A PROBABILISTIC PREDICTION OF BEACH NOURISHMENT PROJECT LIFETIMES

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td></td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td></td>
<td>viii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td></td>
<td>ix</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Objective and Scope</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Summary of the Fenwick Island, Delaware Beach Nourishment Project</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>FACTORS CONTROLLING THE LIFETIME OF A BEACH NOURISHMENT PROJECT</td>
<td>6</td>
</tr>
<tr>
<td>2.1</td>
<td>Existing Coastal Processes</td>
<td>7</td>
</tr>
<tr>
<td>2.1.1</td>
<td>Wave Environment</td>
<td>7</td>
</tr>
<tr>
<td>2.1.2</td>
<td>Other Existing Factors</td>
<td>13</td>
</tr>
<tr>
<td>2.2</td>
<td>Beach Equilibrium Profiles</td>
<td>14</td>
</tr>
<tr>
<td>2.3</td>
<td>Beach Nourishment Project Sediment Quality</td>
<td>18</td>
</tr>
<tr>
<td>2.4</td>
<td>Size and Shape of the Beach Nourishment Project</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>PREDICTING LIFETIMES USING HISTORICAL RECESSION RATES AND VOLUMETRIC EROSIONAL RATES</td>
<td>45</td>
</tr>
<tr>
<td>3.1</td>
<td>Lifetime Based on Historical Shoreline Recession Rates</td>
<td>46</td>
</tr>
<tr>
<td>3.1.1</td>
<td>Calculations Assuming Identical Pre- and Post-Fill Equilibrium Profiles</td>
<td>48</td>
</tr>
</tbody>
</table>
3.1.2 Calculations Considering Differing Pre- and Post-Fill Equilibrium Profiles .................................... 62

3.2 Lifetime Based on Historical Volumetric Erosional Rates (i.e., Sediment Budget Analysis) .................. 67

3.3 Comparison of Results to DNREC’s Field Data from the Fenwick Island Project ............................... 72

4 USING A ONE-LINE NUMERICAL MODEL TO PREDICT THE PERFORMANCE OF A BEACH NOURISHMENT PROJECT ... 75

4.1 Outline of the One-Line Numerical Model ......................... 75

4.2 Application of the Numerical Model to the Fenwick Island Beach Nourishment Project .................. 77

5 PROBABILISTIC PREDICTION OF BEACH NOURISHMENT PROJECTS’ LIFETIMES USING AN ANALYTIC SOLUTION ... 95

5.1 Outline of the Probabilistic Prediction Model .................. 96

5.2 Generating Wave Time Series ................................ 104

5.2.1 AR(2) Model ........................................... 105

5.2.2 Alternative Numerical Procedure ......................... 117

5.3 Application of the Model to the Fenwick Island Beach Nourishment Project 118

6 SUMMARY AND CONCLUSIONS .................................. 141

Appendix

A DNREC SHORELINE POSITION DATA FOR FENWICK ISLAND BEACH NOURISHMENT PROJECT ........... 143

B DNREC VOLUME CHANGE DATA FOR FENWICK ISLAND BEACH NOURISHMENT PROJECT ................ 146

C CUMULATIVE DISTRIBUTION FUNCTIONS OF THE PERCENTAGE OF ORIGINAL BEACHFILL PROFILE WIDTH REMAINING AT VARIOUS TIMES: ESTIMATED FOR THE FENWICK ISLAND BEACH NOURISHMENT PROJECT 149

