiment conducted by Kearney and Kobayashi (2000) with a 1/2 slope stone revetment. The measurement procedures of the free surface elevation, runup and velocities are explained in the subsequent sections. The obtained data were used to validate the numerical wave model and the probabilistic runup model.

3.1 EXPERIMENTAL SETUP FOR 1/5 SLOPE TESTS

The experiment was conducted in a wave flume in the Ocean Engineering Laboratory of the University of Delaware, that was 33 m long, 0.6 m wide and 1.5 m high as shown in Fig. 3-1. An impermeable smooth beach with a 1/34.4 slope was installed in the flume. Angular stone was placed on a 1/5 impermeable slope to simulate an idealized cobble beach. The nominal diameter and porosity of the stone were $D_{n50} = 3.4$ cm and $n_p = 0.5$,
respectively. The vertical thickness of the stone layer was 14 cm. No stone movement was detected during the experiment.

Irregular waves, based on the TMA spectrum, were generated in a burst of 429.6 s using a piston-type wave paddle. The sampling rate was 20 Hz for all the time series measured in the experiment. The initial transient of 20 s in each burst was removed for subsequent data analyses.

![Runup Wire Diagram](image)

<table>
<thead>
<tr>
<th>Instrument</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
<th>G7</th>
<th>ADV1</th>
<th>ADV2</th>
<th>ADV3</th>
</tr>
</thead>
<tbody>
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<td>1.308</td>
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<td>6.588</td>
<td>6.878</td>
<td>4.268</td>
<td>5.908</td>
<td>6.478</td>
</tr>
</tbody>
</table>

**Fig. 3-1: Experimental setup for 1/5 slope tests.**

Thirty tests were conducted for the 1/5 slope experiment. The still water depth $d_i$ at the toe of the slope was varied from 16.6 cm to 24.6 cm with an increment of 2 cm. The spectral peak period $T_p$ was approximately 1.5, 2.3 and 3.0 s. Table 1 lists the measured values of $T_p$ and $H_{rms}$ at wave gauge 1 located in the still water depth $d_i$, as well as the incident root-mean-square wave height ($H_{rms,i}$), and the average reflection coefficient $r$ defined as $r = \left( \frac{H_{rms}}{H_{rms,i}} \right)$, where $(H_{rms})_r$ and $(H_{rms})_i$ are the reflected and incident root-mean-square wave heights obtained using wave gauges 1 – 3 (Kobayashi et al., 1990). $H_{rms}$ includes both incident and reflected waves. The difference between $H_{rms}$ and $(H_{rms})_i$ was less than 5% because $r = 0.16 - 0.24$ and wave gauge 1 was located at the horizontal distance
of 6.3 m seaward of the toe of the 1/5 slope. The root-mean-square wave height $H_{rms} = \sqrt{8} \sigma_\eta$ was selected to be as large as feasible without any wave breaking in the vicinity of the wavemaker located in the still water depth $d_h = (d_t + 31)$ cm. For the specified $d_t$ and $T_p$, two tests were performed to check the variability of $H_{rms}$ resulting from the generation of large waves. Table 1 lists only one of the two repeated tests for brevity but both tests are included in the following data analysis.

<table>
<thead>
<tr>
<th>Test</th>
<th>$d_t$ [cm]</th>
<th>$d_r$ [cm]</th>
<th>$T_p$ [s]</th>
<th>$H_{rms}$ [cm]</th>
<th>$(H_{rms})_i$ [cm]</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.53</td>
<td>10.04</td>
<td>9.62</td>
<td>0.17</td>
</tr>
<tr>
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<td>2.31</td>
<td>10.47</td>
<td>10.07</td>
<td>0.20</td>
</tr>
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</tr>
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<td>2.31</td>
<td>9.66</td>
<td>9.25</td>
<td>0.18</td>
</tr>
<tr>
<td>R18C1</td>
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<td>3.09</td>
<td>7.19</td>
<td>6.98</td>
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</tr>
<tr>
<td>R20A1</td>
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<td>38.9</td>
<td>1.53</td>
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<td>10.71</td>
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<td>R20B1</td>
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</tr>
<tr>
<td>R20C1</td>
<td>20.6</td>
<td>38.9</td>
<td>2.96</td>
<td>7.43</td>
<td>7.21</td>
<td>0.22</td>
</tr>
<tr>
<td>R22A1</td>
<td>22.6</td>
<td>40.9</td>
<td>1.53</td>
<td>11.43</td>
<td>11.16</td>
<td>0.17</td>
</tr>
<tr>
<td>R22B1</td>
<td>22.6</td>
<td>40.9</td>
<td>2.31</td>
<td>12.08</td>
<td>11.53</td>
<td>0.19</td>
</tr>
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<td>R22C1</td>
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<td>40.9</td>
<td>2.90</td>
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</tr>
<tr>
<td>R24A1</td>
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<td>11.59</td>
<td>11.29</td>
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</tr>
<tr>
<td>R24B1</td>
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<td>42.9</td>
<td>2.31</td>
<td>12.22</td>
<td>11.72</td>
<td>0.20</td>
</tr>
<tr>
<td>R24C1</td>
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<td>42.9</td>
<td>2.90</td>
<td>7.96</td>
<td>7.59</td>
<td>0.23</td>
</tr>
</tbody>
</table>

$d_t$ = toe depth; $d_r$ = still water depth at wave gauge 1; $T_p$ = spectral peak period; $H_{rms}$ = root-mean-square wave height; $(H_{rms})_i$ = incident $H_{rms}$; $r$ = average reflection coefficient.

### 3.2 EXPERIMENTAL SETUP FOR 1/2 SLOPE TESTS

The 1/2 slope tests were reported concisely by Kearney and Kobayashi (2000) but they are summarized here for the sake of completeness. The experiment was conducted in a wave tank that was 30 m long, 2.44 m wide, and 1.5 m high. A plywood beach with a 1/32.1 slope and a stone revetment with a 1/2 slope were installed in the tank. The nominal diameter of the stone was $D_{n50} = 3.2$ cm and the stone porosity, which was not measured, is assumed to be the same as $n_p = 0.5$ for the stone used in the 1/5 slope experiment. The thickness of the
permeable layer was approximately 14 cm. A runup wire was placed at a distance of $\delta_r = 2.5$ cm above the 1/2 stone revetment. Ten wave gauges were arranged like in Fig. 2-1 to measure the irregular wave transformation from outside the surf zone to the toe of the 1/2 slope. Wave gauge 1 was located 14.8 m seaward of the toe. One ADV was also placed at the toe. Twenty-seven tests were conducted for the spectral peak periods $T_p = 1.5$, 2.4 and 4.7 s of the TMA spectra and nine different toe depths $d_t = 4 - 20$ cm with an increment of 2 cm. The repeatability of each test was checked but the repeated test results were not reported. The duration of each test and the sampling rate were 400 s and 20 Hz for $T_p = 1.5$ and 2.4 s and 800 s and 10 Hz for $T_p = 4.7$ s. The initial transition of 1,200 data points was removed before the data analyses. The wave characteristics and the reflection coefficient $r$ at wave gauge 1 for the 27 tests are listed in Table 2. The values of $r$ were slightly larger for the 1/2 slope than for the 1/5 slope.
Table 2: Wave characteristics at wave gauge 1 for the 1/2 slope experiment.

