

**HYDRAULIC RESPONSE AND ARMOR LAYER  
STABILITY ON COASTAL STRUCTURES**

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## ABSTRACT

The predictive capabilities for the hydraulic response and armor layer stability on rubble mound structures have improved significantly because of extensive laboratory experiments performed since the publication of the Coastal Engineering Manual (CEM), Part VI: Design of Coastal Project Elements. The preliminary design of rubble mound structures is based on empirical formulas derived from laboratory experiments on wave runup, reflection, overtopping, and transmission as well as armor layer stability. Site-specific hydraulic model testing is performed to integrate different design elements and optimize the entire design. A comprehensive numerical model for the prediction of the hydraulic response and armor layer stability has also been developed to supplement the empirical formulas and hydraulic model testing.

The progress of the coastal structure design since the publication of CEM is summarized in this report. The hydraulic input variables required for the recent empirical formulas are explained first. The estimation of the representative period of incident irregular waves is not straightforward if the toe of a structure is located inside the surf zone on the beach seaward of the structure toe. The need to account for irregular (not regular) wave breaking is pointed out for the prediction of the representative wave height. The recent empirical formulas for wave runup, reflection, overtopping, and transmission are presented in sequence. These formulas developed using a large number of data points are applicable to wider ranges of design situations than those presented in CEM. However, the formulas include more dimensionless parameters and may be harder to interpret. The formulas are explained physically to facilitate their applications.

The recent formulas for armor layer stability on the seaward slope of rubble mound structures with no or little wave overtopping include the effects of shallow water depth and oblique waves. New formulas have also been developed for a slope with a berm, toe armor stability, and armor layer damage progression. For low-crested structures including submerged structures, the effects of the crest height and width on armor stability on the seaward slope, crest, and landward slope have been quantified. A simple formula has also been developed for armor layer stability on the entire structure.

Hydraulic model testing is recommended to combine the different design elements and optimize the entire structure design. The hydraulic response and armor stability can also be examined in integrated manners using the cross-shore numerical model CSHORE which is described concisely at the end of the report. Finally, the improved design methods will need to be utilized to develop innovative structures for storm damage reduction and navigation channel maintenance.

## 1. INTRODUCTION

Coastal storm damage has been increasing mostly due to the recent growth of coastal population and assets. Coastal rubble mound structures such as breakwaters, revetments, and jetties were constructed to reduce coastal storm damage risk as well as to protect ports and navigation channels. Shore Protection Manual (SPM) published in 1977 and 1984 provided technical guidelines for the design of coastal rubble mound structures in the world. Updated design methods for coastal rubble mound structures were published in Part VI: Design of Coastal Project Elements in Coastal Engineering Manual (CEM) but are now obsolete.

### 1.1 Background

A meeting of Jeff Melby, John Winkelman, and Nobu Kobayashi was held in the Ocean Engineering Laboratory of the University of Delaware on December 16, 2014. The goal of this meeting was to outline how to revise CEM Part VI: Design of Coastal Project Elements and write an updated and focused manual for the design of coastal rubble mound structures. The essential items required in this Engineering Manual (EM) were discussed and listed in the 3-page meeting minutes taken by Jeff Melby.

In order to define the scope of the work required to write a cutting-edge manual in the world of coastal engineering, Nobu Kobayashi reviewed the following four manuals and wrote a 9-page summary of the strength and weakness of each of the four manuals:

- **CEM Part VI:** Design of Coastal Project Elements (635 pages) published incrementally during 2001 – 2003 and 2006.
- **The Rock Manual:** The Use of Rock in Hydraulic Engineering (1,254 pages) published jointly by France, Netherlands, and United Kingdom in 2007.
- **EurOtop:** Wave Overtopping of Sea Defenses and Related Structures; Assessment Manual (185 pages) published jointly by United Kingdom, Netherlands, and Germany in 2007.
- **Technical Standards and Commentaries for Port and Harbour Facilities in Japan** (980 pages) published in Japanese in 2007 and translated into English in 2009.

These manuals can be regarded to represent the state of our knowledge in the world up to 2007 or earlier because it takes a few years to write a comprehensive manual.

### 1.2 Outline

The preparatory work above indicated the need to update Chapter 5: Fundamentals of Design in CEM Part VI (316 pages) before the other chapters which rely on the design fundamentals. This report updates 5.2 Structure Hydraulic Response (45 pages) and 5.3 Rubble-Mound Structures Loading and Response (83 pages). The hydraulic response includes wave runup, reflection, overtopping, and transmission. The structure response includes armor layer and toe stability for rubble mound structures with different degrees of wave overtopping and transmission. These sections presented a collection of empirical formulas for wave runup, reflection, overtopping, and transmission as well as armor stability and damage. These empirical formulas are not included in this report for brevity.

The accuracy and applicability of an empirical formula normally improve with additional data. More recent formulas tend to include more dimensionless parameters for

wider applications. However, empirical formulas cannot be extrapolated beyond data ranges. The empirical formulas are useful for the preliminary design of different elements of a prototype structure. The detailed and integrated design of the actual structure will require hydraulic model testing in a laboratory. Numerical models are relatively new as compared to hydraulic model tests and empirical formulas. Numerical models can be used to reduce the number of hydraulic model tests as well as to estimate the quantities which are difficult to measure in hydraulic model testing. The coastal structure design can be optimized by the combined use of empirical formulas, hydraulic model testing, and numerical modeling.

This report is organized as follows. First, site specific conditions (bathymetry, water levels, and wind waves) are discussed concisely to explain the hydraulic input variables required for empirical formulas. Second, recent empirical formulas for wave runup, reflection, overtopping, and transmission are presented separately, although the hydrodynamic processes on and through a rubble mound structure are interdependent. Third, recent empirical formulas for armor layer stability and damage are presented for high-crested and low-crested rubble mound structures where low-crested structures include submerged structures. Fourth, hydraulic model testing and numerical modeling are discussed in order to examine the hydraulic and structure responses simultaneously. Finally, this report is concluded with additional work required to complete an engineering manual for the design of coastal rubble mound structures. The application of the numerical model CSHORE to low-crested breakwaters is presented in Appendix.

This report is written in compartmentalized manners to facilitate future changes. References are given in each section. Equations are numbered in each section.

## **2. SITE SPECIFIC CONDITIONS**

The design of a coastal structure against wind waves requires the data of bathymetry, water levels, and waves in the vicinity of the structure. The numerical prediction and probabilistic analysis of regional storm surge and wind waves have improved significantly for the last decade (e.g., Melby et al. 2011; Smith et al. 2012). The regional models for storm surge and wind waves assume fixed bottom and do not predict bathymetric changes in the vicinity of the structure. If a structure is located in very shallow water, the bathymetric change can be of the same order of magnitude as the water level change during a storm.

### **2.1 Bathymetry**

Digital bathymetry data are now available for most coastal areas in the U.S. The assumption of fixed bottom is appropriate if the bottom elevation change is much smaller than the water depth. This is normally the case in the offshore area with the water depth exceeding about 10 m. In the nearshore area with the water depth less than about 10 m, the bottom elevation change may not be negligible in comparison to the water depth. Storm tide (sum of storm surge and tide) may be assumed to be invariant in the cross-shore direction but the water depth on the evolving bottom influences wave transformation and breaking. Seasonal and long-term bathymetry changes cannot be predicted numerically and need to be measured at a project site. On the other hand, the bathymetry change during a storm can be predicted using numerical models that simulate wave transformation and breaking, wave-induced setup and current, and sediment transport under the combined wave and current action.

In short, the prediction of storm surge and wind waves has become accurate enough to be affected by the error of the fixed bottom approximation.

## 2.2 Water Levels

The surface of the sea varies with time due to tides and has been measured using tide gauges. The periodic rising and falling of the tidal water level due to gravitational attraction of the moon and sun can be predicted using tidal gauge records. The mean sea level at a specific location is the average sea surface elevation for all tidal stages. Storm surge is a rise above normal tidal water level due to wind stress acting on the water surface and atmospheric pressure reduction. Wind also generates waves on the sea surface whose elevation is determined by tides and storm surge. Tide gauges are normally located in sheltered areas and do not include the wave-induced water level increase (wave setup) inside surf zones on beaches. Wave setup may be in the range of 10 – 20% of the offshore significant wave height (e.g., Kobayashi et al. 2003). Wave setup on a natural beach depends on the bathymetry of the entire surf zone (e.g., Raubenheimer et al. 2001).

Relative mean sea level at a specific location results from the combined changes in water and land levels. Lowering in the land level appears as a sea level rise in the tide gauge record. Relative sea level rise is already significant at locations of extreme land subsidence. Long-term tide gauge records are used to estimate the past rate of relative sea level rise. The future rate will be larger than the past rate if the eustatic (global) sea level rise accelerates. Future sea level rise is accounted for in the life-cycle simulation for U.S Federal projects (e.g., Melby et al. 2011).

## 2.3 Wind Waves

Water waves in the ocean include various waves of different time and length scales. Wind-generated waves with periods of the order of 10 s are important in predicting the hydraulic response and damage on a rubble mound structure. Directional spectral wave models such as SWAN (Booij et al. 1999) and STWAVE (Smith et al. 2001) predict the wave energy as a function of frequency and direction. For the design of a coastal structure, the incident directional wave spectrum is normally represented by the spectral significant wave height  $H_{mo}$ , spectral peak period  $T_p$ , and peak spectral wave direction  $\theta_p$ . Spectral parameters such as spectral width and directional spreading as well as a group of individual wave heights in the time domain have also been examined but are not included in present empirical design methods.

The empirical formulas in the subsequent sections require the representative wave height, period, and direction at the toe of the structure as well as the still water depth below storm tide at the structure toe. Nowadays, the representative wave height and direction are the spectral significant wave height  $H_{mo}$  and predominant spectral wave direction  $\theta_p$ . The representative wave period is the spectral peak period  $T_p$  in the U.S. and the spectral wave period  $T_{m-1,0} = m_{-1} / m_0$  in Europe where the spectral moment  $m_n$  with  $n = -1$  and 0 is defined as

$$m_n = \int_0^{\infty} f^n S(f) df \quad (2.1)$$



where  $f$  = frequency with  $f^{-1}$  = wave period; and  $S(f)$  = frequency spectrum. The spectral wave period is a better representative period if the frequency spectrum does not have a dominant spectral peak (van Gent 2001). This is the case with double-peaked spectra and broken wave spectra in the inner surf zone.

For standard single-peaked spectra  $T_{m-1,0}$  and  $T_p$  are uniquely related. The ratio  $T_p/T_{m-1,0}$  is 1.17 and 1.11 for Pierson-Moskowitz and JONSWAP spectra, respectively. The European overtopping manual (Pullen et al. 2007) suggested the use of  $T_p = 1.1 T_{m-1,0}$  if the value of  $T_{m-1,0}$  is unavailable. Kobayashi et al. (2013) analyzed the cross-shore variations of  $T_p$  and  $T_{m-1,0}$  using the data of van Gent (2001). The ratio between  $T_p$  and  $T_{m-1,0}$  outside the surf zone was in the range of 0.98 – 1.41 for field data on a barred beach and 0.70 – 1.45 for laboratory data on uniform slopes. The suggested value of 1.1 was reasonable outside the surf zone in view of the uncertainty of the wave periods in field conditions. The wave periods inside the surf zone are affected by the decay of wind waves due to wave breaking and the generation of infragravity waves with periods in the range of 20 to 200 s (e.g., Guza and Thornton 1985). The wave periods can increase considerably on a gently-sloping beach inside the surf zone seaward of the toe of a structure. The peak period  $T_p$  varied more inside the surf zone than the spectral period  $T_{m-1,0}$ . The assumption of constant wave period, which is normally made for regular waves, is more applicable to  $T_{m-1,0}$  than  $T_p$ . Directional spectral wave models such as SWAN and STWAVE do not include infragravity waves and may not predict  $T_p$  and  $T_{m-1,0}$  inside the surf zone accurately.

In the U.S., the values of  $H_{mo}$ ,  $T_p$  and  $\theta_p$  at an offshore location outside the surf zone are reported. If the toe of the structure is located outside the surf zone, a directional spectral wave model can be used to compute the values of  $H_{mo}$ ,  $T_p$ ,  $T_{m-1,0}$ , and  $\theta_p$  at the structure toe. To predict diffracted waves in the sheltered zone behind a breakwater, the directional spreading about the predominant wave direction  $\theta_p$  must be included in the wave diffraction computation (Goda 2000). If the toe of the structure is located inside the surf zone, the value of  $T_{m-1,0}$  estimated using  $T_p = 1.1 T_{m-1,0}$  outside the surf zone may crudely be assumed to remain constant inside the surf zone. The values of  $H_{mo}$  and  $\theta_p$  at the structure toe may be computed using an appropriate wave transformation model that includes wave refraction and shoaling with irregular wave breaking. The maximum wave height  $H_{max}$  in the irregular wave train with the significant wave height  $H_{mo}$  is slightly less than  $2H_{mo}$  (Goda 2000). The maximum wave may break in the water depth of about  $H_{max}$ . As a result, irregular waves should be regarded to be breaking if the water depth at the structure toe is less than about  $2H_{mo}$  where  $H_{mo}$  is the computed value at the toe.