REFERENCES ..................................................... 171
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Delaware Coast and Beachfill Location</td>
<td>5</td>
</tr>
<tr>
<td>1.2</td>
<td>Close-up of Beachfill Location with Profile Lines</td>
<td>5</td>
</tr>
<tr>
<td>2.1</td>
<td>Seasonal Wave Rose Showing Wave Directions (Dalrymple and Mann, 1985)</td>
<td>8</td>
</tr>
<tr>
<td>2.2</td>
<td>Monthly Littoral Drift for WIS Station 65: (-) denotes littoral drift to the north</td>
<td>11</td>
</tr>
<tr>
<td>2.3</td>
<td>Monthly Littoral Drift for WIS Station 66: (-) denotes littoral drift to the north</td>
<td>12</td>
</tr>
<tr>
<td>2.4</td>
<td>Profile Adjustment</td>
<td>16</td>
</tr>
<tr>
<td>2.5</td>
<td>Example of Profile Adjustment (DNREC)</td>
<td>17</td>
</tr>
<tr>
<td>2.6</td>
<td>Profile 2500 ft. North of Stateline 11/11/85 (Dalrymple and Mann, 1985)</td>
<td>19</td>
</tr>
<tr>
<td>2.7</td>
<td>Profile 2500 ft. North of Stateline 11/26/88 (DNREC)</td>
<td>20</td>
</tr>
<tr>
<td>2.8</td>
<td>Seasonal Profile Adjustments (Dick and Dalrymple, 1983)</td>
<td>20</td>
</tr>
<tr>
<td>2.9</td>
<td>Beach Profile Factor, $A$, vs. Sediment Diameter, $D$ (Dean, 1983, 1990; Modified from Moore, 1982)</td>
<td>21</td>
</tr>
<tr>
<td>2.10</td>
<td>Equilibrium Beach Profiles for Sand Sizes of 0.2 mm and 0.6 mm</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>[ A(D=0.2\text{mm}) = 0.1m^3 \quad A(D=0.6\text{mm}) = 0.2m^3 ] (Dean, 1983)</td>
<td></td>
</tr>
<tr>
<td>2.11</td>
<td>Initial Volume Losses Due to Loss of Fines</td>
<td>23</td>
</tr>
<tr>
<td>2.12</td>
<td>Effect of Nourishment Material Scale Parameter, $A_F$, on Width of Resulting Dry Beach. Four Examples of Decreasing $A_F$. (Dean, 1983, 1990)</td>
<td>26</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>2.13</td>
<td>Beach Slope vs. Dimensionless Fall Velocity (Dalrymple, Thompson, 1976)</td>
<td>29</td>
</tr>
<tr>
<td>2.14</td>
<td>Plot of $K$ vs. $D$ (Dean, 1983, 1989; modified from Dean, 1978)</td>
<td>32</td>
</tr>
<tr>
<td>2.15</td>
<td>Definition Sketch (Dean, 1983, 1989)</td>
<td>32</td>
</tr>
<tr>
<td>2.16</td>
<td>Shoreline Diffusivity, $G$, as a Function of Breaking Wave Height</td>
<td>35</td>
</tr>
<tr>
<td>2.17</td>
<td>Evolution of an Initially Rectangular Beach Planform on an Otherwise Straight Beach (Dean, 1983)</td>
<td>36</td>
</tr>
<tr>
<td>2.18</td>
<td>Profile N10+00 August 22, 1988 (DNREC)</td>
<td>38</td>
</tr>
<tr>
<td>2.19</td>
<td>Profile N10+00 September 19, 1988 Showing Transport of Sand Northward from Ocean City (DNREC)</td>
<td>38</td>
</tr>
<tr>
<td>2.20</td>
<td>Example of Evolution of Initially Rectangular Nourished Beach Planform. Example for Project Length, $\ell$, of 4 miles and Effective Wave Height, $H$, of 2 feet and Initial Nourished Beach Width of 100 ft (Dean, 1983, 1989)</td>
<td>41</td>
</tr>
<tr>
<td>2.21</td>
<td>Fraction of Material Remaining in Front of Location Placed for Several Wave Heights, $H$, and Project Lengths, $\ell$. Effect of Longshore Transport (Dean, 1983)</td>
<td>41</td>
</tr>
<tr>
<td>2.22</td>
<td>Percentage of Material Remaining in Region Placed vs. the Parameter $1/\eta$ (Dean, 1983, 1989)</td>
<td>42</td>
</tr>
<tr>
<td>2.23</td>
<td>% of Material Remaining in Designated Area of Length, $\ell + 2\Delta$. Rectangular Beach Fill of Length, $\ell$ (Dean, 1983)</td>