<table>
<thead>
<tr>
<th>Test</th>
<th>$d_t$ [cm]</th>
<th>$d_s$ [cm]</th>
<th>$T_p$ [s]</th>
<th>$H_{rms}$ [cm]</th>
<th>$(H_{rms})_i$ [cm]</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A20</td>
<td>20.0</td>
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<td>4.7</td>
<td>16.4</td>
<td>15.8</td>
<td>0.33</td>
</tr>
<tr>
<td>A18</td>
<td>18.0</td>
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<td>4.7</td>
<td>16.0</td>
<td>15.1</td>
<td>0.32</td>
</tr>
<tr>
<td>A16</td>
<td>16.0</td>
<td>62.1</td>
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<td>14.7</td>
<td>15.3</td>
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<td>14.3</td>
<td>14.5</td>
<td>0.28</td>
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<td>13.9</td>
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<td>13.7</td>
<td>14.0</td>
<td>0.28</td>
</tr>
<tr>
<td>A8</td>
<td>8.0</td>
<td>54.1</td>
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<td>13.7</td>
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</tr>
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<td>12.4</td>
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<td>0.26</td>
</tr>
<tr>
<td>A4</td>
<td>4.0</td>
<td>50.1</td>
<td>4.7</td>
<td>12.7</td>
<td>12.5</td>
<td>0.26</td>
</tr>
<tr>
<td>B20</td>
<td>20.0</td>
<td>66.1</td>
<td>2.4</td>
<td>11.4</td>
<td>13.8</td>
<td>0.36</td>
</tr>
<tr>
<td>B18</td>
<td>18.0</td>
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<td>2.4</td>
<td>10.9</td>
<td>13.4</td>
<td>0.36</td>
</tr>
<tr>
<td>B16</td>
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<td>62.1</td>
<td>2.4</td>
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<td>2.4</td>
<td>9.9</td>
<td>12.9</td>
<td>0.35</td>
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<td>9.9</td>
<td>12.6</td>
<td>0.34</td>
</tr>
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<td>9.2</td>
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<tr>
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<td>9.1</td>
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<td>0.26</td>
</tr>
<tr>
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<td>20.0</td>
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<td>1.5</td>
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<td>0.23</td>
</tr>
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<td>C18</td>
<td>18.0</td>
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<td>1.5</td>
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<td>C16</td>
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<td>60.1</td>
<td>1.5</td>
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<td>1.5</td>
<td>8.8</td>
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</tr>
<tr>
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<td>8.6</td>
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</table>

$d_t =$ toe depth; $d_s =$ still water depth at wave gauge 1; $T_p =$ spectral peak period; $H_{rms} =$ root-mean-square wave height; $(H_{rms})_i =$ incident $H_{rms}; r =$ average reflection coefficient.

3.3 FREE SURFACE AND RUNUP MEASUREMENTS

For each test, seven capacitance-type wave gauges and a runup wire were used to measure the time series of $\eta$ and $\eta_r$ for the 1/5 slope experiment at the locations indicated in Fig. 3-1 where $x = 0$ at wave gauge 1. Wave gauges 1 – 3 were located immediately outside the surf zone and used to separate the incident and reflected waves using linear wave theory (Kobayashi et al. 1990). Wave gauges 4 – 7 measured the irregular breaking wave transformation on the gentle slope and the seaward edge of the 1/5 porous slope. Calibration of all the wave gauges was performed frequently to ensure the reliability of the measurements.
A capacitance-type runup wire was placed parallel to the 1:5 porous slope to measure the shoreline oscillations above the slope as shown in Fig. 3-1. The vertical height $\delta_r$ of the runup wire above the slope was approximately 2 cm. The wire measures the instantaneous elevation $\eta_r(t)$ above SWL of the intersection between the wire and the free surface. The runup wire was calibrated simultaneously as the wave gauges. Because the runup wire was much longer than the wire used for the wave gauges, an extra capacitor was added in series in such a way that the entire length of the wire could be used for the runup measurement. Due to this modification, a third order fit was used for the runup wire calibration.

Table 3 shows the mean and standard deviation of $\eta$ at wave gauge 6, located at the toe of the porous slope, and the shoreline elevation $\eta_r$ above SWL measured by the runup wire as well as $R_{1/3} = \text{significant runup height above SWL}$ and $R_{2\%} = \text{runup height corresponding to 2\% exceedance probability}$. Wave setup $\bar{\eta}$ at the toe of the 1:5 porous slope is in the range of 0-0.3 cm, while $\bar{\eta}_r$ is in the range of 1-3 cm. The standard deviations $\sigma_{\eta}$ and $\sigma_r$ are on the same order of magnitude for the 1:5 porous slope.

<table>
<thead>
<tr>
<th>Test</th>
<th>Wave Gauge 6</th>
<th>Wave Runup Wire</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{\eta}$ [cm]</td>
<td>$\sigma_{\eta}$ [cm]</td>
</tr>
<tr>
<td>R16A1</td>
<td>0.28</td>
<td>2.88</td>
</tr>
<tr>
<td>R16B1</td>
<td>0.29</td>
<td>2.92</td>
</tr>
<tr>
<td>R16C1</td>
<td>0.04</td>
<td>2.40</td>
</tr>
<tr>
<td>R18A1</td>
<td>0.26</td>
<td>3.14</td>
</tr>
<tr>
<td>R18B1</td>
<td>0.16</td>
<td>2.94</td>
</tr>
<tr>
<td>R18C1</td>
<td>0.05</td>
<td>2.41</td>
</tr>
<tr>
<td>R20A1</td>
<td>0.23</td>
<td>3.38</td>
</tr>
<tr>
<td>R20B1</td>
<td>0.19</td>
<td>3.34</td>
</tr>
<tr>
<td>R20C1</td>
<td>-0.01</td>
<td>2.48</td>
</tr>
<tr>
<td>R22A1</td>
<td>0.25</td>
<td>3.49</td>
</tr>
<tr>
<td>R22B1</td>
<td>0.17</td>
<td>3.53</td>
</tr>
<tr>
<td>R22C1</td>
<td>0.04</td>
<td>2.49</td>
</tr>
<tr>
<td>R24A1</td>
<td>0.21</td>
<td>3.66</td>
</tr>
<tr>
<td>R24B1</td>
<td>0.06</td>
<td>3.82</td>
</tr>
<tr>
<td>R24C1</td>
<td>0.01</td>
<td>2.56</td>
</tr>
</tbody>
</table>
3.4 VELOCITY MEASUREMENTS

Three 3D acoustic Doppler velocimeters (ADV) were used to measure fluid velocities at the approximate middle between the still water level and the beach. ADV3 located at the toe of the 1:5 slope was lowered for the tests with \( d = 16.6 \) and \( 18.6 \) cm to avoid exposure to air during wave rundown. The cross-shore locations of the three ADVs are given in Fig. 3-1. The measured vertical and cross-flume velocities appeared to be dominated by turbulent velocities and were much smaller than the cross-shore velocity \( u \) which was dominated by the wave component. Only the horizontal velocity \( u \) is considered hereafter.
CHAPTER 4 PROBABILITY DISTRIBUTIONS

The statistics of the measured time series of the free surface and shoreline elevations as well as the cross-shore horizontal velocity are examined in this chapter. The measured probability density functions of these variables are compared with the exponential gamma distribution and the Gaussian distribution in the same way as in Kobayashi et al. (1998). A brief description of the exponential gamma distribution is given first. The distributions for the measured wave heights and runup heights are also compared with the Rayleigh and Weibull distributions, where the setup effect on the runup height is examined.