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### 3. WAVE RUNUP

Wave runup on coastal structures and beaches determines the landward extent of wave action and needs to be predicted for coastal flood-risk mapping such as that produced by the U.S. Federal Emergency Management Agency (Crowell et al. 2010). The crest height of a coastal structure required for little wave overtopping is estimated using empirical formulas for wave runup. The definition of wave runup depends on methods used to measure and analyze the moving shoreline on a structure or beach. The prototype water depth on a slope used to define wave runup may be less than 1 – 2 cm (Pullen et al. 2007) or up to 10 cm (van Gent 2001). Wave runup on relatively steep coastal structures may not be affected much by the 10-cm water depth difference.

#### 3.1 Impermeable Slopes

Most laboratory experiments on wave runup were conducted for smooth impermeable slopes partly because earthen dikes (levees) were designed for no or little wave overtopping during a severe storm. It is also easier to measure the shoreline movement on smooth impermeable slopes. For an inclined impermeable coastal structure that allows no or little wave overtopping during a severe storm, it is customary but somewhat arbitrary to use the runup height  $R_{2\%}$  above the still water level (SWL) which is exceeded by 2% of incident irregular waves at the toe of the structure. If the structure crest height above SWL equals  $R_{2\%}$ , only 2% of incident individual waves at the toe will overtop the crest. A number of empirical formulas were proposed to estimate  $R_{2\%}$  and promising formulas were improved using additional data sets. The recent formulas include a few dimensionless constants that can be

calibrated for different applications. No formula is universal and an appropriate formula will need to be selected for a specific application.

The formula for  $R_{2\%}$  on inclined impermeable slopes in the EurOtop manual (Pullen et al. 2007) is based on a large number of international data sets. This formula is rewritten as

$$\frac{R_{2\%}}{\gamma H_{mo}} = \min \left[ 1.65 \xi_{m-1,0}, \left( 4.0 - \frac{1.5}{\sqrt{\xi_{m-1,0}}} \right) \right] \quad (3.1)$$

where min indicates the smaller value of the two values in the square brackets. The runup height  $R_{2\%}$  is of the order of the spectral significant wave height  $H_{mo}$  at the toe of the slope.

The surf similarity parameter  $\xi_{m-1,0}$  is the normalized slope defined as

$$\xi_{m-1,0} = \frac{\tan \alpha}{\sqrt{H_{mo} / L_o}} ; L_o = \frac{g T_{m-1,0}^2}{2\pi} \quad (3.2)$$

where  $\alpha$  = angle of the seaward slope from the horizontal;  $L_o$  = deepwater wavelength based on the spectral wave period  $T_{m-1,0}$  at the toe of the slope. The reduction factor  $\gamma$  in Eq. (3.1), which is unity for wave runup on a smooth uniform slope under normally incident waves, is separated into

$$\gamma = (\gamma_f \gamma_b \gamma_\beta) \leq 1 \quad (3.3)$$

where  $\gamma_f$ ,  $\gamma_b$ , and  $\gamma_\beta$  are the reduction factors associated with slope roughness, a berm near SWL, and oblique waves, respectively.

Eq. (3.1) is a modified version of the formula of van Gent (2001) who proposed the use of  $T_{m-1,0}$  in the surf similarity parameter in Eq. (3.2) for the empirical prediction of wave runup and overtopping. Eq. (3.1) does not include the effect of water depth at the toe of the slope probably because the use of  $H_{mo}$  and  $T_{m-1,0}$  at the toe implicitly accounts for the spectral wave transformation including irregular wave breaking on the beach seaward of the toe of the slope. Eq. (3.1) predicts the monotonic increase of  $R_{2\%} / (\gamma H_{mo}) = 0.8$  to 3.3 with the increase of  $\xi_{m-1,0} = 0.5$  to 5.0 which may be regarded as a typical range for inclined impermeable structures. For  $\gamma = 1$ ,  $R_{2\%}$  is of the order of  $2H_{mo}$ .

Seaward slopes of earthen dikes are normally covered by grass, asphalt, or fitted concrete blocks. The roughness reduction factor is taken as  $\gamma_f = 1.0$  for these smooth slopes. The slope may be regarded to be smooth if its surface protrusion height is small in comparison to the water depth of 1 – 10 cm used to define wave runup. Discrete roughness elements (concrete blocks or ribs) placed near the dike crest may reduce  $\gamma_f$  to the range of 0.75 – 0.85 (Pullen et al. 2007) but the data were limited to urban dikes or embankments in Europe. In the U.S., the beach in front of the dike might be nourished to reduce  $H_{mo}$  at the dike toe and  $R_{2\%}$  on the dike slope.

Eq. (3.2) is applicable to a uniform slope landward of the toe of the slope located below SWL. A seaward dike slope may include a berm with different slopes seaward and

landward of the berm. Formulas developed for uniform slopes are assumed to be applicable to equivalent uniform slopes that represent actual composite slopes. Several intuitive methods have been proposed to estimate the equivalent uniform slope  $\tan \alpha$  for different composite slopes (e.g., Kobayashi and Jacobs 1985; van Gent 2001; Pullen et al. 2007; Mase et al. 2013). The equivalent uniform slope needs to be devised so that the berm reduction factor in  $\gamma_b = 1$  in Eq. (3.3). The EurOtop manual presents complicated methods to estimate  $\tan \alpha$  and  $\gamma_b$  for different composite slopes. No single method is expected to work for different composite slopes because wave dynamics depends on the actual bottom geometry. If the bottom geometry deviates from a uniform slope noticeable, hydraulic model testing or numerical modeling should be performed using the actual bottom whose roughness and permeability may vary spatially.

The reduction factor  $\gamma_\beta$  for oblique directional (short-crested) waves may be estimated as

$$\begin{aligned} \gamma_\beta &= 1.0 && \text{for } |\beta| \leq 20^\circ \\ \gamma_\beta &= 1 - 0.0022|\beta| && \text{for } 20^\circ < |\beta| \leq 80^\circ \\ \gamma_\beta &= 0.82 && \text{for } |\beta| > 80^\circ \end{aligned} \quad (3.4)$$

where  $\beta$  is the incident wave angle  $\theta_p$  in degrees at the toe of the slope. For normally incident waves,  $\beta = 0^\circ$ . Eq. (3.4) is based on the EurOtop manual but allows the sudden jump of  $\gamma_b = 1.0$  to 0.96 at  $|\beta| = 20^\circ$  because of the scatter of about 10% of data points used to obtain Eq. (3.4). The range of  $\gamma_\beta = 0.82 - 1.0$  for directional random waves turns out to be relatively small perhaps because directional waves include waves of nearly normal incidence.

The accuracy of Eq. (3.1) is discussed in the EurOtop manual. The scatter of data points about the fitted equation (3.1) is assumed to be expressed by the normal distribution with the mean value corresponding to the value calculated using Eq. (3.1). The standard deviation of the scattered data points for smooth uniform slopes ( $\gamma = 1$ ) is 7% of the mean value. This implies that the error of Eq. (3.1) is within about 20%. The 20% error for wave runup prediction is probably close to the limit of any empirical formula in view of the very small water depth associated with the upper limit of uprushing water on smooth uniform slopes. This accuracy limit is also applicable to the cross-shore numerical model CSHORE (Kobayashi et al. 2013) which uses the offshore wave conditions as input.

Eq. (3.1) predicts only the runup height  $R_{2\%}$  of 2% exceedance probability. For the case of no or little wave overtopping, the exceedance probability  $P$  of the individual runup height  $R$  above SWL is normally given by the Rayleigh distribution

$$P(R) = \exp \left[ -2 \left( \frac{R}{R_{1/3}} \right)^2 \right] \quad ; \quad R_{2\%} = 1.4R_{1/3} \quad (3.5)$$

where  $R_{1/3}$  = significant runup height defines as the average of 1/3 highest values of  $R$ . The relation between  $R_{2\%}$  and  $R_{1/3}$  can be obtained using Eq. (3.5) with  $P = 0.02$  for  $R = R_{2\%}$ . Kobayashi et al. (2008) included wave setup at the toe of the slope explicitly in Eq. (3.5) because the numerical model CSHORE predicts the cross-shore variation of wave setup. For most laboratory experiments, the prototype and model beaches may not be in geometric

similitude, resulting in dissimilitude of wave setup. If the toe of the slope is located well inside the surf zone, Eq. (3.5) may need to be modified to include wave setup explicitly. For the case of significant wave overtopping, Eq. (3.5) overpredicts the value of  $R$  for small  $P$  because wave runup is reduced by wave overtopping (Kobayashi and de los Santos 2007).

Wave runup on natural beaches has been investigated mostly by coastal scientists using video techniques (e.g., Stockdon et al. 2006). The resolution of the digitized shoreline was typically 5 – 15 cm in the vertical. The time series of the shoreline elevation for a duration (e.g., 17 minutes) of approximately constant tide level was used to obtain the mean (wave setup) and standard deviation  $\sigma$ . A spectral analysis was used to obtain  $\sigma$  and estimate the components in  $\sigma$  of wind waves (period  $T < 20$  s) and infragravity waves ( $T > 20$  s). The 2% runup height  $R_{2\%}$  was estimated as the sum of the mean and  $2\sigma$  which is the 2% exceedance limit of the Gaussian (normal distribution) shoreline oscillation. This definition of  $R_{2\%}$  is different from that used for individual runup heights in Eqs. (3.1) and (3.5). Understandably, Eq. (3.1) with  $\gamma = 1$  does not predict  $R_{2\%}$  on natural beaches. Melby et al. (2012) evaluated and calibrated existing empirical formulas using the data sets of Stockdon et al. (2006). In short, wave runup on natural beaches is discussed briefly to point out different swash hydrodynamics on coastal structures with steep slopes and natural beaches with gentle slopes. If the toe of a coastal structure is located in very shallow water, wave runup on the structure is affected by surf dynamics on the beach seaward of the toe.

### 3.2 Permeable Slopes

Wave runup on the seaward slope of a rubble mound structure influences the degree of wave overtopping and transmission. van der Meer and Stam (1992) proposed empirical formulas for wave runup heights for different exceedance probabilities using about 250 tests conducted in a wave flume. Two layers of rock with a uniform slope in the range of  $\cot \alpha = 1.5 - 4$  were placed on an impermeable or permeable core in relatively deep water for almost all tests. The effect of toe depth on wave runup was not clear in limited shallow-water tests. Pullen et al. (2007) fitted Eq. (3.1) to the measured values of  $R_{2\%}$  in these tests with  $\gamma_b = 1$ ,  $\gamma_\beta = 1$  and  $\gamma = \gamma_f$  in Eq. (3.3). It was necessary to vary  $\gamma_b$  as a function of  $\xi_{m-1,0}$  and Eq. (3.1) was modified as

$$\frac{R_{2\%}}{H_{mo}} = \min \left[ 1.65 \gamma_f \xi_{m-1,0}, \gamma_{f \text{ surge}} \left( 4.0 - \frac{1.5}{\sqrt{\xi_{m-1,0}}} \right) \right] \quad (3.6)$$

The calibrated values of the reduction factor  $\gamma_f$  for breaking waves are

$$\begin{aligned} \gamma_f &= 0.55 && \text{for two layers of rock on impermeable core} \\ \gamma_f &= 0.40 && \text{for two layers of rock on permeable core} \end{aligned} \quad (3.7)$$

The reduction factor  $\gamma_{f \text{ surge}}$  for surging waves was expressed as

$$\begin{aligned} \gamma_{f \text{ surge}} &= \gamma_f && \text{for } \xi_{m-1,0} \leq 1.8 \\ \gamma_{f \text{ surge}} &= \gamma_f + (\xi_{m-1,0} - 1.8)(1 - \gamma_f) / 8.2 && \text{for } 1.8 < \xi_{m-1,0} < 10 \\ \gamma_{f \text{ surge}} &= 1.0 && \text{for } \xi_{m-1,0} \geq 10 \end{aligned} \quad (3.8)$$

where the upper limit of  $(R_{2\%} / H_{mo}) = 1.97$  was imposed for the permeable core. Eq. (3.8) implies that the reduction factor had to be increased from  $\gamma_f = 0.55$  or  $0.40$  to  $1.0$  with the increase of  $\xi_{m-1,0} = 1.8$  to  $10$ . The measured values of  $(R_{2\%} / H_{mo})$  for these tests with  $\xi_{m-1,0} = 1 - 8$  were about  $0.9 - 3$  for the impermeable core and about  $0.8 - 2.4$  for the permeable core.

Available wave runup data for permeable slopes are limited because wave overtopping and transmission are regarded to be more important for the design of rubble mound structures. Nevertheless, Eq. (3.6) is useful in estimating the crest height required for little wave overtopping.