<td>43</td>
</tr>
<tr>
<td>2.24</td>
<td>Calculated Evolution of a Beach Nourishment Project (Dean, 1983)</td>
<td>44</td>
</tr>
<tr>
<td>3.1</td>
<td>Fenwick Island Beach Nourishment Project: Progression of Shoreline Position with Time; BF10/88→4/89 (DNREC)</td>
<td>49</td>
</tr>
<tr>
<td>3.2</td>
<td>Fenwick Island Beach Nourishment Project: Progression of Shoreline Position with Time; 4/89→6/90 (DNREC)</td>
<td>50</td>
</tr>
<tr>
<td>3.3</td>
<td>Fenwick Island Beach Nourishment Project: Volume of Fill Sand Remaining as a Function of Time (DNREC)</td>
<td>51</td>
</tr>
</tbody>
</table>
3.4  Profile Showing Beach Recession and Accretion During First 1 1/2 Months ......................................................... 53
3.5  Relating Recession Rates to Volumetric Erosional Rates ................................................................. 54
3.6  Cut/Fill Cell Areas Along a Profile ................................................................. 56
3.7  Profile Showing Lack of Depth of Closure .................................................................................. 57
3.8  Beachfill Lifetime vs. Recession Rate ($d_c = 20, 25, 30, 35$ ft) ........................................ 61
3.9  Variation of Non-Dimensional Shoreline Advancement $\hat{\nu}_{\nu}^{N}$, with $A'$ and $V'$. Results Shown for $\frac{h}{B} = 2.0$ (Dean, 1990) ......................................................... 64
3.10 Variation of Non-Dimensional Shoreline Advancement $\hat{\nu}_{\nu}^{N}$, with $A'$ and $V'$. Results Shown for $\frac{h}{B} = 4.0$ (Dean, 1990) ......................................................... 65
3.11 Fenwick Island Project: Beachfill Lifetime vs. Volumetric Erosional Rate .............................................. 71
3.12 Fenwick Island Nourishment Project: Average Shoreline Advancement Beyond Pre-Project Position ................................................................. 73
4.1  Computational Scheme Used in Numerical Method (Dean, 1989) ........................................... 76
4.2  Probability Density Function Estimate of the Deep Water Wave Heights, $H_o$ for WIS Station 66 ................................................................. 81
4.3  Comparison of 1 Year Beachfill Planforms: Effective Wave Conditions as Constant vs. WIS Data Applied as a Time Series ......................................................... 84
4.4  Comparison of 1 Year Beachfill Planforms: Mean Wave Conditions as Constant vs. WIS Data Applied as a Time Series ......................................................... 85
4.5  One-Line Numerical Model Simulation Assuming Initially Rectangular Planform ($\Delta y_o=66.28$ ft) ................................................................. 87
4.6  One-Line Numerical Model Simulation Using the DNREC Survey Data As Initial Conditions ................................................................. 88
4.7  Comparison of the First Two Years of Shoreline Simulation Using the One-Line Numerical Model with the DNREC Field Survey Data ($X=0→-15000$) ......................................................... 90
4.8 Comparison of the First Two Years of Shoreline Simulation Using the One-Line Numerical Model with the DNREC Field Survey Data (X=0→-6000) ........................................... 91