4.1 EXPONENTIAL GAMMA DISTRIBUTION

The measured normalized free surface elevation \( \eta = \left( \eta - \bar{\eta} \right) / \sigma_\eta \) and the normalized cross-shore horizontal velocity \( u = \left( u - \bar{u} \right) / \sigma_u \) are compared with the exponential gamma
distribution described in the report by Orzech and Kobayashi (1997). The exponential gamma distribution for \( \eta \) is expressed as

\[
f(\eta) = \left[ \Gamma(a) \right]^{1/4} \sqrt{\psi'(a)} \exp(-a\psi) \exp\left[-\exp(-y)\right]
\]

(15)

\[
y = \sqrt{\psi'(a)} \eta - \psi(a)
\]

(16)

where \( a \) = shape parameter, \( \Gamma \) = gamma function, \( \psi \) = digamma function and \( \psi' \) = trigamma function. The relationship between the skewness, \( s \), and the shape parameter \( a \) is given by

\[
s = -\psi''(a) \left[ \psi'(a) \right]^{3/2}
\]

(17)

where \( \psi'' \) = tetragamma function. The kurtosis of, \( K \), is given by

\[
K = \psi'''(a) \left[ \psi'(a) \right]^{3} + 3
\]

(18)

where \( \psi''' \) = pentagamma function. The gamma and related functions are explained by Abramowitz and Stegun (1972) and tabulated by Gran (1992).

The Gaussian and the exponential distributions are special cases of the exponential gamma distribution when the skewness \( s \) equals 0 and 2, respectively:

\[
\text{Gaussian: } f(\eta) = \frac{1}{\sqrt{2\pi}} \exp\left(-\eta^2 / 2\right) \text{ for } s = 0
\]

(19)

\[
\text{Exponential: } f(\eta) = \exp\left[-(\eta + 1)\right] \text{ for } s = 2
\]

(20)

### 4.2 FREE SURFACE AND SHORELINE ELEVATIONS

The statistics of the measured time series of the free surface and shoreline elevations \( \eta \) and \( \eta_r \) are examined first. The normalized free surface elevation \( \eta \) is defined as
\[ \eta_i = \frac{\eta - \eta}{\sigma_{\eta}} \]  

(21)

where \( \eta \) and \( \sigma_{\eta} \) are the mean and standard deviation of \( \eta \). For the runup elevation \( \eta_r \), \( \eta \) and \( \sigma_{\eta} \) are replaced by \( \bar{\eta} \), and \( \sigma_r \). The mean and standard deviation of \( \bar{\eta} \) are zero and unity.

Kobayashi et al. (1998) compared the measured probability density functions for \( \eta \) and \( u \), with the Gaussian and exponential gamma distributions using the measured values of mean, standard deviation and skewness.

Fig. 4-1 and Fig. 4-2 show the measured probability density functions at wave gauges 1 - 7 and the runup wire for the 30 tests in the 1/5 slope experiment. Wave gauges 1, 2 and 3 are located just seaward of the surf zone, wave gauge 4 and 5 approximately in the middle of the surf zone, wave gauge 6 at the toe of the porous slope and wave gauge 7 on the permeable slope. The shoreline elevation above SWL, measured by the runup wire is analyzed in the same way as the free surface elevations. The theoretical exponential gamma distribution is also plotted in Fig. 4-1 and Fig. 4-2 using the average value of the measured skewness, \( s \), for all the tests at each wave gauge location. It should be noted that the agreement with the data for each specific test is better when the measured skewness for the specific test is utilized instead of the average value of the skewness for all the tests at the same wave gauge location. The Gaussian distribution is also plotted in Fig. 4-1 and Fig. 4-2 for comparison. Fig. 4-1 and Fig. 4-2 show the skewed wave profiles in the surf zone with peaked crests and flat troughs caused by wave nonlinearities. The agreement for the exponential gamma distribution is good. Nevertheless, the adoption of the exponential gamma distribution for prediction purposes is not convenient because the skewness is hard to estimate accurately. The Gaussian distribution is a good first approximation for the free surface elevation well outside the surf zone.
zone and the shoreline elevation. The Gaussian distribution of the shoreline elevation is made implicitly in Eqs. (13) and (14).

Fig. 4-1: Measured and computed probability density functions of normalized free surface elevation $\eta$, at wave gauges 1 - 4 for all the 1/5 slope tests.
Fig. 4-2: Measured and computed probability density functions of normalized free surface elevation $\eta_s$ at wave gauges 5 - 7 and by runup wire for all the 1/5 slope tests.
4.3 CROSS-SHORE VELOCITY

The statistics of the measured time series of the cross-shore horizontal velocity \( u \) is examined in the same manner as the free surface elevation. The normalized cross-shore horizontal velocity \( u_* \) is defined as

\[
u_* = \frac{u - \bar{u}}{\sigma_u}
\]

(22)

where \( \bar{u} \) and \( \sigma_u \) are the measured mean and standard deviation of \( u \).

Fig. 4-3: Measured and computed probability density functions of normalized cross-shore horizontal velocity \( u_* \) at ADV1, ADV2 and ADV3 for all the 1/5 slope tests.
Fig. 4-3 shows the comparison between the measured probability density function of the normalized cross-shore horizontal velocity \( u \) and the exponential gamma distribution using the average value of the measured skewness, \( s_m \), for all the tests at each velocimeter location. The measured skewness, \( s_m \), is in the range of 0 - 0.50 and the Gaussian distribution is a good approximation. It is noted that the skewness is smaller for the horizontal velocity than for the free surface, implying that the nonlinearity decreases downward from the free surface. The Gaussian distribution of the horizontal velocity \( u \) is assumed to derive Eq. (4).

### 4.4 WAVE HEIGHT DISTRIBUTION

The measured exceedance probability distributions of the zero-upcrossing wave height \( H \) at each wave gauge location is compared with the Rayleigh and Weibull distributions given by

\[
\text{Rayleigh: } P = \exp\left[-2 \left(\frac{H}{H_{1/3}}\right)^2\right] \\
\text{Weibull: } P = \exp\left[-\beta \left(\frac{H}{H_{1/3}}\right)^\alpha\right]
\]

with \( P = \) exceedance probability, \( H_{1/3} = \) measured significant wave height for each time series, \( \alpha = \) shape parameter and \( \beta \) is a nondimensional parameter related to the scale parameter of the Weibull distribution. Eq. (24) reduces to Eq. (23) when \( \alpha = 2 \) and \( \beta = 2 \).

Fig. 4-4 and Fig. 4-5 compare the measured exceedance probability distributions of the wave heights \( H \) at wave gauges 1 - 7 with the Rayleigh distribution given by Eq. (23). These figures include all the data points from the 30 tests in the 1/5 slope experiment.

Fig. 4-6 and Fig. 4-7 compare the measured exceedance probability distributions of \( H \) with the Weibull distribution given by Eq. (24) at wave gauges 1-7 for the test R20B1, as
an example. The Rayleigh distribution is also plotted for reference. The parameters $\alpha$ and $\beta$ are estimated for each measured distribution using the maximum likelihood method. Table 4 shows the estimates of the Weibull parameters $\alpha$ and $\beta$ for the different 1/5 slope tests at each wave gauge location. Due to the fact that $\alpha$ and $\beta$ are fitted for each distribution, the agreement with the measured distribution is slightly better for the Weibull distribution. However, for prediction purposes, it is difficult to use the Weibull distribution because $\alpha$ and $\beta$ are hard to estimate in advance.

As a whole, the Rayleigh distribution is a good approximation except that the scatter of data points is large for very small $P$ partly because the test duration of 429.6 s was not very long.
Fig. 4-4: Measured and Rayleigh exceedance probability distributions of wave heights at wave gauges 1 - 4 for all the 1/5 slope tests.
Fig. 4-5: Measured and Rayleigh exceedance probability distributions of wave heights at wave gauges 5 - 7 for all the 1/5 slope tests.