### 3.3 References

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## 4. WAVE REFLECTION

The mechanism of wave reflection from a coastal structure is not clearly understood but may be related to incident wave uprush and downrush on the seaward slope of the structure. The total reflection coefficient  $K_r$  for irregular waves is normally defined as

$$K_r = \left( \frac{E_r}{E_i} \right)^{0.5} = \frac{(H_{mo})_r}{H_{mo}} \quad (4.1)$$

where  $E_r$  = total (all frequencies and directions) reflected wave energy;  $E_i$  = total incident wave energy;  $(H_{mo})_r$  = spectral significant wave height of the reflected wave frequency spectrum; and  $H_{mo}$  = spectral significant wave height of the incident wave frequency spectrum without the subscript  $i$  for brevity. The reflection coefficient can also be defined as a function of frequency using the reflected and incident wave frequency spectra. The reflection coefficient generally increased with the decrease of the frequency (e.g., Wurjanto and Kobayashi 1993). The value of  $K_r$  is normally estimated at the toe of the structure using linear wave theory. If the structure toe is located inside the surf zone, the estimated value of  $K_r$  may not be very accurate. The value of  $K_r$  increases landward inside the surf zone because of the landward decrease of the incident wave energy caused by wave breaking (e.g., Baquerizo et al. 1997). As a result, the reflection coefficients are normally measured outside the surf zone on horizontal bottoms where  $E_i$  and  $H_{mo}$  in Eq. (4.1) vary little spatially. Existing empirical formulas for  $K_r$  assume normally incident waves or do not include the influence of the incident wave angle.

Goda (2000) listed the following typical ranges of reflection coefficients using available data:

$$\begin{aligned} K_r &= 0.7 - 1.0 \text{ for emerged vertical wall with varying wave overtopping} \\ K_r &= 0.5 - 0.7 \text{ for submerged vertical wall with varying wave transmission} \\ K_r &= 0.3 - 0.6 \text{ for rock slopes with } \cot \alpha = 2 - 3 \\ K_r &= 0.3 - 0.5 \text{ for slopes of concrete armor units} \\ K_r &= 0.05 - 0.2 \text{ for natural beaches} \end{aligned} \quad (4.2)$$

The reflection coefficient for a conventional rubble mount structure with no or little wave overtopping is expected to be less than about 0.6.

#### 4.1 Impermeable Structures

Seelig and Ahrens (1995) developed empirical formulas for normally incident wave reflection from non-overtopping structures using available laboratory data. The surf similarity parameter  $\xi_p$  based on the spectral peak period  $T_p$  in Eq. (3.2) instead of  $T_{m-1,0}$  was used to separate breaking and non-breaking waves on the structures. Formulas for smooth impermeable slopes are omitted because of the limited data. For non-breaking waves ( $\xi_p > 4$ ) on smooth slopes, the values of  $K_r$  are about 0.8. For breaking waves ( $\xi_p < 2.5$ ) on smooth slopes,  $K_r$  increases about 0.2 at  $\xi_p = 1.5$  to about 0.5 at  $\xi_p = 2.5$ .

For rough impermeable slopes, Seelig and Ahrens (1995) expressed  $K_r$  for non-breaking waves as

$$K_r = \left[ 1 + \lambda^{1.57} \exp(e) \right]^{-1} ; \quad \lambda = d_i \cot \alpha / L_p \text{ for } \xi_p > 4 \quad (4.3)$$

with

$$e = 2.29 \left[ (\cot \alpha)^{0.3} (D_{n50} / L_p) (1 + H_{mo} / d_t)^{1.5} + P^{0.4} / (\cot \alpha)^{0.7} \right]$$

where  $d_t$  = water depth at the toe of the structure;  $\alpha$  = slope angle from the horizontal;  $L_p$  = linear wave length based on  $d_t$  and  $T_p$ ;  $D_{n50} = (M_{50} / \rho_r)^{1/3}$  = nominal rock diameter based on the median rock mass  $M_{50}$  and density  $\rho_r$ ;  $H_{mo}$  = spectral significant wave height at the structure toe;  $P$  = notional permeability factor with  $P = 0.1$  for rough (two layers of rock) impermeable slopes (van der Meer and Stam 1992). For breaking waves,  $K_r$  is given by

$$K_r = 1 - \exp \left[ \left( -0.06 \xi_p^{2.4} - 0.5 \frac{H_{mo}}{d_t} \right) f \right] \quad \text{for } \xi_p < 2.5 \quad (4.4)$$

with

$$f = \left[ 0.16 + (0.5P - 0.45) \ln \left( \frac{D_{n50}}{H_{mo}} \right) \right] \leq 1 \quad \text{for } \left( \frac{D_{n50}}{H_{mo}} \right) \leq 1$$

where  $f$  is the correction factor introduced to the formula for smooth slopes for which  $f = 1$ . Eq. (4.4) is limited to the condition of  $D_{n50} \leq H_{mo}$ . For transitional waves ( $2.5 < \xi_p < 4$ ). The values of  $K_r$  based on Eqs. (4.3) and (4.4) multiplied by  $(\xi_p - 2.5)/1.5$  and  $(4 - \xi_p)/1.5$ , respectively, are added to obtain an interpolated value.

Seelig and Ahrens (1995) compared their formulas with impermeable slope data ( $P = 0.1$ ) and permeable slope data ( $P = 0.4 - 0.6$  depending on filter and core rock sizes relative to armor rock size  $D_{n50}$ ). The measured and predicted values of  $K_r$  for irregular waves are in the range of about 0.1 – 0.7. The difference between the measured and predicted values is less than about 0.1.

## 4.2 Permeable Structures

Davidson et al. (1996a) measured wave reflection coefficients seaward of a low-crested rubble mound breakwater in England before and after a reduction of the seaward slope of the structure. The measurements were performed during the time of no or little wave overtopping. This slope reduction ( $\cot \alpha = 0.82$  to 1.55) decreased maximum reflection coefficients for very large values of  $\xi_p$  from about 0.6 to about 0.5. An empirical formula was proposed to explain the field data.

Davidson et al. (1996b) combined their field data with some of the laboratory data used by Seelig and Ahrens (1995) and performed a multiple regression analysis of 780 data points. Their formula is expressed as

$$K_r = 0.298 \log(\xi_p) - 0.011 \frac{D_{n50}}{H_{mo}} - 0.321 \log \left( \sqrt{\frac{D_{n50}}{L_o}} \cot \alpha \right) - 0.191P - 0.358 \left( \frac{d_t}{L_o} \right) - 0.049 \quad (4.5)$$



where  $L_o$  = deepwater wavelength based on  $T_p$ . The permeability factor was  $P = 0.6$  for the field data and  $P = 0.04 - 0.6$  for the laboratory data. The difference between the measured and predicted values of  $K_r$  is mostly within 0.1.

The formulas of Davidson et al. (1996b) and Seelig and Ahrens (1995) may look different but express  $K_r$  as a function of the dimensionless parameters based on  $H_{mo}$ ,  $T_p$ ,  $\cot \alpha$ ,  $D_{n50}$ ,  $d_t$  and  $P$ . Both formulas predict  $K_r$  in the range of about 0.1 – 0.7 within the error of about 0.1. It is desirable to compare the two formulas using all the data sets employed by Davidson et al. (1996b) and Seelig and Ahrens (1995). The difference between the two predicted values might turn out to be about 0.1.

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## 5. WAVE OVERTOPPING

The crest height of a coastal structure is nowadays designed using allowable wave overtopping criteria instead of the wave runup height of an allowable (e.g., 2%) exceedance probability. The degree of wave overtopping is related to damage on the structure crest and landward area but no reliable method exists to predict damage initiation and progression during an entire storm. The intermittent wave overtopping events during the peak of a storm are simply represented by the average wave overtopping rate  $q$  per unit alongshore length of the structure crest. The rate  $q$  can be measured reliably in a laboratory experiment by collecting the overtopped water volume for a specified duration. However, it is difficult to correlate  $q$  with damage partly because it is difficult to quantify resistance of the area (e.g., bare, grassed or paved soil) affected by wave overtopping. As a result, available guidelines for allowable wave overtopping rates are crude. For example, the EurOtop manual (Pullen et al. 2007) recommends

$q = 10^{-3} m^3 / s / m$  : Grass and/or clay may start to erode on crest and landward slopes

$q = 10^{-2} m^3 / s / m$  : Significant overtopping for dikes and embankments.

Some overtopping for rubble mound breakwaters.

$q = 10^{-1} m^3 / s / m$  : Dike crest and landward slopes need to be protected by asphalt or concrete.

Transmitted waves may be generated landward of rubble mound breakwaters.

This order-of-magnitude guideline may be too crude but  $q$  is sensitive to the crest elevation which needs to be designed.

### 5.1 Impermeable Structures

The EurOtop manual (Pullen et al. 2007) presented separate formulas to predict the average overtopping rate  $q$  for simple slopes and vertical seawalls. Goda (2009) proposed unified formulas for both simple slopes and vertical walls with smooth impermeable surfaces. The unified formulas for normally incident waves were developed using 715 data points from 11 data sets for vertical walls and 1,452 data points from 27 data sets for uniform slopes in the range of  $\cot \alpha = 1 - 7$  with  $\alpha =$  slope angle from the horizontal. The effects of the toe depth and seabed slope were included in the formulas. The unified formulas were shown to perform better than the separate formulas in the EurOtop manual for the selected data sets. As a result, the formulas of Goda (2009) are presented in the following.

The average wave overtopping rate  $q$  is expressed as

$$\frac{q}{\sqrt{gH_{mo}^3}} = q_* = \exp \left[ - \left( A + B \frac{R_c}{H_{mo}} \right) \right] \quad \text{for } R_c \geq 0 \quad (5.1)$$

where  $g =$  gravitational acceleration;  $H_{mo} =$  spectral significant wave height at the toe of the structure;  $q_* =$  normalized rate;  $R_c =$  crest height above the still water level (SWL);  $A$  and  $B =$  dimensionless parameters related to the slope angle  $\alpha$ , toe depth  $d_t$ , and seabed slope angle  $\theta$  from the horizontal. The formulas are limited to the constant slopes  $\tan \theta$  and  $\tan \alpha$ , positive or zero toe depth  $d_t$ , and positive or zero crest height  $R_c$ . It is noted that Goda (2009) disregarded the difference between  $H_{mo}$  and  $H_{1/3} =$  significant wave height based on a zero-upcrossing method at the toe of the structure. The parameters  $A$  and  $B$  are given by

$$A = A_0 \tanh \left\{ (0.956 + 4.44 \tan \theta) \left[ \frac{d_t}{H_{mo}} + 1.242 - 2.032 (\tan \theta)^{0.25} \right] \right\} \quad (5.2)$$

$$B = B_0 \tanh \left[ (0.822 - 2.22 \tan \theta) \left( \frac{d_t}{H_{mo}} + 0.578 + 2.22 \tan \theta \right) \right] \quad (5.3)$$

for  $\cot \theta = 10 - 1000$  and  $(d_t / H_{mo}) = 0 - 23$ .  $A_0$  and  $B_0$  are expressed as

$$A_0 = 3.4 - 0.734(\cot \alpha) + 0.239(\cot \alpha)^2 - 0.0162(\cot \alpha)^3 \quad (5.4)$$

$$B_0 = 2.3 - 0.5(\cot \alpha) + 0.15(\cot \alpha)^2 - 0.011(\cot \alpha)^3 \quad (5.5)$$

for vertical walls with  $\cot \alpha = 0$  and uniform slopes with  $\cot \alpha = 1 - 7$ .

Eqs. (5.1) - (5.5) do not include the spectral wave period or peak period at the toe of the structure. The wave period affects the irregular wave transformation on the seabed slope and  $H_{mo}$  at the structure toe. This is convenient because it is difficult to predict the cross-shore change of the wave period inside the surf zone. The wave height  $H_{mo}$  at the structure toe with  $d_t = 0$  is positive because of wave setup inside the surf zone. The presence of the seabed slope  $\tan \theta$  in Eqs. (5.2) and (5.3) implies that the effect of the wave transformation on the beach cannot be taken into account completely by the use of the wave conditions at the toe in the formulas.

Goda (2009) plotted the ratios  $R_q$  between the measured and estimated values of the normalized rate  $q_*$  as a function of the estimated  $q_* < 1.0$  for all the data points for vertical walls and uniform slopes. The deviation from  $R_q = 1$  (perfect agreement) increased with the decrease of  $q_*$  from unity. Only the order of magnitude of  $q_*$  can be predicted for  $q_* < 10^{-3}$ . The approximate lower and upper limits of  $R_q$  were expressed as

$$\begin{aligned} q_*^{1/3} < R_q < q_*^{-1/4} & \text{ for vertical walls} \\ q_*^{2/5} < R_q < q_*^{-1/3} & \text{ for uniform slopes} \end{aligned} \quad (5.6)$$

The initiation of wave overtopping and the small overtopping rates are difficult to predict numerically as well because of the very small water depth in the upper limit of wave uprush (Kobayashi et al. 2013).