4.9 One-Line Numerical Model: Beach Planform Which is Entirely Landward of its Pre-Project Location .................................................. 92

4.10 One-Line Numerical Model: Beach Planform Which is Only Partially Landward of its Pre-Project Location .................................. 94

5.1 Fenwick Island Beach Nourishment Project ST50+00: Shoreline Change When Exposed to the First Month of the WIS Data Time Series 101

5.2 Fenwick Island Beach Nourishment Project ST50+00: Shoreline Change When Exposed to Constant Wave Conditions ......................... 102

5.3 Correlation of Consecutive G Values (Time Lag of 3 Hours) in the WIS Data Set .................................................. 108

5.4 Correlation of G Values with a Time Lag of 6 Hours in the WIS Data Set 108

5.5 Probability Distribution Function of G Values Derived from the WIS Data at Station 66 .................................................. 111

5.6 Probability Distribution Function of the Power Transformed G Values Using shift = 0.1 and n=1/5 ........................................... 112

5.7 Check of Adequacy of the AR(2) Model Developed for Modelling WIS Station 66 Data by Establishing the Independence of the Individual Random Components ................................... 114

5.8 Comparison of the PDF of the Shoreline Diffusivity Parameters: WIS Calculated G Values and AR(2) Model Generated G Values ... 116

5.9 Comparison of the Probability Distributions of Shoreline Diffusivity Parameters, G, in the Time Series: WIS Data Set and the Borgman, Scheffner Model Simulation .......................... 119

5.10 Cumulative Distribution Function of Percentage of Original Planform Width Remaining (ST20+00 after 5 years) ......................... 121

5.11 Envelope of Possible Shoreline Locations after 1 Year for the Fenwick Island Beach Nourishment Project through 100 Simulations ... 128
5.12 Envelope of Possible Shoreline Locations after 5 Years for the Fenwick Island Beach Nourishment Project through 100 Simulations ... 129

5.13 Envelope of Possible Shoreline Locations after 15 Years for the Fenwick Island Beach Nourishment Project through 100 Simulations ... 130

5.14 Fraction of Original Beachfill Volume Remaining within the Project Limits with Varying Probability of Occurrence for the Fenwick Island Beach Nourishment Project (Subtracting “Negative” Volumes) ................................................................. 131

5.15 Fraction of Original Beachfill Volume Remaining within the Project Limits with Varying Probability of Occurrence for the Fenwick Island Beach Nourishment Project (Disregarding “Negative” Volumes) ................................................................. 132

5.16 Fraction of Original Beachfill Volume Remaining within the Project Limits with Varying Probability of Occurrence for the Fenwick Island Beach Nourishment Project with DNREC Field Survey Data (denoted with ◦’s) ................................................................. 133

5.17 Simulated Shoreline Planform after 1 Year for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width ................................................................. 135

5.18 Simulated Shoreline Planform after 5 Years for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width ................................................................. 136

5.19 Simulated Shoreline Planform after 10 Years for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width ................................................................. 137

5.20 Simulated Shoreline Planform after 15 Years for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width ................................................................. 138

5.21 Simulated Shoreline Planform after 20 Years for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width ................................................................. 139
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Annual Sediment Transport for Delaware-Maryland Coast (Dalrymple and Mann, 1985)</td>
<td>10</td>
</tr>
<tr>
<td>2.2</td>
<td>Volume Change Profiles 100–160 AD(10/88)–AD(11/26/88)</td>
<td>24</td>
</tr>
<tr>
<td>2.3</td>
<td>Sediment Data Prior and After Beach Nourishment (DNREC, DGS)</td>
<td>28</td>
</tr>
<tr>
<td>2.4</td>
<td>Relative Changes in Beach Slope Due to Changing Sand Size</td>
<td>29</td>
</tr>
<tr>
<td>2.5</td>
<td>Values of $G$ for Representative Breaking Wave Heights (Dean, 1983)</td>
<td>34</td>
</tr>
<tr>
<td>2.6</td>
<td>Comparison of Cumulative Percentage Losses from Rectangular and Tapered Fill Planforms (Dean, 1983)</td>
<td>43</td>
</tr>
<tr>
<td>3.1</td>
<td>Initial Beach Recessions for the Profiles of the Beachfill at Fenwick Island, 1988</td>
<td>47</td>
</tr>
<tr>
<td>3.2</td>
<td>Historical Recession Rates for Fenwick Island, Delaware</td>
<td>52</td>
</tr>
<tr>
<td>3.3</td>
<td>Depth of Closure Values for the Delaware Coast</td>
<td>58</td>
</tr>
<tr>
<td>3.4</td>
<td>Lifetime Estimates of the Fenwick Island Beachfill for Various Recession Rates and Depths of Closure</td>
<td>60</td>
</tr>
<tr>
<td>3.5</td>
<td>Historical Volumetric Erosional Rates for Fenwick Island, Delaware</td>
<td>68</td>
</tr>
<tr>
<td>3.6</td>
<td>Beachfill Lifetimes for Various Volumetric Erosional Rates</td>
<td>70</td>
</tr>
</tbody>
</table>
ABSTRACT