Table 4: Estimates of Weibull parameters at each wave gauge for all the 1/5 slope tests.

<table>
<thead>
<tr>
<th>TEST</th>
<th>Gauge 1</th>
<th>Gauge 2</th>
<th>Gauge 3</th>
<th>Gauge 4</th>
<th>Gauge 5</th>
<th>Gauge 6</th>
<th>Gauge 7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\alpha$</td>
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<tr>
<td>R16A1</td>
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<td>2.09</td>
<td>2.05</td>
<td>2.11</td>
<td>2.02</td>
<td>2.47</td>
</tr>
<tr>
<td>R16B1</td>
<td>2.12</td>
<td>2.05</td>
<td>2.11</td>
<td>2.03</td>
<td>2.10</td>
<td>2.08</td>
<td>2.29</td>
</tr>
<tr>
<td>R16C1</td>
<td>1.68</td>
<td>2.04</td>
<td>1.64</td>
<td>2.02</td>
<td>1.70</td>
<td>2.04</td>
<td>1.70</td>
</tr>
<tr>
<td>R18A1</td>
<td>2.14</td>
<td>2.06</td>
<td>1.95</td>
<td>1.91</td>
<td>2.05</td>
<td>2.00</td>
<td>2.43</td>
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<tr>
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<td>2.02</td>
<td>2.15</td>
<td>2.03</td>
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<td>2.05</td>
<td>1.67</td>
<td>2.08</td>
<td>1.76</td>
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<td>2.05</td>
<td>2.12</td>
<td>2.14</td>
<td>2.18</td>
<td>2.01</td>
<td>2.19</td>
</tr>
<tr>
<td>R20B1</td>
<td>2.11</td>
<td>2.01</td>
<td>2.04</td>
<td>2.03</td>
<td>2.05</td>
<td>2.07</td>
<td>2.13</td>
</tr>
<tr>
<td>R20C1</td>
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<td>1.90</td>
<td>2.09</td>
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<td>2.05</td>
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<td>2.10</td>
<td>2.04</td>
<td>2.21</td>
</tr>
<tr>
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<td>2.11</td>
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<td>2.05</td>
<td>2.04</td>
<td>2.00</td>
<td>2.08</td>
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<td>2.01</td>
<td>1.84</td>
<td>2.07</td>
<td>1.88</td>
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<td>2.02</td>
<td>2.04</td>
<td>2.07</td>
<td>2.08</td>
<td>2.04</td>
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<td>2.05</td>
<td>2.02</td>
<td>2.11</td>
<td>2.22</td>
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<td>2.04</td>
<td>2.03</td>
<td>2.05</td>
<td>2.03</td>
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</tbody>
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Fig. 4-6: Measured, Rayleigh and Weibull exceedance probability distributions of wave heights at wave gauges 1 - 4 for test R20B1 on 1/5 slope.
Fig. 4-7: Measured, Rayleigh and Weibull exceedance probability distributions of wave heights at wave gauges 5 - 7 for test R20B1 on 1/5 slope.

4.5 WAVE RUNUP DISTRIBUTION

Fig. 4-8 and Fig. 4-9 compare the measured probability distributions of zero-upcrossing runup heights with the Rayleigh distribution given by Eq. (13) which accounts for the mean $\bar{\eta}_r$ for the runup height $R$ above SWL, and the standard Rayleigh distribution

$$P(R) = \exp\left[-2\left(R/R_{1/3}\right)^2\right],$$

with no regard to the wave setup. Fig. 4-8 includes all the data
points from the 30 tests in the 1/5 slope experiment, while Fig. 4-9 all the data for the 27 tests in the 1/2 slope experiment. The Rayleigh distribution is a good approximation except for the scatter of data points for the exceedance probabilities of the order of 0.01 or less. The agreement for the standard Rayleigh distribution without the wave setup is worse for the 1/5 slope but is similar for the 1/2 slope because the wave setup is appreciable on the 1/5 slope and should be accounted for.

In the same manner as in Figs. 4-6 and 4-7 for the wave height distributions, Fig. 4-10 compares the measured exceedance probability distributions of runup heights as a function of \( \left( R - \bar{\eta}_r \right) / \left( R_{1/3} - \bar{\eta}_r \right) \) and \( R / R_{1/3} \) for test R20B1 with Weibull and Rayleigh distributions. Table 5 lists the Weibull parameters \( \alpha \) and \( \beta \) estimated using the maximum likelihood method. The values of \( \alpha \) and \( \beta \) are not too different from two when the wave setup is accounted for.

Fig. 4-8: Measured and Rayleigh exceedance probability distributions of runup heights as a function of \( \left( R - \bar{\eta}_r \right) / \left( R_{1/3} - \bar{\eta}_r \right) \) and \( R / R_{1/3} \) for all the 1/5 slope tests.
Fig. 4-9: Measured and Rayleigh exceedance probability distributions of runup heights as a function of $(R - \bar{\eta})/(R_{1/3} - \bar{\eta})$ and $R/R_{1/3}$ for all the 1/2 slope tests.

Table 5: Estimates of Weibull parameters for runup heights in the 1/5 slope experiment.

<table>
<thead>
<tr>
<th>TEST</th>
<th>$(R - \bar{\eta})/(R_{1/3} - \bar{\eta})$</th>
<th>$R/R_{1/3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>R16A1</td>
<td>2.13</td>
<td>1.89</td>
</tr>
<tr>
<td>R16B1</td>
<td>1.68</td>
<td>1.97</td>
</tr>
<tr>
<td>R16C1</td>
<td>1.42</td>
<td>1.91</td>
</tr>
<tr>
<td>R18A1</td>
<td>1.64</td>
<td>2.03</td>
</tr>
<tr>
<td>R18B1</td>
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<td>R18C1</td>
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</tr>
<tr>
<td>R24C1</td>
<td>1.51</td>
<td>1.95</td>
</tr>
</tbody>
</table>
Fig. 4-10: Measured, Rayleigh and Weibull exceedance probability distributions of runup heights as a function of \( \frac{(R - \bar{\eta}_r)}{(R_{1/3} - \bar{\eta}_r)} \) and \( \frac{R}{R_{1/3}} \) for test R20B1 on 1/5 slope.

Fig. 4-11 shows the relationship between the 2% runup height \( R_{2\%} \) and significant runup height \( R_{1/3} \) for each of the 57 tests where Eq. (13) yields

\[
R_{2\%} - \bar{\eta}_r = 1.4 \left( R_{1/3} - \bar{\eta}_r \right) 
\]

(25)

It is noted that \( R_{2\%} \) is used for the design of the crest height of a dike in The Netherlands (van Gent 2001). Fig. 4-11 indicates that Eq. (25) based on the Rayleigh distribution with the wave setup effect slightly overpredicts \( R_{2\%} \) for given \( R_{1/3} \) and \( \bar{\eta}_r \). The degree of the overprediction increases when the measured values of \( R_{2\%} \) and \( R_{1/3} \) are compared with \( R_{2\%} = 1.4 R_{1/3} \) without the wave setup effect, as can be seen in Fig. 4-12.
Fig. 4-11: Relationship between $(R_{25\%} - \bar{\eta}_r)$ and $(R_{1/3} - \bar{\eta}_r)$ for 1/5 and 1/2 slope tests.