To include the effect of oblique waves, Goda (2009) replaced  $R_c$  in Eq. (5.1) by  $R_c / \gamma_\beta^*$  with  $\gamma_\beta^* < 1$  where the reduction factor  $\gamma_\beta^*$  increases the crest height in Eq. (5.1). Using 341 data points from five data sets,  $\gamma_\beta^*$  was expressed as a function of the incident wave angle  $\beta$  in degrees.

$$\gamma_\beta^* = 1 - 0.0096|\beta| + 0.000054\beta^2 \quad \text{for } |\beta| < 80^\circ \quad (5.7)$$

which predicts  $\gamma_\beta^* = 0.58$  for  $\beta = 80^\circ$ .

Eq. (5.1) is limited to the positive or zero crest height  $R_c$  above SWL. For the case of  $R_c < 0$  only on the seaward side of a coastal structure (not a submerged structure), steady overflow combined with wave overtopping occurs on the structure crest. Hughes and Nadal (2009) conducted 27 tests in a wave flume and measured the average flow rate of combined

wave overtopping and surge overflow of a levee of a seaward slope of  $\cot \alpha = 4.25$ . The measured rate  $q$  was expressed as

$$\frac{q}{\sqrt{gH_{mo}^3}} = 0.034 + 0.53 \left( \frac{-R_c}{H_{mo}} \right)^{1.58} \quad \text{for } R_c < 0 \text{ and } \cot \alpha = 4.25 \quad (5.8)$$

The second term on the right hand side of Eq. (5.8) related to steady flow over a broad-crested weir becomes larger than 0.034 for  $(-R_c / H_{mo}) > 0.2$ . Eq. (5.8) might become applicable to other uniform slopes if 0.034 is replaced by  $\exp(-A)$  with  $A$  given by Eq. (5.2) in view of Eq. (5.1) with  $R_c = 0$ . The parameter  $A$  may be in the range of 2–4 and  $\exp(-A) = 0.018 - 0.135$  for  $A = 2 - 4$ . A similar crude approximation is described in the EurOtop manual (Pullen et al. 2007).

van der Meer and Bruce (2014) provided new physical insights into the formulas of wave overtopping rates for sloping and vertical structures in the EurOtop formula (Pullen et al. 2007). Recent data for very steep slopes were added to improve the formulas for the continuous range of  $\cot \alpha$  where Eqs. (5.4) and (5.5) were developed using the data of  $\cot \alpha = 0$  and  $\cot \alpha = 1-7$  with a gap between  $\cot \alpha = 0$  and 1. Unfortunately, they did not compare Eqs. (5.1) - (5.5) with the very steep slope data. Fine-tuning of existing empirical formulas will continue as more data become available but the formulas become more complicated and harder to interpret.

## 5.2 Permeable Structures

Wave overtopping and seepage cause water flux on and through a rubble mound structure (e.g., Kobayashi and de los Santos 2007). To measure only the wave overtopping rate  $q$ , a rubble mound with a horizontal crest is placed in front of a vertical wall where the crest elevations of the mound and wall are set to be level. Available data of  $q$  for permeable structures are less than those for smooth impermeable slopes and walls.

The EurOtop manual (Pullen et al. 2007) expressed  $q$  for uniform slopes with  $\cot \alpha = 1.5$  in the form

$$\frac{q}{\sqrt{gH_{mo}^3}} = 0.2 \exp \left( -2.6 \frac{R_c}{\gamma_f \gamma_\beta H_{mo}} \right) \quad \text{for } \cot \alpha = 1.5 \quad (5.9)$$

where  $\gamma_f$  = reduction factor for roughness and permeability; and  $\gamma_\beta$  = reduction factor for oblique waves on permeable slopes. Eq. (5.9) is limited to the typical crest width of  $3D_{n50}$  and relatively deep water at the toe of the slope. Eq. (5.1) reduces to Eq. (5.9) if  $A = 1.6$  and  $B = 2.6 / (\gamma_f \gamma_\beta)$  where  $A$  and  $B$  depend on the structure slope  $\tan \alpha$ , seabed slope  $\tan \theta$ , and relative depth  $d_t / H_{mo}$  in Eqs. (5.2) and (5.3).

The number of data points used to estimate  $\gamma_f$  in Eq. (5.9) was small and the data scatter was large. For two layers of rock on the slope of  $\cot \alpha = 1.5$ ,  $\gamma_f = 0.55$  and 0.40 for

impermeable and permeable cores, respectively, which are the same as those recommended for  $R_{2\%}$  in Eq. (3.6). The reduction factor  $\gamma_\beta$  in Eq. (5.9) is given by

$$\gamma_\beta = 1 - 0.0063|\beta| \quad \text{for } |\beta| < 80^\circ \quad (5.10)$$

which predicts  $\gamma_\beta = 0.50$  for  $|\beta| = 80^\circ$ . The difference between Eqs. (5.7) and (5.10) may be within the scatter of data points for the impermeable and permeable structures.

Eq. (5.9) is limited to uniform slopes with a crest width of  $3D_{n50}$ . The EurOtop manual discusses the effects of a wide armored crest and a wave wall at the landward end of the armor layer as well as composite slopes and berms, including berm breakwaters. However, it is not possible to devise simple correction factors in Eq. (5.9) for complicated permeable structures because the number of design variables becomes too large for any empirical formula. Hydraulic model testing will be required for such structures. The EurOtop manual also describes the scale and model effect on the wave overtopping rate  $q$ . The scale effect appears to be negligible for  $q > 10^{-3} \text{ m}^3 / \text{s} / \text{m}$  (1 liter/s/m) which may cause erosion of grassed clay on the crest and landward slope of a dike.

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## 6. WAVE TRANSMISSION

Wave transmission over and through rubble mound structures has been investigated in relation to shore-parallel breakwaters constructed for protection of beaches and marshes. These detached or offshore breakwaters need to satisfy the shore protection requirement based on simple criteria such as allowable wave overtopping and transmission. The crest of the breakwater is normally low and narrow to reduce the construction cost. Submerged breakwaters have become popular for their aesthetics and effectiveness in triggering wave breaking without eliminating the landward flow of water. The crest of a submerged breakwater needs to be wide enough to reduce wave transmission sufficiently.

Laboratory experiments on low-crested breakwaters including oblique wave tests in a wave basin were performed for the European Project DELOS (Kramer et al. 2005). The

oblique wave tests were conducted to measure only waves transmitted over and through low-crested breakwaters without diffracted waves. van der Meer et al. (2005) developed empirical formulas for the wave transmission coefficient  $K_t$  for normally incident waves using 2,337 data points from 7 data sets. The coefficient  $K_t$  is defined as

$$K_t = (H_{mo})_t / H_{mo} \quad (6.1)$$

where  $(H_{mo})_t$  = transmitted significant wave height; and  $H_{mo}$  = incident significant wave height at the toe of the structure. Eq. (6.1) implicitly assumes the same water depth at the locations of the measurements of  $(H_{mo})_t$  and  $H_{mo}$ . This assumption is satisfied if a low-crested breakwater is located on a horizontal bottom. Their two formulas for narrow and wide crests with imposed lower and upper limits of  $K_t$  predicted  $K_t$  within an error of about 0.2 where the error is defined as the difference between the measured and predicted values of  $K_t$ .

van der Meer et al. (2005) analyzed the wave basin tests of Kramer et al. (2005). The measured values of  $K_t$  for unidirectional (long-crested) and directional (short-crested) waves were practically the same for the incident wave angle  $\beta < 60^\circ$ . The effect of the angle  $\beta < 60^\circ$  on  $K_t$  was small in comparison to the scatter of data points. van der Meer et al. (2005) also examined wave transmission over smooth impermeable low-crested structures and proposed a separate formula for  $K_t$  that included the effect of the angle  $\beta < 70^\circ$  in the form of  $(\cos \beta)^{2/3}$ .

The formulas of wave transmission over and through low-crested breakwaters proposed by Goda and Ahrens (2008) using 851 data points and adjusted by Tomasicchio et al. (2011) using 3,327 data points are presented in the following. The formulas are limited to normally incident waves on rubble mound structures but estimate the wave transmission coefficients over and through the permeable structures separately. The separate estimation may be more useful in designing the height and crest width of a high-crested breakwater as well.

### 6.1 Waves over Low-Crested Structures

Using a Japanese design diagram, Goda and Ahrens (2008) expressed the wave transmission coefficient  $(K_t)_{\text{over}}$  over a low-crested structure in the range of 0 – 1.0 as follows:

$$(K_t)_{\text{over}} = \left\{ 1.0 - \exp \left[ a \left( \frac{R_c}{H_{mo}} - R_o \right) \right] \right\} \geq 0 \quad (6.2)$$

with

$$a = 0.248 \exp \left[ -0.384 \ln \left( \frac{B_{\text{eff}}}{L_o} \right) \right] \quad (6.3)$$

$$R_0 = \max \left[ 0.6, \min \left( 0.8, \frac{H_{mo}}{D_{eff}} \right) \right] \quad \text{for } D_{eff} > 0 \quad (6.4)$$

$$R_0 = 1.0 \quad \text{for } D_{eff} = 0 \quad (6.5)$$

where  $H_{mo}$  = incident spectral significant wave height at the toe of the structure;  $R_c$  = structure crest height above the still water level ( $R_c < 0$  for a submerged structure);  $L_o$  = deepwater wavelength based on the spectral peak period  $T_p$  given by  $L_o = gT_p^2 / (2\pi)$  with  $g$  = gravitational acceleration;  $B_{eff}$  = effective crest width for wave transmission; and  $D_{eff}$  = effective diameter of rock or concrete armor units. In Eq. (6.4), max (min) indicates the larger (smaller) value of the two values in the square (round) brackets. The constants 0.6 and 0.8 in Eq. (6.4) are the values adjusted by Tomasicchio et al. (2011) from 0.5 and 1.0, respectively, in Goda and Ahrens (2008).

The value of  $(K_t)_{over}$  given by Eq. (6.2) with Eq. (6.3) decreases with the increase of the normalized crest height ( $R_c / H_{mo}$ ), which may be in the ranges of  $-3$  to  $3$ , and with the increase of the normalized crest width ( $B_{eff} / L_o$ ), which may be in the range of  $0.02$  to  $0.2$  with the corresponding range of  $a = 0.46$  to  $1.11$ . The dimensionless parameter  $R_0$  is related to the value of  $(K_t)_{over}$  with  $R_c = 0$  in Eq. (6.2) when the structure crest is at the still water level (SWL). For an impermeable structure,  $D_{eff} = 0$  and  $R_0 = 1.0$  according to Eq. (6.5). For  $R_c = 0$ ,  $R_0 = 1$  and  $a = 0.46 - 1.11$ , Eq. (6.2) yields  $(K_t)_{over} = 0.37 - 0.67$ . For a permeable structure,  $D_{eff} > 0$  and  $R_0 = 0.6 - 0.8$ . For  $R_c = 0$ ,  $R_0 = 0.6 - 0.8$ , and  $a = 0.46 - 1.11$ , Eq. (6.2) gives  $(K_t)_{over} = 0.24 - 0.59$ . The wave transmission coefficient over a permeable structure with its crest at SWL is roughly  $0.4$ .

The effective width  $B_{eff}$  in Eq. (6.3) is estimated as

$$\begin{aligned} B_{eff} &= \text{structure width at SWL} && \text{for } R_c > 0 \\ B_{eff} &= 0.9 \times (\text{crest width}) + 0.1 \times (\text{bottom width}) && \text{for } R_c = 0 \\ B_{eff} &= 0.8 \times (\text{crest width}) + 0.2 \times (\text{bottom width}) && \text{for } R_c < 0 \end{aligned} \quad (6.6)$$

The effective width is assumed intuitively to correspond to the structure width at the level of 10% (20%) below the crest for zero freeboard with  $R_c = 0$  (submerged breakwaters with  $R_c < 0$ ). The effective diameter  $D_{eff}$  is given by

$$D_{eff} = D_{n50} = \left( \frac{M_{50}}{\rho_r} \right)^{1/3} \quad (6.7)$$

where  $D_{n50}$  = nominal diameter of rock armor unit;  $M_{50}$  = median rock mass; and  $\rho_r$  = rock density. The characteristics of underlayer and core materials are not accounted for in Eq.

(6.7). For concrete armor units,  $M_{50}$  and  $\rho_r$  in Eq. (6.7) are replaced by the armor unit mass and concrete density, respectively.