The purpose of this thesis is to investigate various methods of predicting “lifetimes” of beach nourishment projects. Here, the “lifetime” is defined as the time it takes any location within the nourishment project limits to erode landward of its pre-placement location. First, important parameters affecting a beachfill’s performance are briefly overviewed. Next, several models are used to simulate the evolution of beach nourishment projects. The first two models extrapolate historic shoreline recession and volumetric erosional rates, respectively, to estimate the lifetime. The third prediction model is a one-line numerical model developed by Dean (1989). The final model uses an analytic solution to the evolution of an initially rectangular beachfill planform to predict a lifetime. Many realistic time series of wave conditions are developed as input and applied individually to this analytic solution which enables a probability to be attached to each realization of the model simulation. An AR(2) and a numeric simulation model developed by Borgman and Scheffner (1990), were used to generate the wave time series as input, with Borgman and Scheffner’s model performing best. Each of the four beachfill evolution models were applied to the data available for the first two years after placement of the Fenwick Island, Delaware project completed in the fall of 1988.

The historic recessional and volumetric erosional models predict lifetimes of the Fenwick Island project at 14.8 and 15.4 years, respectively. Dean’s numerical model predicts that this same project will last 17 years. Finally, the analytic model is used, and deemed the most applicable because of the probability associated with its results. This probabilistic model predicts that it is 75% probable that the Fenwick Island project will last at least 12.1 years. Shoreline locations of varying probability of occurrence for this model are also presented.
Chapter 1

INTRODUCTION

1.1 Background

Beach erosion is a major concern for many coastal communities. With projected rises in sea level, the threat of loss of property to the invading oceans is ever-increasing. For many of these towns, the beach and dunes are the last line of defense against the severe storms originating from the sea. The beach is at the same time vital to many of these towns’ economies, which often rely on tourism whose basis is dependent on the availability of an adequate recreational beach. Therefore, coastal communities are faced with the task of implementing a plan to maintain and protect their beaches. It becomes the engineers’ duty to develop well-designed solutions to combat the erosional problems these towns face. Many questions need to be answered: What methods are most effective in combating beach erosion? Which methods are most economical? What environmental impacts will the shore protection methods have?

There are various types of structures which can be used as shoreline protection. These structures can be split into two general categories: “hard” structures and “soft” structures. “Hard” structures include groins, seawalls, offshore breakwaters, and revetments. “Soft” structures are synonymous with beach nourishment (beachfill).

“Hard” structures, in an attempt to protect a shoreline, interrupt the natural coastal processes. Groins prevent the alongshore drift of sand, breakwaters reduce the intensity of the nearshore wave climate (and thus also reducing the wave-induced currents), and seawalls and revetments fortify the eroding shoreline by placing a “solid” barrier between the water and the sand. Recently, the trend has been to avoid such structures in many situations. One reason is the adverse effects they may have on neighboring
beaches. For example, groins must necessarily starve downdrift beaches of sand thus causing them to erode. Seawalls prevent the on-offshore transfer of sand thus causing erosion in the toe of the structure. More importantly, however, because the natural processes are interrupted, increasing environmental concerns have drastically reduced the use of these “hard” structures. In fact, several coastal states have passed legislation prohibiting the use of such structures.

With such limitations on “hard” structures, and the realization of the importance of the recreational and protective functions of a beach, an increased interest in beach nourishment, a “soft” structure, for shore protection has developed. Beach nourishment is the placement of large quantities of compatible sand, usually on an eroding shoreline, to advance the shoreline seaward. The advantages gained by choosing beach nourishment are numerous. First, the new wider beach serves as shore protection. Second, since there are no interruptions of the natural coastal processes, the environmental effects of such a project are minimized. Third, the recreational benefits of the beach are retained. Finally, the project does not adversely affect any neighboring beaches; in fact, it often serves as a “feeder” beach for any of the neighboring downdrift beaches. Such is the case at the Fenwick Island, Delaware beach nourishment project. Fenwick Island, which is not at the end of a littoral drift cell, is a source area for the northerly littoral transport in Delaware, thus groins would not prove as helpful as they are in other Delaware sites, e.g., Rehoboth Beach.