Fig. 4-12: Relationship between $R_{25\%}$ and $R_{1/3}$ for 1/5 and 1/2 slope tests.
Fig. 4-13 examines the accuracy of Eq. (14) for $\tan \theta = 1/5$ and 1/2. The slope correction term $(\sigma, \tan \theta)$ improves the agreement in comparison with $(R_{1/3} - \bar{\eta}_r) = 2\sigma_r$ based on the Gaussian shoreline fluctuations. This slope correction will not be valid when the slope becomes steeper and the slope angle $\theta$ approaches 90°.

![Chart](image)

**Fig. 4-13:** Empirical formula $(R_{1/3} - \bar{\eta}_r) = 2.2\sigma_r$ and $2.5\sigma_r$ for 1/5 and 1/2 slopes.
CHAPTER 5  COMPARISON WITH EXPERIMENTS

In this chapter, the comparisons of the numerical model with the experiments are shown. First, the measured and predicted cross-shore variations of $\bar{\eta}$, $\sigma_{\eta}$, $\bar{u}$ and $\sigma_u$ for the 15 pairs of the two repeated tests for the 1/5 slope experiment are presented. Second, the permeability effects on $\bar{\eta}$, $\sigma_{\eta}$, $\bar{u}$ and $\sigma_u$ as well as the sensitivities of the numerical model to the roller effect, the breaker ratio parameter and bottom friction factor on the porous slope are examined. Last, the measured wave runup statistics for the 1/5 and 1/2 slope experiments are compared with the results of the time-averaged probabilistic model.

5.1 CROSS-SHORE WAVE TRANSFORMATION

Fig. 4-8 - Fig. 4-13 indicate that Eqs. (13), (14) and (25) may be used to predict the wave runup heights $R_{1/3}$ and $R_{2\eta}$ as well as the value of $R$ for the specified exceedance probability $P$ if the mean $\bar{\eta}$, and standard deviation $\sigma_{\eta}$ of the shoreline fluctuations can be
predicted accurately by the numerical model based on Eqs. (1) - (12). For each of the 57 tests, the bottom profiles \( z_b(x) \) and \( z_p(x) \) and the measured values of \( T_p, \bar{\eta} \) and \( \sigma_\eta \) at \( x = 0 \) are specified as input to the numerical model. The output of the numerical model includes the cross-shore variations of the variables in Eqs. (1) – (10), the reflected wave height given by Eq. (11), and the values of \( \bar{\eta}_r \) and \( \sigma_r \) in Eq. (12). The only empirical parameter calibrated here is the breaker ratio parameter \( \gamma \) in Eq. (10). Use is made of \( \gamma = 0.7 \) calibrated by Kobayashi et al. (2005) who compared the numerical model with the experiment in which the 1/5 permeable slope in Fig. 3-1 was replaced by a submerged porous breakwater. The increase of \( \gamma \) causes the landward shift of irregular wave breaking on the gently sloping beach. The calibration of \( \gamma \) is made by comparing the measured and computed cross-shore variations of \( \bar{\eta}, \sigma_\eta, \bar{u} \) and \( \sigma_u \). The overall agreement is good except for the 1/2 slope tests with \( T_p = 1.5 \) and 2.4 s. For these tests, \( \gamma = 0.8 \) is used to improve the agreement but the reason for this increase of \( \gamma \) is not clear. The other empirical parameters calibrated by Kobayashi et al. (2005) are the slope adjustment factor \( b=3 \) in Eq. (10), the bottom friction factor on the stone slope, \( f_b = 0.01 \) in Eq. (14), and the turbulent porous flow resistance factor \( \beta_0 = 5 \) in Eq. (8). These values are used here in the subsequent analysis without any additional calibration. In the following, the roller effect is not included to reduce the number of the empirical parameters except in Section 5.3.

Fig. 5-1 - Fig. 5-15 compare the measured and predicted cross-shore variations of \( \bar{\eta}, \sigma_\eta, \bar{u} \) and \( \sigma_u \) and the bottom profiles \( z_b(x) \) and \( z_p(x) \) for the 15 pairs of the two repeated tests (runs 1 and 2) for the 1/5 slope experiment. The tests were repeatable, indicating the reliability of the data. The computed results in the region of small water depth above SWL are reproducible, implying the numerical stability in the region of numerical
difficulties. The numerical model slightly overpredicts the wave setup \( \bar{\eta} \) near the shoreline. The agreement for \( \sigma_\eta \) is good partly because the measured \( \sigma_\eta \) did not vary much. The comparisons for \( \bar{\mu} \) and \( \sigma_x \) are somewhat ambiguous because the computed \( \mu \) is the depth-averaged velocity but the measured \( \mu \) corresponded to the mid-depth elevation. The agreement for \( \bar{u} \) is better than expected from the previous comparison for a submerged breakwater by Kobayashi et al. (2005). The standard deviation \( \sigma_u \) is overpredicted slightly. On the other hand, the comparisons with the 27 tests in the 1/2 slope experiment were presented in the report by Kearney and Kobayashi (2001) who used the earlier version of this numerical model by Johnson and Kobayashi (1998). The present numerical model has also been compared with the 27 tests and the agreement has been found to be very similar because the major improvement here is essentially limited to the region near the shoreline where \( \eta \) and \( \mu \) were not measured.
Fig. 5-1: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R16A1 and R16A2 on 1/5 slope.
Fig. 5-2: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R16B1 and R16B2 on 1/5 slope.
Fig. 5-3: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R16C1 and R16C2 on 1/5 slope.
Fig. 5-4: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R18A1 and R18A2 on 1/5 slope.
Fig. 5-5: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $\bar{u}$ above bottom profile $z_b$ for tests R18B1 and R18B2 on 1/5 slope.
Fig. 5-6: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R18C1 and R18C2 on 1/5 slope.
Fig. 5-7: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R20A1 and R20A2 on 1/5 slope.
Fig. 5-8: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R20B1 and R20B2 on 1/5 slope.
Fig. 5-9: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R20C1 and R20C2 on 1/5 slope.
Fig. 5-10: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R22A1 and R22A2 on 1/5 slope.
Fig. 5-11: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R22B1 and R22B2 on 1/5 slope.
Fig. 5-12: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R22C1 and R22C2 on 1/5 slope where the measured $\sigma_u$ by ADV1 was not reliable.
Fig. 5-13: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R24A1 and R24A2 on 1/5 slope.
Fig. 5-14: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R24B1 and R24B2 on 1/5 slope.
Fig. 5-15: Measured and predicted cross-shore variations of mean and standard deviation of $\eta$ and $u$ above bottom profile $z_b$ for tests R24C1 and R24C2 on 1/5 slope.
5.2 EFFECTS OF POROUS LAYER

To examine the permeability effects on $\bar{\eta}$, $\sigma_{\eta}$, $\bar{u}$ and $\sigma_{u}$, computation is also made for the 1/5 slope tests with no permeable layer by specifying $z_p = z_b$ and $h_p = 0$ in Eqs. (6) and (7). The empirical parameters used in the computations are $\gamma = 0.7$, $\beta_o = 5$, $b = 3$ and $f_b = 0.01$. The permeability effects tend to increase with the increase of the toe depth $d_t$ and with the decrease of the period $T_p$. Consequently, the computed results for the test with the largest $d_t = 24.6$ cm and the smallest $T_p = 1.59$ s are presented in Fig. 5-16 where $H_{ims} = 11.59$ cm at $x = 0$. The cross-shore variations above the 1/5 slope are shown to differentiate the computed variations on the porous and impermeable slopes. The wave setup $\bar{\eta}$ reaches a higher elevation as it approaches tangential to the impermeable slope. The standard deviation $\sigma_{\eta}$ decreases landward more gradually because of no energy dissipation due to porous flow resistance. The return current $\bar{u}$ is negative (offshore) on the impermeable slope, whereas $\bar{u}$ becomes positive above the still water shoreline on the porous slope because of infiltration and return flow inside the porous layer. The computed mean discharge velocity $\bar{v}$ is of the order of $-0.3$ cm/s and the corresponding standard deviation $\sigma_v$ is about $1$ cm/s as shown in Fig. 5-19. Obviously, $\bar{v}$ and $\sigma_v$ are zero for the impermeable slope. The standard deviation $\sigma_u$ of the horizontal velocity $u$ above the slope is much larger than $\sigma_v$ and decreases landward more gradually on the impermeable slope.