## 6.2 Waves through Rubble Mound Structures

Goda and Ahrens (2008) adopted a Japanese formula for the transmission coefficient  $(K_t)_{thru}$  of waves passing through sloped rubble mounds

$$(K_t)_{thru} = \left[ 1 + C \left( \frac{H_{mo}}{L} \right)^{0.5} \right]^{-2} ; \quad C = 3.45 \left( \frac{B_{eff}}{D_{eff}} \right)^{0.65} \quad (6.8)$$

where  $L$  = wavelength based on  $T_p$  and water depth  $d_t$  at the toe of the structure. For an impermeable structure with  $D_{eff} = 0$ , the parameter  $C$  becomes infinitely large and  $(K_t)_{thru} = 0$ . Eq. (6.8) developed for the case of negligible wave transmission over the structure is multiplied by a correction factor for a low structure height in the next section. The constant 3.45 for  $C$  is the value adjusted by Tomasicchio et al. (2011) from 1.135 in Goda and Ahrens (2008). The adjustment of the factor of 3 may be large but  $(K_t)_{thru}$  is normally small in comparison to  $(K_t)_{over}$  for low-crested permeable structures.

The value of  $(K_t)_{thru}$  given by Eq. (6.8) decreases with the increase of the wave steepness  $(H_{mo}/L)$  and the value of  $(B_{eff}/D_{eff})$  which may be regarded to represent the number of armor units across the structure width. For a typical range of  $(B_{eff}/D_{eff}) = 3 - 10$ ,  $C = 7 - 15$  and  $(K_t)_{thru} = 0.25 - 0.42$  for  $(H_{mo}/L) = 0.04$ .

## 6.3 Combined Wave Transmission

The transmitted waves over and through a permeable structure may be correlated but the correlation coefficient is unknown. The wave energy transmitted over the structure and the wave energy transmitted through the structure are added to estimate the combined wave transmission coefficient  $(K_t)_{all}$  whose upper limit is unity.

$$(K_t)_{all} = \left[ (K_t)_{over}^2 + K_h^2 (K_t)_{thru}^2 \right]^{0.5} \leq 1.0 ; \quad K_h = \frac{h_c}{d_t + H_{mo}} \leq 0.8 \quad (6.9)$$

where  $K_h$  = correction factor related to the crest height  $h_c$  of the structure above the bottom relative to the wave transmission height  $(d_t + H_{mo})$  through the structure with  $d_t$  = toe depth at the structure toe. The upper limit of  $K_h$  was 1.0 in Goda and Ahrens (2008) and adjusted to 0.8 in Tomasicchio et al. (2011). It is noted that  $K_h (K_t)_{thru}$  should be regarded as the actual wave transmission coefficient through the permeable structure with the specified height  $h_c$  which is neglected in Eq. (6.8).



Tomasicchio et al. (2011) compared the measured and predicted values of  $(K_t)_{all}$  for 3,327 data points from 33 data sets. The correlation coefficient was 0.9 and the standard deviation of the difference between the measured and predicted values was 0.10. In short, Eq. (6.9) predicted  $(K_t)_{all}$  within an error of about 0.2. The original formula of Goda and Ahrens (2008) tended to overpredict  $(K_t)_{all}$  for  $(K_t)_{all} < 0.4$  and its correlation coefficient was 0.83 for the 3,327 data points.

## 6.4 References

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## 7. ARMOR LAYER STABILITY ON SEAWARD SLOPE

The stability of rock armor units of a two-layer thickness on the seaward slope of a traditional rubble mound structure with no or little wave overtopping has been investigated experimentally to quantify several effects that were not included in earlier formulas developed before the year of 2000. These effects include the shallow toe depth, incident wave angle, and a berm on the seaward slope. The stability of toe armor units has also been investigated in more detail. The progression of armor layer damage under time-varying storm surge and wind waves has been examined more extensively. These improvements are summarized in the following. This summary is limited to rock armor units because almost all rubble mound structures in the U.S. are constructed of quarry rock. New concrete armor units have been invented since 2000. The detailed information of such concrete armor units may be available from the inventors.

### 7.1 Shallow Toe Depth

The important parameter for the armor stability is the stability number  $N_{mo}$  or  $N_s$  defined as

$$N_{mo} = \frac{H_{mo}}{\Delta D_{n50}} ; N_s = \frac{H_s}{\Delta D_{n50}} ; \Delta = \frac{\rho_r - \rho}{\rho} ; D_{n50} = \left( \frac{M_{50}}{\rho_r} \right)^{1/3} \quad (7.1)$$

where  $H_{mo}$  = spectral significant wave height;  $H_s$  = significant wave height based on a zero-upcrossing method;  $\rho_r$  = rock density;  $\rho$  = water density;  $\Delta$  = (rock specific gravity - 1);  $D_{n50}$  = nominal rock diameter; and  $M_{50}$  = median rock mass. The incident significant wave

height  $H_{mo}$  or  $H_s$  at the toe of the rubble mound structure is normally used to represent the incident irregular waves. The stability number can be regarded as the ratio between the wave force  $\rho g H_{mo} D_{n50}^2$  or  $\rho g H_s D_{n50}^2$  and the submerged rock weight  $(\rho_r - \rho) g D_{n50}^3$ .

The Hudson formula (Hudson 1959) for the regular wave height  $H$  can be expressed as

$$\frac{H}{\Delta D_{n50}} = (K_D \cot \alpha)^{1/3} \quad (7.2)$$

where  $K_D$  = stability coefficient; and  $\alpha$  = angle of the seaward slope from the horizontal. For no or little damage of rock armor units,  $K_D$  is in the range of 2 – 4 (Shore Protection Manual 1984). For typical values of  $\Delta = 1.6$  and  $\cot \alpha = 2$ ,  $(H / D_{n50}) = 2.5 - 3.2$ . The rock diameter  $D_{n50}$  is of the order of  $H / 3$ . The upper limit of  $D_{n50}$  of quarry rock may be about 2 m. Concrete armor units may be necessary if the design wave height exceeds about 6 m. The Hudson formula is useful for an initial rough estimate of  $D_{n50}$  and  $M_{50}$ . A more refined estimate of  $D_{n50}$  is required because the cost of a rubble mound structure tends to be sensitive to the required size of armor units. Since the Hudson formula was based on regular wave experiments, an effort was made to replace  $H$  in Eq. (7.2) by an equivalent irregular wave height (Shore Protection Manual 1984) before irregular wave experiments became standard. However, the need of irregular wave experiments was obvious to obtain a refined estimate of  $D_{n50}$ .

van der Meer (1987) conducted 262 irregular wave tests with the stability number  $N_s = 0.8 - 4.4$  and  $\cot \alpha = 1.5 - 6$ . The eroded area  $A_e$  of the armor layer with  $D_{n50} = 3.6$  cm was measured after the seaward slope was exposed to a specified irregular wave train consisting of  $N$  waves with  $N = 1,000$  and 3,000. The damage  $S$  defined as  $S = A_e / D_{n50}^2$  was used to represent the degree of the alongshore averaged slope deformation where  $S = 2 - 3$  for damage initiation and  $S = 8 - 17$  for failure (underlayer exposure to direct wave attack). The stability number  $N_s$  was expressed as a function of  $S / \sqrt{N}$ ,  $\cot \alpha$ ,  $H_s / L_o$  with  $L_o$  = deepwater wavelength based on the mean wave period  $T_m$ , and notational (fitted) permeability coefficient  $P$  with  $P = 0.1$  for an impermeable core,  $P = 0.4$  or 0.5 (no filter layer) for a permeable core, and  $P = 0.6$  for a homogeneous structure. The effects of armor grading, incident wave spectrum shape, and incident wave groupiness on the stability numbers  $N_s$  were not apparent. The 262 tests were limited to the horizontal bottom seaward of the uniform slope whose toe depth was larger than  $3 H_s$ . van der Meer (1988) conducted additional 16 tests on the seabed slope  $\cot \theta = 30$  with  $\theta$  = seabed slope angle from the horizontal. The shallow water depth effect on  $N_s$  was included using the ratio  $H_{2\%} / H_s$  with  $H_{2\%}$  = wave height exceeded by 2% of incident irregular waves measured at the toe of the slope. For the toe depth exceeding  $3 H_s$ ,  $H_{2\%} / H_s = 1.4$  was assumed on the basis of the Rayleigh wave height distribution. However, the accurate prediction of  $H_{2\%} / H_s$  for breaking waves is difficult for actual applications of the formula of van der Meer (1988).

Most of the rubble mound structures in the U.S. are exposed to depth-limited breaking waves during storms. Melby and Kobayashi (1998) measured damage progression and variability on a conventional rubble mound exposed to depth-limited breaking waves on a seabed slope of  $\cot \theta = 20$ . They simplified the empirical formula of van der Meer (1988) which does not include the seabed slope affecting irregular wave breaking in front of the mound. The simplified formula was manipulated for the prediction of the alongshore-averaged damage under time-varying wave conditions and water levels. The alongshore-damage variability was analyzed and shown to be significant. This damage variability may be related to local mound breaching but has not been incorporated in the rubble mound design yet.

van Gent et al (2003) used 207 tests conducted on seabed slopes with  $\cot \theta = 30$  and 100 in front of mound slopes with  $\cot \alpha = 2$  and 4 to evaluate the formula of van der Meer (1988). The use of the spectral wave period  $T_{m-1,0}$  instead of the mean period  $T_m$  improved the agreement for the 207 tests. The re-calibrated formulas for plunging and surging waves are expressed as

$$\frac{S}{\sqrt{N}} = \left[ \frac{1}{8.4} N_s \xi_{m-1,0}^{0.5} P^{-0.18} \left( \frac{H_{2\%}}{H_s} \right) \right]^5 \quad \text{for } \xi < \xi_c \quad (7.3)$$

$$\frac{S}{\sqrt{N}} = \left[ \frac{1}{1.3} N_s \xi_{m-1,0}^{-P} P^{0.13} (\tan \alpha)^{0.5} \left( \frac{H_{2\%}}{H_s} \right) \right]^5 \quad \text{for } \xi \geq \xi_c \quad (7.4)$$

with

$$\xi_{m-1,0} = \tan \alpha \left( \frac{g T_{m-1,0}^2}{2\pi H_s} \right)^{0.5} \quad ; \quad \xi_c = \left[ \frac{8.4}{1.3} P^{0.31} (\tan \alpha)^{0.5} \right]^{1/(P+0.5)} \quad (7.5)$$

The constants 8.4 and 1.3 in Eqs. (7.3) and (7.4) for plunging and surging waves, respectively, were 8.68 and 1.4 in the original equations of van der Meer (1988). The standard deviation  $\sigma$  of the difference between the measured and predicted values of  $S/\sqrt{N}$  with  $N = 1,000$  and  $3,000$  ( $\sqrt{N} = 32$  and  $55$ ) was  $\sigma = 0.167$  for the original equations and  $\sigma = 0.109$  for Eqs. (7.3) – (7.5). The use of  $H_{m0}$  instead of  $H_s$  in Eqs. (7.3) – (7.5) resulted in  $\sigma = 0.107$ . The spectral significant wave height  $H_{m0}$  and period  $T_{m-1,0}$  can be used to predict the damage  $S$  using Eqs. (7.3) – (7.5) which were limited to  $N_s = 0.6 - 4.3$ ,  $(H_{2\%}/H_s) = 1.21 - 1.42$ , and  $(H_s/d_t) = 0.15 - 0.78$  with  $d_t =$  toe depth. The ratio  $(H_{2\%}/H_s)$  is difficult to estimate for practical applications. The use of  $(H_{2\%}/H_s) = 1.3$  in Eqs. (7.3) and (7.4) would change  $S$  by a factor of  $(1.21/1.3)^5 = 0.70$  to  $(1.42/1.3)^5 = 1.55$ . This change may be smaller than  $\sigma\sqrt{N} = 3.4 - 5.9$ .

van Gent et al. (2003) proposed the following simple formula:

$$\frac{S}{\sqrt{N}} = \left[ 0.57 N_s (\tan \alpha)^{0.5} \left( 1 + \frac{D_{n50\text{core}}}{D_{n50}} \right)^{-1} \right]^5 \quad (7.6)$$

where  $D_{n50\text{core}}$  = nominal diameter of the core stone which is zero for an impermeable core. The ratio  $D_{n50\text{core}} / D_{n50}$  was in the range of 0.0 – 0.3 for their 207 tests. The standard deviation  $\sigma$  for Eq. (7.6) was  $\sigma = 0.109$  and the same as Eqs. (7.3) and (7.4). The most important term in Eq. (7.6) is  $N_s^5$  and the other terms related to the seaward slope  $\tan \alpha$  and permeability are secondary. The error of the predicted damage  $S$  may be of the order of 100% but the stability number  $N_s$  for given  $S$  can be predicted more accurately because  $N_s$  is proportional to  $S^{0.2}$  in Eqs.,(7.3). (7.4) and (7.6) which are limited to normally incident waves.