For this study it is necessary to define what is meant by a beach nourishment project’s lifetime. Often data is presented in a misleading manner in order to establish either an overly-optimistic or overly-pessimistic view of a project’s success or failure. The question needed to be answered is: when is the sand actually lost? There are many methods used in determining this; some make more sense than others. In this report, the lifetime begins when the fill is placed and ends when the entire volume of fill sand is gone from the active beach system (i.e., when the beach returns to the state that existed before the beachfill was placed). Sand which moves downdrift along the coast to an area not originally part of the project area can be considered lost (especially by the community
which financed the beachfill project for their beaches alone). The sand is not really lost, but it no longer contributes to the length of beach for which it was meant. Sand which is transported offshore is not considered lost either. Here it is important to remember that the beach does not only include the beach above the Mean Water Level (MWL). This part of the beach which is above water and visible makes up only a fraction of the total beach profile. The offshore sand, out for approximately 10 meters, plays an active role in the evolution of the beach. This sand can be transported onshore during periods of beach accretion or be used to form bars which act as natural protection during winter storms.

1.2 Objective and Scope

The purpose of this study was to develop a method of determining how long a given beach nourishment project could be expected to last, i.e., its "lifetime." First, a brief overview of controlling factors affecting such lifetimes is presented. The study concentrates on factors which can be calculated and controlled in the design process. Next, several methods of predicting beach nourishment project lifetimes are discussed in detail. The methods used are historical recession and volumetric erosional rates, a one-line model, and an analytic solution to beach planform evolution. The analytic model is further modified in order that a degree of probability may be attached to a given lifetime prediction. After each model is presented and applied to values associated with the Fenwick Island, Delaware beach nourishment project of 1988, the results are compared to actual field data collected through a monitoring program performed by the Delaware Department of Natural Resources and Environmental Control (DNREC).

1.3 Summary of the Fenwick Island, Delaware Beach Nourishment Project

Here a brief overview of the beach nourishment project placed at Fenwick Island, Delaware in the Fall of 1988 is presented. This beach has been an area of high erosion (Dalrymple and Mann, 1985; Dick and Dalrymple, 1983) and the beach fill project was designed to restore the bathing beach for both shore protection and recreational purposes. The beachfill location is shown in Figure 1.1 and Figure 1.2. It should be noted here that the Fenwick Island beachfill was actually just a northward extension of the Ocean City,
Maryland beachfill project which took place throughout most of 1988. The total length of the combined beachfills, extending from the Ocean City Inlet north to Fenwick Island, was approximately 9.3 miles, containing 3.7 million yd$^3$ of fill sand. However, this study only concerns the approximately 333,500 yd$^3$ of sand placed on 6000 ft of beach extending northward from the Delaware/Maryland state line.
Figure 1.1: Delaware Coast and Beachfill Location

Figure 1.2: Close-up of Beachfill Location with Profile Lines
Chapter 2

FACTORS CONTROLLING THE LIFETIME OF A
OF A BEACH NOURISHMENT PROJECT

There are many factors which help determine the lifetime of a beach nourishment project. The emphasis here is on factors which can be controlled and manipulated in the design and placement of beachfill projects. Obviously, many factors, such as the wave climate (i.e., heights, frequencies and directions) and weather (i.e., storm frequency and severity) are important in determining how long a beachfill will last, but cannot be controlled by the design engineers.

Beachfills are used to help replace the sand that has for many years been eroded away by waves and currents. However, an important point to remember in analyzing a beachfill project is that the beachfill is not a cure to the erosional problem but is only an "aspirin" to give temporary relief to the eroding beach. The placed beachfill does very little, if anything, to change the erosive conditions that caused the need for the beachfill in the first place. The environmental factors, such as waves, storms and currents, remain the same and intact. The beachfill is not an attempt to reverse the erosional processes, but an attempt to go back in time to a wider beach which existed before years of erosion took place. Because the erosional problems are not cured the beachfill erodes just as the beach it relieved did. After many years of erosion, the beachfill will have eroded away, and the beach originally in place will once again exist. However, this beach will continue to erode unless another beachfill is placed. Thus, a cycle of placing beachfills is set in motion. The cycle cannot stop unless the beach is surrendered to the ocean or a better method of beach protection is developed; one in which the erosional processes are altered.