Fig. 5-17 shows the computed cross-shore variations of $n, a, Q$ and $\sigma_u$. The ratio $n$ between the group and phase velocities in Eq. (2) is almost unity in the shallow water on the 1/5 slope. The coefficient $a$ in Eq. (10) increases landward with the decrease of $\bar{h}$ and significantly increases $D_b$ due to intense wave breaking on the 1/5 slope. The fraction $Q$ of
breaking waves in Eq. (10) increases rapidly over the 1/5 slope and indicates the breaking of all waves in the region \( x > 6.9 \) m. The ratio \( \sigma_\ast = \sigma_\ast / \bar{h} \) increases landward but is less than unity due to the empirical adjustment for \( \sigma_\ast > \sigma_\ast_c = \gamma / \sqrt{8} \) discussed below Eq. (10).

Fig. 5-18 shows the cross-shore variations of \( S_{\sigma_\ast}^\ast = S_{\sigma_\ast} / \rho g \) and \( \tau_h^\ast = \tau_h / \rho g \) involved in the momentum equation in Eq. (1). \( S_{\sigma_\ast} \) decreases rapidly due to wave breaking in the region of \( Q = 1 \) and causes the increase of the wave setup. \( \tau_h \) is negative because \( \bar{u} < 0 \) except above the still water shoreline on the porous slope because of infiltration into the permeable layer. \( |\tau_h| \) is somewhat larger on the impermeable slope due to the larger \( \sigma_a \) on the impermeable slope.

Fig. 5-20 shows the computed cross-shore variations of \( F^\ast = F / \rho g \), \( D_b^\ast = D_b / \rho g \), \( D_r^\ast = D_r / \rho g \) and \( D_f^\ast = D_f / \rho g \) involved in the energy equation in Eq. (1). The wave energy flux \( F \) decreases landward due to the energy dissipation but the difference between the values on the porous and impermeable slopes is small. This is because the energy dissipation rate \( D_b \) due to wave breaking increases on the impermeable slope for which the energy dissipation rate \( D_r \) due to porous flow resistance is zero. The energy dissipation rate \( D_f \) due to the bottom friction is of the order of 0.02 \( D_b \). The computed results in Fig. 5-16 - Fig. 5-20 may not be very accurate but indicate the interconnected nature of the variables in Eqs. (1) – (10).
Fig. 5-16: Permeability effects on $\bar{\eta}$, $\sigma_\eta$, $\bar{u}$ and $\sigma_u$ for test R24A1.
Fig. 5-17: Permeability effects on \( n, a, Q \) and \( \sigma \) for test R24A1.
Fig. 5-18: Permeability effects on $S^*_a$ and $\tau^*_b$ for test R24A1.
Fig. 5-19: Permeability effects on $\bar{v}$ and $\sigma_v$ for test R24A1.
Fig. 5-20: Permeability effects on wave energy flux $F = \rho g F^*$ and dissipation rates $D_b = \rho g D_b^*$, $D_r = \rho g D_r^*$, and $D_f = \rho g D_f^*$ due to wave breaking, porous flow resistance, and bottom friction, respectively.
5.3 SENSITIVITY TO ROLLER EFFECT

In this section the sensitivity of the computed results to the roller effect is investigated. Fig. 5-21 shows the measured and computed $\overline{\eta}$, $\sigma_\eta$, $\overline{u}$, and $\sigma_u$ for test R20B1 where $IROLL = 1$ and $IROLL = 0$ indicate the computed results with and without the roller volume flux $q_r$ in Eqs. (2) and (6). The empirical parameters used in the computations are $\gamma = 0.7$, $\beta_o = 5$, $b = 3$ and $f_o = 0.01$. The inclusion of the roller volume flux $q_r$ increases significantly the wave setup $\overline{\eta}$ near the still water shoreline, while the effect on $\overline{\eta}$ over the gentler impermeable beach slope is negligible. The standard deviations of the free surface elevation and horizontal velocity $\sigma_\eta$ and $\sigma_u$ decrease landward more gradually with $IROLL = 1$ near the still water shoreline where the wave energy flux $F = \rho g C_\epsilon \sigma_\eta^2$ in the adopted wave energy equation (1) has no effect of $q_r$. The mean horizontal velocity $\overline{u}$ becomes more negative (seaward), on the porous slope due to the contribution of the roller volume flux $q_r$ in Eq. (6), whereas the onshore mean velocity $\overline{u}$ above the still water shoreline is reduced.

The inclusion of the roller effect with the additional empirical parameter $\beta_r$ did not necessarily improve the agreement as was the case for sand beaches (Kobayashi et al., 2005). Consequently, the roller effect has been neglected for the rest of the computed results in this report.
Fig. 5-21: Measured and computed $\bar{\eta}$, $\sigma_\eta$, $\bar{u}$ and $\sigma_u$ for test R20B1 where IROLL = 0 and 1 indicate the computed results with and without the roller volume flux $q_r$. 
5.4 SENSITIVITY TO BREAKER RATIO PARAMETER

The breaker ratio parameter $\gamma$ in Eq. (10) is varied here to analyze the sensitivity of the computed results to this parameter. Table 6 shows the estimated values of the breaker ratio parameter $\gamma$ using the empirical formula by Battjes and Stive (1985)

$$\gamma = 0.5 + 0.4 \tanh \left( 33s_o \right)$$  \hspace{1cm} (26)

with $s_o = H_{rmso} / L_{op} = \text{deep-water wave steepness}$. $H_{rmso}$ and $L_{op}$ are the equivalent deep water root-mean-square wave height and wavelength, respectively, based on linear wave shoaling for periodic waves with frequency $1/T_p$. The estimated values are in the range of 0.6 - 0.8 for all the 1/5 slope tests. Consequently, computations are made with $\gamma = 0.6, 0.7$ and 0.8. Fig. 5-22 shows the comparisons between the measured and computed $\bar{\eta}, \sigma_{\bar{\eta}}, \bar{u}$ and $\sigma_{\bar{u}}$ for test R20B1 for $\gamma = 0.6, 0.7$ and 0.8. The decrease of $\gamma$ reduces the depth-limited wave height $H_m$ in Eq. (10), and increases $Q$ and $D_\eta$ in Eq. (10). This increase of $D_\eta$ in Eq. (1) results in the decrease of $\sigma_{\bar{\eta}}, |\bar{u}|$ and $\sigma_{\bar{u}}$ and the increase of $\bar{\eta}$ on the impermeable beach. However, $\bar{\eta}$ is reduced slightly on the permeable 1/5 slope due to the decrease of $\sigma_{\eta}$. As a whole, $\gamma = 0.7$ predicts $\sigma_{\eta}$ better for most of the 1/5 slope tests and $\gamma = 0.7$ is used for all the 1/5 slope tests.
Table 6: Estimated breaker ratio parameter $\gamma$ using Battjes and Stive (1985).