## 7.2 Oblique Waves

The stability of rock and concrete armor layers on rubble mound structures under oblique wave attack was investigated in several experimental studies performed in wave basins. van Gent (2014) conducted experiments in a wave basin of 50-m length and 50-m width in order to synthesize the earlier findings and provide more definitive formulas for the increase of the armor layer stability with the increase of the incident wave angle  $\beta$  in the range of  $\beta = 0 - 90^\circ$ . The experiments were limited to the horizontal bottom and the toe depth  $d_t$  exceeding  $2.4 H_s$  (no wave breaking seaward of the structure toe). The reduction factor  $\gamma_\beta$  for the required armor size  $D_{n50}$  for the wave angle  $\beta > 0^\circ$  relative to that for normally-incident waves ( $\beta = 0^\circ$ ) was defined as

$$\gamma_\beta = \left[ N_s(\beta = 0^\circ) / N_s(\beta) \right] = D_{n50}(\beta) / D_{n50}(\beta = 0^\circ) \quad (7.7)$$

where the stability number  $N_s$  defined in Eq. (7.1) is proportional to  $D_{n50}^{-1}$ . The reduction factor  $\gamma_\beta$  was expressed as

$$\gamma_\beta = (1 - C_\beta)(\cos \beta)^2 + C_\beta \quad \text{for} \quad 0 \leq \beta \leq 90^\circ \quad (7.8)$$

with

$$\begin{aligned} C_\beta &= 0.42 \quad \text{for rock slopes with short-crested waves} \\ C_\beta &= 0.35 \quad \text{for rock slopes with long-crested waves} \end{aligned} \quad (7.9)$$

where  $\gamma_\beta$  decreases from 1.0 for  $\beta = 0^\circ$  to  $C_\beta$  for  $\beta = 90^\circ$ . Eq. (7.9) is applicable to rock slopes only because for the concrete cubes tested in the experiments,  $C_\beta = 0$  and 0.35 for the cubes placed in a single layer and double layers, respectively.

For the application of Eqs. (7.7) – (7.9), it is required to estimate the stability number  $N_s$  and corresponding diameter  $D_{n50}$  for  $\beta = 0^\circ$  first. van Gent (2014) compared the damage

data for  $\beta = 0^\circ$  with Eq. (7.3) where the data were limited to the surf similarity parameter  $\xi_{m-1,0} = 2.2 - 3.5$ , the number of waves,  $N = 1,000$ , and the rock slopes of  $\cot \alpha = 1.5$  and  $2.0$  with  $P = 0.1$  (impermeable core) and  $P = 0.5$  (no filter layer on a permeable core). The damage  $S$  was not measured always probably because the profile measurement in a wave basin is laborious. The number  $N_{OD}$  of displaced units over the alongshore distance of  $D_{n50}$  was measured for all the tests. The agreement of Eq. (7.3) with the data was deemed acceptable under the assumption of  $S = N_{OD}$ . Eq. (7.8) was based on the limited number (probably less than 50) of tests with  $N_{OD} = 2 - 20$ . The limited data indicates that the wave angle effect is negligible for  $\beta < 15^\circ$  ( $\gamma_\beta > 0.95$ ) and noticeable for  $\beta > 30^\circ$  ( $\gamma_\beta < 0.85$ ). It is necessary to confirm Eq. (7.8) and calibrate  $C_\beta$  for conventional rubble mound structures ( $P = 0.4$ ) exposed to depth-limited breaking waves ( $d_t < 2H_s$ ) on a sloping seabed during storms.

Pratt et al. (2003) conducted an experiment in a wave basin to measure damage development on rubble mound jetties. The jetties were arranged in a parallel configuration to form an inlet and were exposed to four storm series composed of waves only and waves on a steady ebb current. A precision laser profiler was used to measure the three-dimensional profile on the head and connecting regions along the jetty length. For the test with waves head-on to the jetties, the incident irregular waves shoaled on the seabed with  $\cot \theta = 34$  and the significant wave height at the toe of the jetty head was about a half of the toe depth of 17 or 20 cm. Damage development on the jetty head under head-on waves was similar to that on the breakwater trunk under normally-incident waves in the experiment by Melby and Kobayashi (1998). Damage decreased landward along the jetty. The damage decrease along the jetty might be explained qualitatively using Eq. (7.8) where  $\beta = 0^\circ$  ( $\gamma_\beta = C_\beta$ ) landward along the jetty trunk. The unique damage data by the U.S. Army Engineer Research and Development Center could be used to establish the damage reduction factor  $\gamma_\beta$  along the jetty.

### 7.3 Slope with a Berm

van Gent (2013) investigated rock stability of a rubble mound breakwater with a non-reshaping statically stable berm in contrast to a berm breakwater whose berm is reshaped into a new statically or dynamically stable profile under design wave conditions. Berm breakwaters have been studied by a number of researchers (e.g., Lykke Andersen and Burcharth 2009) but have not been adopted much perhaps because of the difficulty in predicting the long-term performance of a berm breakwater with movable rock units. A breakwater with a statically stable berm may not be adopted either but a berm could be installed to upgrade an aging rubble mound structure and reduce wave overtopping. In any case, his experiment clarifies the damage variation on the seaward slope of a breakwater. His empirical formulas illustrate the difficulty in developing general formulas for non-uniform slopes because of the increased number of structural variables.

The wave-flume experiment of van Gent (2013) was limited to rock slopes of  $\cot \alpha = 2$  and  $4$  on a permeable core ( $P = 0.5$ ) exposed to the significant wave height  $H_s \leq 24$  cm at the toe of the slope with the toe depth  $d_t = 70$  cm ( $d_t > 2.9H_s$ ). A horizontal berm was

located at an elevation of  $(-h_b)$  above the still water level (SWL). The berm width  $B$  was 0, 20, 40 and 80 cm where the tests for  $B = 0$  were used to measure the damage  $S$  in the absence of the berm. The upper slope above the berm and the lower slope below the berm were the same in order to estimate the damage  $S$  for  $B = 0$  using Eqs. (7.3) and (7.4) limited to the constant slope  $\tan \alpha$ . The diameter  $D_{n50}$  of the rock armor layer was 2 cm (1.4 cm) on the upper slope and 4 cm (2.8 cm) on the berm and lower slope, respectively, for  $\cot \alpha = 2$  ( $\cot \alpha = 4$ ). The selected values of  $D_{n50}$  suggest higher rock stability on the upper slope and on the gentler slope of  $\cot \alpha = 4$ . A wire mesh was used to fix the rock units on the berm and lower slope in a test to measure the upper slope profile change. The profile change on the berm and lower slope was measured in another test with the wire mesh on the upper slope. The use of the wire mesh may be acceptable if the total profile change is relatively small. The tests were limited to the damage  $S \leq 20$ .

The measured damage  $S_{up}$  on the upper slope and the damage  $S_{bs}$  on the berm and lower slope were compared with the damage  $S(B = 0)$  on the entire uniform slope computed using Eq. (7.3) for plunging waves where  $N = 1,000$ ,  $N_s = 2 - 8$  ( $H_s = H_{mo}$ ),  $\xi_{m-1,0} = 1.2 - 4.2$ ,  $P = 0.5$ , and  $(H_{2\%} / H_s) = 1.4$  for these tests. This intuitive approximation may be necessary but the stability of rock units on the actual seaward slope with the berm is not analyzed explicitly.

The damage  $S_{up}$  on the upper slope was expressed as

$$S_{up} = \gamma_{berm} \gamma_{pos} S(B = 0) \quad \text{for upper slope} \quad (7.10)$$

with

$$\gamma_{berm} = 1 - 0.15 \xi_{m-1,0} \left( \frac{B}{H_s} \right) \quad \text{with } \gamma_{berm} \geq 0 \quad (7.11)$$

$$\gamma_{pos} = \left[ 0.25 + 0.125 \cot \alpha \left( 1 + \frac{h_b}{H_s} \right) \right]^5 \quad \text{with } 0 \leq \gamma_{pos} \leq 1 \quad (7.12)$$

where the reduction factors  $\gamma_{berm}$  and  $\gamma_{pos}$  are related to the berm width  $B$  and position  $h_b$  ( $h_b < 0$  for the berm above SWL). The upper slope damage  $S_{up}$  above the berm decreases with the increase of  $B$  and with the decrease of  $h_b$  (higher berm elevation) where  $\gamma_{pos} = 0$  and  $S_{up} = 0$  for  $h_b = -(1 + 2 \tan \alpha) H_s$ . The upper slope is not damaged if the berm is located at the elevation of  $2 H_s$  and  $1.5 H_s$  above SWL for  $\cot \alpha = 2$  and 4, respectively. On the other hand,  $\gamma_{pos} = 1$  for  $h_b = (6 \tan \alpha - 1) H_s$ . Damage on the upper slope is not affected by the berm position if the depth  $h_b$  above the berm exceeds  $2 H_s$  and  $0.5 H_s$  for  $\cot \alpha = 2$  and 4, respectively.

The damage  $S_{bs}$  on the berm and lower slope was expressed as

$$S_{bs} = (\gamma_{bs})^5 S(B = 0) \quad \text{for berm and lower slope} \quad (7.13)$$

with

$$\gamma_{bs} = 1 - 0.12 \frac{B}{H_s} \frac{h_b}{H_s} \quad \text{for } h_b \geq 0 \quad (7.14)$$

$$\gamma_{bs} = 1 + 0.02 \xi_{m-1,0} \frac{B}{H_s} \frac{h_b}{H_s} \quad \text{for } h_b < 0 \quad (7.15)$$

where the reduction factor  $\gamma_{bs}$  ( $0 < \gamma_{bs} \leq 1$ ) includes both  $B$  and  $h_b$ . For  $h_b = 0$ ,  $\gamma_{bs} = 1$  and  $S_{bs} = S(B = 0)$ , implying that the damage on the berm and lower slope is approximately the same as that of the entire slope with no berm. For  $h_b = 0$  and  $\cot \alpha = 2 - 4$ , Eq. (7.12) yields  $\gamma_{pos} = 0.03 - 0.24$  and the damage on the upper slope is predicted to be small.

The predicted values of  $S_{up}$  and  $S_{bs}$  using Eqs. (7.10) and (7.13) were slightly less accurate than  $S(B = 0)$  predicted using Eq. (7.3) in van Gent et al. (2003). The damage prediction error of about 100% is typical for uniform seaward slopes with no wave overtopping. The prediction error appears to increase with the complexity of the structure geometry. van Gent (2013) provided a formula for the landward recession of the seaward edge of the berm which was assumed to be proportional to  $S_{bs}$  predicted by Eq. (7.13). The berm recession (local erosion) is harder to predict accurately than the eroded area (integrated erosion) and damage  $S, S_{up}$  and  $S_{bs}$ .

#### 7.4 Toe Armor Stability

The toe of the armor layer on a rubble mound structure is supported by a toe mound constructed of rock or concrete units. The stability of the rock mound at the toe was investigated sporadically. Several simple formulas for rock toe stability were proposed. van Gent and van der Werf (2014a) conducted 192 tests in a wave flume and showed that these simple formulas could predict only the order of magnitude of damage measured in the 192 tests. These tests were conducted for the toe of a rock (2.7-cm in diameter) armor layer with  $\cot \alpha = 2$  placed on a permeable core (0.8-cm diameter). The toe was located on a fixed seabed slope of  $\cot \theta = 30$  (no toe scour). The water depth  $d_t$  at the toe was in the range of 20 – 40 cm. The significant wave height  $H_s$  based on individual waves measured at the toe was chosen so that the ratio  $H_s / d_t = 0.22 - 0.81$  and no severe wave breaking occurred before the waves reached the structure ( $H_{2\%} / H_s = 1.4$ ). The toe mound consisted of rock units with the nominal diameter  $D_{n50} = 1.46$  or 2.33 cm. The thickness  $t_t$  of the toe mound was in the range of 3 – 6 cm, corresponding to the range of  $2D_{n50} - 4D_{n50}$ . The crest width  $B_t$  of the toe mound was in the range of 4.4 – 21 cm, corresponding to the range of  $3D_{n50} - 9D_{50}$ . The landward end of the toe mound was connected with the armor layer. The seaward slope of the toe mound was very steep.