Although erosion will continue to take place after a beach is nourished, several factors can be controlled to extend the lifetime of a beach nourishment project. There are
three concepts which can be controlled during the design and placement of a beachfill which are known to influence the lifetime of the beachfill: beach equilibrium profiles, beachfill sediment quality, and shape/size of the beachfill. These three factors, when determined properly, can optimize the lifetime of a beachfill, which along with costs, are often the two major concerns in such a project. When possible, these factors will be discussed for the specific case of the Fenwick Island project.

2.1 Existing Coastal Processes

This section will briefly discuss a few of the “uncontrollable” factors that affect the expected lifetime of a beach nourishment project. Although these factors are essentially out of the hands of the designing engineer, they are nonetheless very important. After all, these factors about to be discussed are the ones, in general, that brought about the need for the beachfill in the first place.

2.1.1 Wave Environment

In most cases, beaches are attacked by waves which approach the beach at an oblique angle, as shown in Figure 2.1 (Dalrymple and Mann, 1985). As shown, in the spring (April to June) the waves generally approach the Delaware coast from the southeast. Because of the obliquity of the wave approach, the net erosional force due to the waves can be split into two components: cross-shore and longshore sediment transport. Cross-shore sediment transport involves moving sand on and offshore. It is believed that cross-shore transport of sand is basically temporary (during storms), and seasonal. The cross-shore sand transport is landward during the summer and fall, and seaward during the winter months along Delaware’s coast (Dick and Dalrymple, 1983). Thus sand moved in cross-shore transport is largely not lost from the beachfill area. Most sand lost from beachfills is due to the longshore sediment transport. This transport, also known as littoral drift, involves moving sand along the beach. Because the sand is removed from the beachfill area, it is considered lost. Deguchi and Sawaragi (1986) found that the amount of sand moved in the longshore direction surpasses the amount of sand transported in the cross-shore direction regardless of the shapes of the shorelines. Therefore, this phenomenon does
Figure 2.1: Seasonal Wave Rose Showing Wave Directions (Dalrymple and Mann, 1985)

not just apply to straight beaches, such as those that characterize most barrier island beaches, such as Fenwick Island.

For example, the waves represented by the wave rose in Figure 2.1 would cause a net transport of sand northward of Fenwick Island during the spring. Plots of actual littoral drift values for Fenwick Island are presented in Dalrymple and Mann (1985). As shown in their Table 2.1, the littoral drift varies greatly from year to year. The extreme variability is even more pronounced when the data obtained from CERC's WIS by evaluating monthly littoral drift is plotted as in Figure 2.2 and Figure 2.3. These two figures show how the monthly littoral drift will vary from year to year. (As a side note, when using these monthly littoral drift calculations it may be best to use the median values (connected by a line in the figures) as the most representative.) What is important to notice from Table 2.1 is not so much the actual values listed, but how much the transport rates vary from year to year for the same location. In fact, for two of the stations, the standard deviation was
greater than the mean transport rate. This shows that often the historical littoral drift rate for an area may or may not be indicative of what will happen to a beach during any given year. The number of storms, severity of storms, and directions from which the storms come, are all very important in determining the net littoral drift of an area for any given year. In fact, it is known that “northeasters,” severe storms that are most prevalent during the winter months, are particularly damaging to Delaware’s Atlantic coast. During such storms, large amounts of sand are transported southward into Maryland and very little of it ever returns to Delaware because of the existing sediment transport tendencies.