<table>
<thead>
<tr>
<th>Test</th>
<th>$H_{rms}$ [m]</th>
<th>$d_1$ [m]</th>
<th>$T_p$ [s]</th>
<th>$s_o$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R16A1</td>
<td>0.100</td>
<td>0.349</td>
<td>1.5</td>
<td>0.029</td>
<td>0.80</td>
</tr>
<tr>
<td>R16B1</td>
<td>0.105</td>
<td>0.349</td>
<td>2.3</td>
<td>0.012</td>
<td>0.65</td>
</tr>
<tr>
<td>R16C1</td>
<td>0.068</td>
<td>0.349</td>
<td>3.0</td>
<td>0.004</td>
<td>0.56</td>
</tr>
<tr>
<td>R18A1</td>
<td>0.105</td>
<td>0.369</td>
<td>1.5</td>
<td>0.031</td>
<td>0.81</td>
</tr>
<tr>
<td>R18B1</td>
<td>0.097</td>
<td>0.369</td>
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<td>0.011</td>
<td>0.64</td>
</tr>
<tr>
<td>R18C1</td>
<td>0.072</td>
<td>0.369</td>
<td>3.0</td>
<td>0.005</td>
<td>0.56</td>
</tr>
<tr>
<td>R20A1</td>
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<td>0.389</td>
<td>1.5</td>
<td>0.032</td>
<td>0.81</td>
</tr>
<tr>
<td>R20B1</td>
<td>0.115</td>
<td>0.389</td>
<td>2.3</td>
<td>0.013</td>
<td>0.67</td>
</tr>
<tr>
<td>R20C1</td>
<td>0.074</td>
<td>0.389</td>
<td>3.0</td>
<td>0.005</td>
<td>0.56</td>
</tr>
<tr>
<td>R22A1</td>
<td>0.114</td>
<td>0.409</td>
<td>1.5</td>
<td>0.034</td>
<td>0.82</td>
</tr>
<tr>
<td>R22B1</td>
<td>0.121</td>
<td>0.409</td>
<td>2.3</td>
<td>0.014</td>
<td>0.67</td>
</tr>
<tr>
<td>R22C1</td>
<td>0.078</td>
<td>0.409</td>
<td>3.0</td>
<td>0.005</td>
<td>0.57</td>
</tr>
<tr>
<td>R24A1</td>
<td>0.116</td>
<td>0.429</td>
<td>1.5</td>
<td>0.034</td>
<td>0.83</td>
</tr>
<tr>
<td>R24B1</td>
<td>0.122</td>
<td>0.429</td>
<td>2.3</td>
<td>0.014</td>
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<td>0.429</td>
<td>3.0</td>
<td>0.005</td>
<td>0.57</td>
</tr>
</tbody>
</table>

$H_{rms}$ = measured root-mean-square wave height at wave gauge 1; $d_1$ = water depth at wave gauge 1; $T_p$ = peak period; $s_o$ = deep water steepness; $\gamma$ = breaker ratio parameter
Fig. 5-22: Sensitivity to breaker ratio parameter $\gamma = 0.6$, 0.7 and 0.8 for test R20B1.
5.5 SENSITIVITY TO BOTTOM FRICTION FACTOR

The bottom friction factor $f_b$ in Eq. (4) is varied to examine the sensitivity of the computed results to this parameter. Fig. 5-23 shows the measured and computed $\bar{\eta}$, $\sigma_\eta$, $\bar{u}$ and $\sigma_u$ for the bottom friction factor $f_b = 0.01$ and 0.05 on the porous slope for test R20B1. The computed results by this time-averaged model are found to be insensitive to $f_b$, mainly because wave breaking and porous flow resistance are dominant in contrast to bottom friction on the 1/5 porous slope.
Fig. 5-23: Sensitivity to bottom friction factor $f_b = 0.01$ and 0.05 on the porous slope for test R20B1.
5.6 WAVE RUNUP STATISTICS

Fig. 5-24 and Fig. 5-25 compare the measured and predicted values of the mean \( \bar{\eta}_r \) of the shoreline elevation \( \eta_r \) for the 1/5 and 1/2 slope experiments, respectively, for the different values of \( T_p \). Fig. 5-26 and Fig. 5-27 compare the standard deviation \( \sigma_r \) for both experiments. The agreement is fair except for \( \bar{\eta}_r \) on the 1/2 slope. The computed \( \bar{\eta}_r \) for \( T_p = 2.4 \) and 4.7 s is too large in comparison with the measured \( \bar{\eta}_r \) of about 1 cm. The effect of this overprediction on \( R_{1/3} \) and \( R_{2\%} \) is minor because \( \sigma_r \) on the 1/2 slope for \( T_p = 2.4 \) and 4.7 s is of the order of 4 cm and much larger than the measured \( \bar{\eta}_r \).

Fig. 5-28 and Fig. 5-29 compare the measured and predicted significant runup heights \( R_{1/3} \) for the 1/5 and 1/2 porous slope experiments for the different values of \( T_p \), where \( R_{1/3} \) is predicted using Eq. (14). The numerical model predicts \( R_{1/3} \) within the error of about 20% partly because the slope correction is included in Eq. (14) empirically.

Fig. 5-30 and Fig. 5-31 compare the measured and predicted 2% runup heights \( R_{2\%} \) where use is made of Eq. (25). The empirical formula of van der Meer and Janssen (1995) is also compared with the data. For the case of normally incident waves on a slope with no berm, this formula can be expressed as

\[
R_{2\%} = 1.5 \xi \gamma_f \gamma_h H_{1/3} \quad \text{with} \quad \xi \leq 2
\]  

(27)

with

\[
\xi = \left( \frac{gT_p^2}{2\pi H_{1/3}} \right)^{0.5} \tan \theta \quad \gamma_f = 1 - 0.03 \left( 4 - \frac{d_f}{H_{1/3}} \right)^2 \quad \text{if} \quad \frac{d_f}{H_{1/3}} < 4
\]  

(28)
where $\xi$ = surf similarity parameter; $H_{1/3}$ = significant wave height at the toe of the slope; $\gamma_f$ = reduction factor due to slope roughness; $\gamma_h$ = reduction factor due to wave breaking on a shallow foreshore which is less than unity if $(d_s / H_{1/3}) < 4$. Eq. (27) implies that $(1.5 \xi)$ is replaced by 3.0 if $\xi > 2$. The reduction factor $\gamma_h$ based on the measured ratio, $H_{25\%} / (1.4 H_{1/3})$, on a foreshore slope of 1/100 with $H_{25\%}$ = 2% wave height is assumed to be valid for the present beach slopes of 1/34.4 and 1/32.1. The reduction factor $\gamma_f$ for a rubble layer with two or more stone diameter thickness was suggested to be in the range of 0.50 – 0.55 for $\xi < 4$ and larger for $\xi > 4$ but $\gamma_f = 0.52$ is used here to obtain the fair agreement shown in Fig. 5-30 and Fig. 5-31. The significant wave height $H_{1/3}$ for each test is obtained from the time series of the free surface elevation measured by the wave gauge at the toe of the slope. For the 1/5 slope tests, $0.97 < \xi < 2.23$ and $0.78 < \gamma_h < 0.91$. For the 1/2 slope tests, $2.85 < \xi < 9.33$ and $0.61 < \gamma_h < 0.86$. Eq. (27) developed originally for $0.5 < \xi < 5$ is applied here for $0.97 < \xi < 9.33$. All the parameters for the 1/5 and 1/2 slope experiments are summarized in Table 7 and Table 8. Fig. 5-30 and Fig. 5-31 indicate that the numerical model and empirical formula predicts $R_{25\%}$ within the error of about 20%. It should be noted that the numerical model uses the measured $H_{mm}$ outside the surf zone instead of the measured $H_{1/3}$ at the toe of the slope.
Fig. 5-24: Measured and predicted mean shoreline elevations $\bar{\eta}_r$ for $1/5$ slope tests.