Rock units on the toe mound displaced over a distance more than one diameter  $D_{50}$  (1.46 or 2.33 cm) were counted after the exposure to 1,000 waves to obtain the number  $N_{OD}$

of displaced units over the alongshore width of  $D_{n50}$ . The measured values of  $N_{OD}$  for the 192 tests were expressed as

$$N_{OD} = 0.032 \left( \frac{t_t}{H_s} \right) \left( \frac{B_t}{H_s} \right)^{0.3} N_s^3 \frac{u_\delta}{\sqrt{gH_s}} \quad \text{for } N_{OD} < 7.3 \quad (7.16)$$

with

$$u_\delta = \frac{\pi H_s}{T_{m-1,0}} \frac{1}{\sinh(k_o h_t)} ; \quad k_o = \frac{(2\pi)^2}{g (T_{m,1-0})^2} \quad (7.17)$$

where  $N_s = (H_s / \Delta D_{n50}) =$  stability number of rock units at the toe with  $\Delta = 1.7$  and  $N_s = 1.2 - 10.5$  in these tests;  $u_\delta =$  representative wave-induced velocity on the toe;  $T_{m,1-0} =$  spectral wave period at the toe;  $k_o =$  deepwater wave number; and  $h_t =$  water depth above the toe given by  $h_t = (d_t - t_t)$ . The number  $N_{OD}$  increased with the increase of  $H_s, T_{m,1-0}, t_t$  and  $B_t$  and with the decrease of  $D_{50}$  and  $h_t$ . The agreement between the measured and predicted values of  $N_{OD}$  was well within a factor of 2.

van Gent and van der Werf (2014b) conducted additional tests for the rock toe mound of a rock armor layer with  $\cot \alpha = 1.5$  (46 tests) and  $\cot \alpha = 4$  (59 tests) as well as for the rock toe mound of a single layer of cubes (smoother surface) with  $\cot \alpha = 2$  (40 tests). The agreement between the measured and predicted values of  $N_{OD}$  for the rock toe mound of the rock armor layer was within a factor of 2 for  $\cot \alpha = 1.5$  and mostly within a factor of 2 for  $\cot \alpha = 4.0$ . For the single-layer cubes, the agreement became good when the constant 0.032 in Eq. (7.16) was replaced by 0.048. The increased wave downrush on the smooth cube surface may have increased the number of displaced rock units on the toe mound by a factor of 1.5.

The required size of rock units for the toe protection of the rock armor layer with  $\cot \alpha = 1.5 - 4$  may be estimated using Eq. (7.16) if breaking waves do not attack the toe rock units directly. The degree of allowable damage to the toe was discussed by van Gent and van der Werf (2014a) but toe scour will need to be estimated to judge allowable toe damage. The interaction of sand and rock units is poorly understood and local toe scour is difficult to estimate at present.

## 7.5 Damage Progression

Damage formulas for an intact rubble mound structure under constant water level and wave conditions may be used to estimate damage during the peak of a storm but cannot be used for a life-cycle analysis of the structure. Damage progression formulas were developed for concrete armor units (Hanzawa et al. 1996) and rock armor units (Melby and Kobayashi 1998). Kobayashi et al. (2003) simulated the virtual performance of rock mound structures in shallow water under combined storm surge and breaking waves for sequence of hurricanes. The computed progression of damage to the armor layer was caused episodically by several major storms but slowed down as the structure aged. The damage increment during a storm was shown to correspond to the damage caused by approximately 1,000 waves during the peak of a hurricane storm. Melby and Kobayashi (2011) upgraded their damage progression



formula using additional tests. The upgraded formula was included in the breakwater-harbor time-dependent life-cycle analysis software CSsim developed by Melby et al. (2011).

The upgraded formula by Melby and Kobayashi (2011) adopted the maximum depth-integrated wave momentum flux,  $(M_F)_{max}$ , at the toe of the structure to represent the wave force in the toe depth  $d_t$ . The normalized momentum flux  $M_*$  based on nonlinear wave theory was approximated as

$$M_* = \frac{(M_F)_{max}}{\rho g d_t^2} = A_0 \left( \frac{d_t}{g T_m^2} \right)^{-A_1} \quad (7.18)$$

with

$$A_0 = 0.639 \left( \frac{H_{mo}}{d_t} \right)^{2.026} ; \quad A_1 = 0.180 \left( \frac{H_{mo}}{d_t} \right)^{-0.391} \quad (7.19)$$

where  $\rho$  = water density;  $g$  = gravitational acceleration;  $H_{mo}$  = spectral significant wave height at the toe; and  $T_m$  = mean wave period at the toe. The new stability number  $N_m$  based on  $M_*$  was defined as

$$N_m = \left( \frac{M_*}{\Delta} \right)^{1/2} \frac{d_t}{D_{n50}} ; \quad \Delta = \frac{\rho_r}{\rho} - 1 \quad (7.20)$$

where  $\rho_r$  = rock density. The formula for damage  $S$  for constant wave conditions was obtained using the data of van der Meer (1988) with the number of waves,  $N = 1,000$  and  $3,000$ .

$$\frac{S}{\sqrt{N}} = (a_m N_m)^5 \quad (7.21)$$

with

$$a_m = \left[ 5P^{0.18} (\cot \alpha)^{0.5} \right]^{-1} \quad \text{for } s_m \geq s_{mc} \quad (7.22)$$

$$a_m = s_m^{P/3} \left[ 5P^{0.18} (\cot \alpha)^{0.5-P} \right]^{-1} \quad \text{for } s_m < s_{mc} \quad (7.23)$$

$$s_{mc} = 0.028 - 0.0035 \cot \alpha \quad \text{for } 1.5 \leq \cot \alpha \leq 6 \quad (7.24)$$

where  $P$  = notational permeability coefficient ( $P = 0.1$  for an impermeable core;  $P = 0.4$  for a traditional multilayer structure; and  $P = 0.6$  for a homogeneous structure) introduced by van der Meer (1987);  $\alpha$  = angle of the seaward slope from the horizontal;  $s_m = H_{mo} / L_m$  with  $L_m$  = wavelength based on  $T_m$  and  $d_t$ ; and  $s_{mc}$  = wave steepness dividing plunging waves ( $s_m \geq s_{mc}$ ) and surging waves ( $s_m < s_{mc}$ ). The coefficients of 5 in Eqs. (7.22) and (7.23) were mean values for the data of van der Meer (1988). The standard deviation was 0.3 for Eq. (7.22) and 0.4 for Eqs. (7.23). Eq. (7.21) based on  $N_m$  given by Eq. (7.20) is similar to Eqs. (7.3) and (7.4) based on the stability number  $N_s$  given by Eq. (7.1) except that  $N_m$  includes the effect of the toe depth  $d_t$  explicitly because the ratio  $H_{2\%} / H_s$  is difficult to estimate for practical applications.

For the prediction of damage progression under time-varying wave conditions and water level, the damage  $S(t_n)$  at the end time  $t_n$  of constant wave conditions and still water level from time  $t_{n-1}$  to time  $t_n$  with  $n = 1, 2, \dots, N_t$  with  $N_t =$  number of time intervals was expressed as

$$S(t_1) = 0.5 + 1.3(N_1)^{0.5} (a_m N_m)_1^5 \quad \text{for } n = 1 \quad (7.25)$$

$$S(t_n) = \left[ (N_e)_{n-1} + N_n \right]^{0.5} (a_m N_m)_n^5 \quad \text{for } n = 2, 3, \dots, N_t \quad (7.26)$$

with

$$(N_e)_{n-1} = \left[ S(t_{n-1}) / (a_m N_m)_n^5 \right]^2 \quad \text{for } n = 2, 3, \dots, N_t \quad (7.27)$$

where  $N_n =$  number of waves during  $t_{n-1}$  to  $t_n$ ;  $(a_m N_m)_n =$  constant value of  $(a_m N_m)$  during  $t_{n-1}$  to  $t_n$ ; and  $(N_e)_{n-1} =$  equivalent number of waves that would cause the damage  $S(t_{n-1})$  under the constant wave forcing  $(a_m N_m)_n$ . The initial condition at time  $t_0$  was taken as  $S(t_0) = 0$  and  $(N_e)_0 = 0$  was not included in Eq. (7.25). For the first time interval during  $t_0$  to  $t_1$ , the damage  $S(t_1)$  was computed using Eq. (7.25) without the constant 0.5. If the computed damage  $S(t_1) > 0.2$ , the constant 0.5 was added to account for the dislodgement of discrete stone units placed initially in unstable positions. This initial small adjustment of stones cannot be avoided for stones placed randomly.

Eqs. (7.25) – (7.27) were compared with 17 tests for traditional multilayer rubble mound structures with  $\cot \alpha = 1.5$  and 2 in relatively shallow water. The test duration was in the range of 2 – 28.5 hours. The damage  $S$  at the end of the test was less than 16. The measured and computed values of  $S$  for each test were plotted as a function of the cumulative number of waves up to almost 61,000 waves. The agreement for all the tests was within a factor of about 2. This upgraded damage progression formula is as accurate as the other formulas limited to constant wave conditions and water level.

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## **8. ARMOR LAYER STABILITY ON CREST AND LANDWARD SLOPE**

If a rubble mound structure is designed to be overtopped significantly, the armor layer on the crest and landward slope will need to be designed against the overtopping flow and the seepage flow through the rubble mound. The stability of the armor layer on the crest and landward slope has been investigated in laboratory experiments but no general formula exists because the number of available tests for the armor stability on the crest and landward slope is small in comparison to the number of design variables such as the crest height and width and the landward slope in addition to the variables associated with the armor stability on the seaward slope for the case of no or little wave overtopping. Furthermore, the armor stability depends on the detailed flows on and through the rubble mound unlike the wave transmission coefficient. As a result, most laboratory experiments were conducted for specific low-crested rubble mound structures and armor units that have been adopted in different countries.

### **8.1 Crest Height Effects on Stability**

Ahrens (1989) conducted 205 tests in a wave tank to examine the stability and wave transmission of reef breakwaters which are low-crested structures of homogeneous stones constructed for beach stabilization and shore protection. The initial trapezoidal profile with a narrow crest of three stone diameters was limited to the seaward and landward slopes of 1/1.5 (vertical/horizontal). For a typical test with severe wave overtopping, the narrow crest was lowered by minor seaward stone movement on the steep seaward slope and major landward stone movement on the crest and landward slope. Ahrens (1989) proposed a stability formula

in terms of the reduced equilibrium (under the action of about 4,000 waves) crest height. This formula has not been adopted widely because the equilibrium crest height depends on the water level and wave conditions which vary with time during storms. In short, it is difficult to predict the long-term deformation of a reef breakwater once stone movement becomes excessive.

Vidal et al. (1992) conducted 35 tests in a wave basin to examine armor rock stability on the front (seaward) slope, crest, and back (landward) slope of low-crested and submerged breakwaters with two layers of armor rock in relatively deep water. The incident irregular waves were normal to the trunk alignment. The front and back slopes were 1/1.5 and the crest width was six stone diameters. A steel frame and a wire mesh were used to measure stone movement in designated sections of the breakwater. The measured stability number  $N_s$  defined in Eq. (7.1) for different damage levels was plotted as a function of the normalized freeboard  $R_d = R_c / D_{50}$  where  $R_c$  = crest elevation above the still water level and  $D_{50}$  = nominal stone diameter with  $D_{n50} = 2.5$  cm for the 35 tests. The value of  $R_d$  is negative for submerged breakwaters. For the 35 tests,  $R_d$  was in the range of  $-2$  to  $2.4$ . The stability number  $N_s$  on the front slope for given damage level increased with the decrease of  $R_d$ . The armor stability increased with the increase of wave overtopping and transmission probably because of the reduction of wave downrush on the steep front slope. On the other hand, the corresponding value of  $N_s$  on the crest decreased with the decrease of  $R_d$  and the increase of wave overtopping of the emerged crest ( $R_d > 0$ ). The value of  $N_s$  for the submerged crest ( $R_d < 0$ ) increased with the decrease of  $R_d$  and the increase of the crest submergence. The minimum value of  $N_s$  occurred when the crest was near the still water level. The value of  $N_s$  on the back slope increased with the decrease of  $R_d$  from  $R_d = 2.4$  perhaps because overtopping flow impinged on the subaerial part of the back slope for  $R_d = 2.4$ . The stability of the entire structure increased with the decrease of  $R_d$ . Vidal et al. (1995) plotted the measured  $N_s$  as a function of  $R_d$  for the front head and back head of the round head of the breakwater. The armor stability on the head increased with the decrease of  $R_d$ .

Kudale and Kobayashi (1996) analyzed the stability of an armor unit on the back slope impinged by overtopping jet of water issuing from an emerged crest. Their stability model could explain the increase of the stability number  $N_s$  for the initiation of damage on the back slope with the decrease of  $R_d$  in the tests of Vidal et al. (1992). The stability model elucidates the damage initiation mechanism on the back slope but is not suited for practical applications because of its computation time and the uncertainties of the drag, lift, and inertia coefficients used in the model.

van Gent and Pozueta (2004) conducted more than 100 tests in a wave flume and measured the eroded area  $A_e$  on the back slope of 1/2 or 1/4 for each test. The crest and front slope of 1/4 were fixed so that damage occurred on the back slope only. The damage  $S$  was defined as  $S = A_e / D_{n50}^2$  with  $D_{n50}$  = nominal diameter of the back slope stone which was the same as the core material. Use was made of 63 tests with  $S = 2 - 50$  and the number of incident waves,  $N < 4,000$ . The measured value of  $S / \sqrt{N}$  in each of the 63 tests was correlated to the horizontal velocity  $U_{1\%}$  on the landward end of the crest exceeded by 1% of

the incident waves where  $U_{1\%}$  was estimated using empirical formulas for wave runup and overtopping. The value of  $S/\sqrt{N}$  is proportional to  $U_{1\%}^6$  in their formula and sensitive to the error of the estimated  $U_{1\%}$ . The agreement between the measured and empirical values of  $S/\sqrt{N}$  for the 63 tests was similar to that of Eqs. (7.3) and (7.4) for the seaward slope with no or little wave overtopping. The empirical formula was not compared with the back slope damage data of Vidal et al. (1992). Melby (2009) modified the formula of van Gent and Pozueta (2004) for his time dependent lift-cycle analysis of breakwaters. In short, this formula based on the 63 tests will need to be evaluated using other data sets before its practical applications. The major problem of this formula is that the most important wave parameter  $U_{1\%}$  is not a measured value unlike the significant wave height  $H_s$  or  $H_{mo}$  used for the prediction of armor stability on the seaward slope.