Fig. 5-25: Measured and predicted mean shoreline elevations $\bar{\eta}_r$ for $1/2$ slope tests.
Fig. 5-26: Measured and predicted standard deviations of shoreline oscillations \( \sigma_r \) for 1/5 slope tests.

Fig. 5-27: Measured and predicted standard deviations of shoreline oscillations \( \sigma_r \) for 1/2 slope tests.
Fig. 5-28: Measured and predicted significant runup heights $R_{1/3}$ for 1/5 slope tests.

Fig. 5-29: Measured and predicted significant runup heights $R_{1/3}$ for 1/2 slope tests.
Fig. 5-30: Measured and predicted 2% runup heights $R_{2\%}$ for 1/5 slope tests.

Fig. 5-31: Measured and predicted 2% runup heights $R_{2\%}$ for 1/2 slope tests.
Table 7: Comparison with empirical formula for 1/5 slope tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>(d_i) [cm]</th>
<th>(T_p) [s]</th>
<th>(H_{1/3}) [cm]</th>
<th>(\xi)</th>
<th>(\gamma_h)</th>
<th>(\gamma_f)</th>
<th>(\gamma_b)</th>
<th>(\frac{R_{2%}}{H_{1/3}})</th>
<th>(R_{1.7%}) [cm]</th>
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<tbody>
<tr>
<td>R16A1</td>
<td>16.6</td>
<td>1.5</td>
<td>11.922</td>
<td>1.107</td>
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<td>0.52</td>
<td>1</td>
<td>0.688</td>
<td>8.197</td>
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<td>13.248</td>
<td>1.050</td>
<td>0.798</td>
<td>0.52</td>
<td>1</td>
<td>0.654</td>
<td>8.660</td>
</tr>
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<td>R20A1</td>
<td>20.6</td>
<td>1.5</td>
<td>14.721</td>
<td>0.997</td>
<td>0.797</td>
<td>0.52</td>
<td>1</td>
<td>0.620</td>
<td>9.121</td>
</tr>
<tr>
<td>R22A1</td>
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<td>1.5</td>
<td>15.052</td>
<td>0.986</td>
<td>0.813</td>
<td>0.52</td>
<td>1</td>
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<td>R24A1</td>
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<td>15.487</td>
<td>0.972</td>
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</tr>
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<td>1.630</td>
<td>0.785</td>
<td>0.52</td>
<td>1</td>
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<td>R18B1</td>
<td>18.6</td>
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<td>2.3</td>
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\[ d_i = \text{toe depth}; \quad T_p = \text{peak period}; \quad H_{1/3} = \text{significant wave height at gauge 6 located at the toe of the porous slope}; \quad \xi = \tan \theta / \sqrt{2\pi H_{1/3} / (g T_p^2)} = \text{surf similarity parameter}; \quad g = \text{acceleration due to gravity}; \quad \gamma_h = \text{reduction factor due to a shallow foreshore}; \quad \gamma_f = \text{reduction factor due to slope roughness}; \quad \gamma_b = \text{reduction factor for a berm}; \quad R_{1.7\%} = \text{2% runup level}. \]
Table 8: Comparison with empirical formula for 1/2 slope tests.

<table>
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<tr>
<th>Test</th>
<th>$d_i$ [cm]</th>
<th>$T_p$ [s]</th>
<th>$H_{1/3}$ [cm]</th>
<th>$\xi$</th>
<th>$\gamma_b$</th>
<th>$\gamma_f$</th>
<th>$\gamma_R$</th>
<th>$\frac{R_{2%}}{H_{1/3}}$</th>
<th>$R_{2%}$ [cm]</th>
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<td>4.7</td>
<td>9.898</td>
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<td>0.612</td>
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<td>1</td>
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<td>0.700</td>
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</table>

$d_i$ = toe depth; $T_p$ = peak period; $H_{1/3}$ = significant wave height at gauge 9 located at the toe of the revetment; $\xi = \tan \theta / \sqrt{2 \pi H_{1/3} / (g T_p^2)}$ = surf similarity parameter; $g$ = acceleration due to gravity; $\gamma_b$ = reduction factor due to a shallow foreshore; $\gamma_f$ = reduction factor due to slope roughness; $\gamma_R$ = reduction factor for a berm; $R_{2\%} = 2\%$ runup level.
5.7 WAVE REFLECTION COEFFICIENT

Fig. 5-32 and Fig. 5-33 compare the measured and predicted reflection coefficient $r$ at $x = 0$ for the 1/5 and 1/2 slope experiments, respectively, where $r$ is predicted using Eq. (11). The numerical model overpredicts $r$ for the 1/5 slope and underpredicts $r$ for the 1/2 slope. The error is of the order of 30%. The wave reflection coefficient estimated from the residual wave energy flux at the still water shoreline may be crude but is useful in estimating the order of magnitude of wave reflection using the time-averaged model.
Fig. 5-32: Measured and predicted wave reflection coefficients $r$ for 1/5 slope tests.

Fig. 5-33: Measured and predicted wave reflection coefficients $r$ for 1/2 slope tests.
CHAPTER 6 CONCLUSIONS

The numerical model developed for the prediction of irregular breaking wave transmission over a submerged porous breakwater by Kobayashi et al. (2005) is extended to predict irregular wave shoaling and breaking on a beach with a gentle slope and irregular wave runup on a permeable slope that represents a cobble beach and a stone revetment. The numerical model based on the time-averaged continuity, momentum and energy equations predicts the cross-shore variations of the mean and standard deviation of the free surface elevation and the horizontal velocities above and inside the permeable layer. The mean and standard deviation of the shoreline oscillations on the permeable slope are estimated using the predicted statistics of the free surface elevation as shown in Fig. 2-2. The probability distribution of individual runup heights above SWL is assumed to be given by the Rayleigh distribution where the effect of wave setup is included. The significant runup height is related empirically to the mean and standard deviation of the shoreline oscillations.

The developed model is compared with the 1/5 slope experiment, described in Chapter 3, and the 1/2 slope experiment of Kearney and Kobayashi (2001). The numerical model is
shown to predict the cross-shore variations of the mean and standard deviation of the measured free surface elevation and horizontal velocity fairly accurately when the breaker ratio parameter $\gamma$ is calibrated for some of the 1/2 slope tests. The computed results with and without the permeable layer are compared to examine the degree of the permeability effects which are found to reduce the wave energy dissipation rate due to wave breaking. Computation are also made to examine the sensitivities of the model to the bottom friction factor, the breaker ratio parameter and the roller volume flux. The numerical model predicts the mean and standard deviation of the measured shoreline oscillations reasonably well except that the mean is overpredicted for some of the 1/2 slope tests. Nevertheless, the numerical model predicts the significant and 2% runup heights within the error of about 20%. This accuracy is similar to the accuracy of available empirical formulas based on the known wave conditions at the toe of the permeable slope. The advantage of this numerical model is that it can predict the irregular wave transformation on a beach of arbitrary profile including a nearshore bar (Kobayashi et al. 2005). Furthermore, the numerical model may be applied to gentler permeable slopes for which the effect of wave setup is not negligible. This numerical model is computationally efficient and easy to use because no numerical difficulty has been experienced in the region of very small water depth.
REFERENCES


*J. Waterw., Port, Coastal, Ocean Eng.*, 113(3), 282-298.