## 8.2 Stability Formula for Entire Structure

Kramer and Burcharth (2003) conducted 69 tests in a wave basin to investigate armor stability on a low-crested breakwater in shallow water under short-crested (directional) irregular waves. The nominal diameter of the armor stone was  $D_{n50} = 3.3$  cm. The breakwater height above the horizontal bottom was 30 cm ( $9 D_{n50}$ ). The front and back slopes were 1/2. The crest width was  $3D_{n50}$  or  $8D_{n50}$ . The crest height  $R_c$  above the still water level ( $R_c < 0$  for submerged crests) was in the range of  $-10$  cm ( $-3D_{50}$ ) to  $5$  cm ( $1.5D_{n50}$ ). The predominant incident wave direction relative to the shore normal was in the range of  $-21^\circ$  to  $26^\circ$ . The measured stability number  $N_s$  for the initiation of damage was plotted as a function of the normalized freeboard  $R_d = R_c / D_{n50}$  in the same way as in Vidal et al. (1992). The effects of the crest width, wave steepness, and incident wave direction on  $N_s$  were secondary in comparison to that of  $R_d$  for the 69 tests. The variations of  $N_s$  as a function of  $R_d$  for the front slope, crest and back slope of the trunk and for the front head and back head of the round head were similar to those for the 35 tests of Vidal et al. (1992, 1995).

Low-crested breakwaters for beach stabilization and shore protection are normally constructed in shallow water parallel to the shoreline and exposed to depth-limited breaking waves during storms. Burcharth et al. (2006) recommended that such a low-crested breakwater should be designed for low or no damage to reduce damage accumulation during the breakwater life time. To estimate the minimum stability number  $N_s$  for the initiation of damage as a function of  $R_d$ , Burcharth et al. (2006) plotted all the data points for the different sections of the trunk and round head by Vidal et al. (1992, 1995) and Kramer and Burcharth (2003). The trunk data points from the wave-flume experiment by Burger (1995) were included in their plot of  $N_s$  as a function of  $R_d$ . The lower bound of  $N_s$  for the initiation of damage was expressed as

$$N_s = 1.36 - 0.23R_d + 0.06R_d^2 \quad ; \quad N_s = \frac{H_s}{\Delta D_{n50}} \quad ; \quad R_d = \frac{R_c}{D_{n50}} \quad (8.1)$$

where  $H_s$  = significant wave height at the toe of the breakwater;  $\Delta$  = (stone specific gravity  $- 1$ );  $D_{n50}$  = nominal stone diameter; and  $R_c$  = crest elevation above the stillwater level. Eq.

(8.1) for the same  $D_{50}$  on the entire structure is limited to the range of  $-3 \leq R_d < 2$ . Since  $R_c$  depends on the water level, the temporal variations of the water level and  $H_s$  during various storms are required to estimate the required value of  $D_{50}$ .

Eq. (8.1) is intended for a preliminary conservative design of the armor stone size of a low-crested breakwater. Eq. (8.1) yields  $N_s = 2.06, 1.36$  and  $1.14$  for  $R_d = -2, 0$  and  $2$ , respectively. Use may be made of  $N_s = 1.2$  for an emerged breakwater with  $R_d = 0 - 2$ . The wave transmission coefficient  $K_t$  estimated using the formulas in Section 6 may become too large for a narrow-crested submerged breakwater with its crest width in the range of  $3 D_{n50}$  to  $8 D_{n50}$ . For  $N_s = 1.2$  and  $\Delta = 1.6$ ,  $D_{n50} = 0.5H_s$  for the initiation of damage. This initial estimate of  $D_{n50}$  can be improved using site-specific model testing. Eq. (8.1) does not allow the spatial variation of  $D_{n50}$  but it is relatively easy to vary the armor stone size on the front slope, crest and back slope in the model testing. Eq. (8.1) is not applicable to wide-crested submerged breakwaters that have been constructed in Japan. For a submerged breakwater with  $R_d \approx -2$ , the crest width of the order of  $20 D_{n50}$  may be necessary to keep the wave transmission coefficient  $K_t$  acceptably small (e.g., Ota et al. 2006).

Eq. (8.1) cannot be used for the prediction of damage accumulation and repair during a life cycle of a low-crested breakwater (Melby 2009). Available damage data are not sufficient to develop a reliable empirical formula for damage initiation and progression. Physical model testing for a specific project is recommended for the detailed design. The numerical model CSHORE may also be incorporated in the time-dependent life-cycle model of Melby (2009). The capability of CSHORE in predicting the large deformation of low-crested breakwaters was evaluated by Kobayashi et al. (2013) using the 205 reef breakwater tests of Ahrens (1989) and the 20-h submerged wide-crested breakwater test of Ota et al. (2006). The capability of CSHORE in predicting damage on the different sections of low-crested breakwaters was assessed by Garcia and Kobayashi (2015) using the 35 deepwater tests of Vidal et al. (1992) and the 69 shallow-water tests of Kramer and Burcharth (2003). The research report by Garcia and Kobayashi is attached in Appendix for those who are interested in the CSHORE assessment results.

### 8.3 References

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## **9. INTEGRATED DESIGN**

The design of a rubble mound structure for given site specific conditions requires the prediction of wave runup, reflection, overtopping and transmission as well as armor layer stability on the entire structure. The empirical formulas in Sections 3 – 8 can be used for the preliminary design of the structure geometry and armor rock size for given project requirements. These formulas are essentially limited to a trapezoidal structure with a horizontal crest and constant seaward and landward slopes located on a horizontal or uniformly-sloping seabed. The detailed design of the structure requires the integration of the different design elements and the optimization of the structure geometry and armor rock size. Physical model testing in a hydraulic laboratory is required for the detailed and integrated design. Hydraulic model testing of rubble mound structures on a fixed bottom is well established (e.g., Hughes 1993) and discussed briefly partly because of the declining use of hydraulic model testing in the U.S.

### **9.1 Hydraulic Model Testing**

Hydraulic response and armor layer stability of a rubble mound structure is tested in a wave flume for its trunk and in a wave basin for its trunk and head. A fixed sloping bottom is installed to simulate incident irregular wave transformation in the vicinity of the structure which is normally located in shallow water. A wave maker is used to generate irregular waves at a sufficient distance from the structure. Nowadays, wave makers have a capability of absorbing most waves propagating seaward. Some wave makers use second-order wave generation techniques to minimize second-order long (infragravity) waves generated by the wave maker. However, wave generation is limited to wind waves. Ocean waves measured in the nearshore zone contain free (not related to local wind waves) long waves (e.g., Elgar et al. 1992) and free long waves are not accounted for in laboratory experiments.

Wave gauges are used to measure free surface elevations at points of interest and separate incident and reflected waves at the toe of the structure. Wave overtopping is measured by collecting overtopped water in a container because the continuous velocity measurement on the structure is difficult due to the small water depth. Damage to the armor layer is quantified by measuring the armor layer profile change or by counting the number of

displaced colored stones. These measurements are now routine. The experimental facility and instrument will need to be maintained and upgraded through series of hydraulic model testing and research experiments.

Hydraulic model testing for a specific project is based on Froud similitude with a length ratio of the order of 1/36 (model/prototype). The incident significant wave height is normally larger than 10 cm to minimize scale effects. The spectral peak period is of the order of 2 s. The model armor stone is selected so that the stability number  $N_s$  is the same in the model and prototype. This implies that the model stone approximately satisfies geometric similitude if the specific gravity is approximately the same for the model and prototype stones. The nominal diameter of the model stone is normally larger than 1 cm. This roughness on the structure causes turbulent flow on the armor layer and limits viscous effects only inside the porous structure. This may explain why small-scale model testing is regarded to be an acceptable replica of the prototype structure. It is noted that sand transport is difficult to simulate in small-scale tests for lack of satisfactory similitude for prototype sand particles that are transported as bed load (stone moves as bed load) as well as suspended load for which the sand fall velocity becomes important.

Once the model experiment is set up in a wave flume or basin, it is easy to change the water level and incident wave conditions. The structure geometry can also be adjusted to optimize the structure geometry and reduce the cost of a new rubble mound structure which is much larger than the model testing cost. The empirical formulas in Sections 3 – 8 are not accurate and versatile enough for the optimization of the structure geometry and armor rock size.

## 9.2 Numerical Modeling

Numerical modeling of hydraulic response and armor layer damage is less reliable than laboratory model testing but requires less time and cost. The cross-shore model CSHORE has been shown to be capable of predicting wave runup, reflection, overtopping and transmission as well as armor layer damage on an entire rubble mound structure of arbitrary geometry located on a beach with or without a bar. The model CSHORE may be used to improve the preliminary design based on the empirical formulas in Sections 3 – 8 and narrow down the scope of hydraulic model testing.

The cross-shore model CSHORE based on the assumption of local alongshore uniformity consists of the following components: a combined wave and current model based on time-average continuity, cross-shore and longshore momentum, and wave energy equations; a permeable layer model to account for porous flow and energy dissipation (not used for an impermeable bottom); a cohesionless (sand, gravel and stone) sediment transport model for bed load and suspended load (computed to be zero for stone); and a probabilistic model for an intermittently wet and dry zone on an impermeable or permeable structure. The details of CSHORE up to 2013 were documented by Kobayashi (2013) and the public-domain CSHORE is managed by Brad Johnson (ERDC-CHL) at <https://sites.google.com/site/cshorecode/>.

Wave runup was predicted within errors of about 20% for permeable slopes (Kobayashi et al. 2008) and for impermeable slopes (Kobayashi et al. 2013a). The wave reflection coefficient  $K_r$  is estimated from the computed cross-shore wave energy flux at the



still water shoreline for the case of no wave overtopping because CSHORE does not account for reflected waves. The measured values of  $K_r$  in the range of 0.1 to 0.4 were predicted within errors of about 40% (Kobayashi and de los Santos 2007, Kobayashi et al. 2008, Kobayashi et al. 2009). Wave overtopping rates were predicted within errors of about 100% for permeable slopes (Kobayashi and de los Santos 2007, Kobayashi et al. 2010a) and for impermeable levees (Kobayashi et al. 2010b). However, zero or small overtopping rates at the threshold of wave overtopping were predicted within a factor of 10 (Kobayashi et al. 2013a). Wave transmission coefficients were predicted within errors of 20% for submerged breakwaters (Kobayashi et al. 2007) but within errors of 100% for low-crested breakwaters with crests near the still water level (Kobayashi et al. 2013b, Garcia and Kobayashi 2015). Armor layer damage was predicted within errors of about 100% for high-crested breakwaters (Kobayashi et al. 2010a), reef breakwaters (Kobayashi et al. 2013b), and low-crested breakwaters (Garcia and Kobayashi 2015). CSHORE predicts the temporal changes of the armor layer profile and hydraulic response such as wave overtopping and transmission.

The cross-shore model CSHORE may not be very accurate but it is robust and versatile. Beach erosion and toe scour seaward of an intact rubble mound structure could be simulated using CSHORE but this option of CSHORE has never been verified. The present version of CSHORE could be extended to allow both beach erosion and structure damage progression. It is desirable to predict the consequences of a severe storm in realistic manners.

### 9.3 References

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## 10. CONCLUSIONS

The hydraulic input variables and recent formulas for hydraulic response (wave runup, reflection, overtopping, and transmission) and armor layer stability on rubble mound structures are presented and explained physically to facilitate their applications. The selected formulas made the most use of available data and are expected to be applicable to wide ranges of design situations. Nevertheless, these formulas are essentially limited to a trapezoidal structure with a horizontal crest and constant seaward and landward slopes located on a horizontal or uniformly-sloping fixed seabed. The need of hydraulic model testing is emphasized in order to optimize the entire structure design. The capabilities and limitations of the cross-shore numerical model CSHORE are summarized as an additional tool for the preliminary and detailed design of a coastal structure.

This report updates 5.2 Structure Hydraulic Response (45 pages) and 5.3 Rubble-Mound Structures Loading and Response (83 pages) in Chapter 5: Fundamentals of Design (316 pages) in Coastal Engineering Manual Part VI: Design of Coastal Project Elements (635 pages). The next phase for the completion of the engineering manual for the design of coastal rubble mound structures may be to outline the contents of this engineering manual on the basis of the 9-page summary explained in Section 1.1.