A brief description of the wave environment for Fenwick Island, DE will be presented here. A more detailed description can be found in Dick and Dalrymple (1983), Dalrymple and Mann (1985), and Jensen (Sep. 1983). The wave data used throughout this report was developed by the Coastal Engineering Research Center (CERC) during its Wave Information Study (WIS). Details of how the data was generated can be found in Jensen (Jan. 1983), and Brooks and Corson (1984). Fenwick Island is located in WIS Station 66. Averaging this stations data over the 20 year long record yields the following results:

\[
\begin{align*}
\text{Significant Wave Height} & \quad (H_s)_{\text{mean}} = 1.65\text{ft} \\
\text{Mean Wave Period} & \quad T_{\text{mean}} = 5.43\text{sec} \\
\text{Mean Wave Direction (from shore normal)} & \quad \theta_{\text{mean}} = 0.001^\circ
\end{align*}
\]

For sediment transport calculations \(H_{rms} = H_s/\sqrt{2}\) was used.

Dean (1989) has found that on long uninterrupted shorelines, the effect of wave direction is relatively unimportant to the long-term performance of a beach nourishment project. Of course, the exception is the case in which the project is adjacent to a structure which interferes with the longshore sediment transport. Note that the mean wave direction is approximately zero, indicating perhaps that this beach may be a nodal point for littoral transport since no general, constant direction is established.

As noted, in general, the largest waves occur during the winter, arriving from the northeast. The summer and fall months tend to be calm with waves arriving generally
Table 2.1: Annual Sediment Transport for Delaware-Maryland Coast (Dalrymple and Mann, 1985)

<table>
<thead>
<tr>
<th>Year</th>
<th>Station 65 Ocean City-Fenwick yd³/yr</th>
<th>Station 66 Fenwick-Rehoboth yd³/yr</th>
<th>Station 67 Rehoboth-Cape Henlopen yd³/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1956</td>
<td>-63720</td>
<td>272426</td>
<td>1131544</td>
</tr>
<tr>
<td>1957</td>
<td>-113292</td>
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<tr>
<td>1958</td>
<td>-176954</td>
<td>-450115</td>
<td>175900</td>
</tr>
<tr>
<td>1959</td>
<td>-172608</td>
<td>-175944</td>
<td>-43191</td>
</tr>
<tr>
<td>1960</td>
<td>-113479</td>
<td>67417</td>
<td>338391</td>
</tr>
<tr>
<td>1961</td>
<td>-96657</td>
<td>100242</td>
<td>245857</td>
</tr>
<tr>
<td>1962</td>
<td>-67646</td>
<td>584679</td>
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<tr>
<td>1963</td>
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<td>35547</td>
<td>153723</td>
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<tr>
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<td>229643</td>
</tr>
<tr>
<td>1965</td>
<td>-88862</td>
<td>83786</td>
<td>167369</td>
</tr>
<tr>
<td>1966</td>
<td>-191221</td>
<td>-20213</td>
<td>83561</td>
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<tr>
<td>1967</td>
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<td>-65215</td>
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<td>-225432</td>
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</tr>
<tr>
<td>1973</td>
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<td>-160474</td>
<td>-210039</td>
</tr>
<tr>
<td>1974</td>
<td>-175128</td>
<td>-36368</td>
<td>-48388</td>
</tr>
<tr>
<td>1975</td>
<td>-243095</td>
<td>136825</td>
<td>666079</td>
</tr>
<tr>
<td>Mean</td>
<td>-153000</td>
<td>56900</td>
<td>274200</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>84530</td>
<td>217610</td>
<td>336178</td>
</tr>
</tbody>
</table>

Note: The negative sign (−) denotes littoral drift to the north.
Figure 2.2: Monthly Littoral Drift for WIS Station 65: (·) denotes littoral drift to the north.
Figure 2.3: Monthly Littoral Drift for WIS Station 66: (-) denotes littoral drift to the north
from the south. Thus the driving force behind the seasonal sediment transport rates are established. Fenwick Island tends to lose large quantities of sand to the south into Ocean City, MD during the winter “northeasters,” and slowly gains sand back from the south during the summer and fall swells.

As shown, although not “designable,” a knowledge of the existing wave climate, which is responsible for the erosional patterns of the beach, can be very useful in planning a nourishment project. Knowing the wave environment and storm severity and frequency of a beach, enables a project to be designed to last the desired length of time. Likewise, knowing the littoral drift of the project location would help in determining the optimal location for the project. For example, placing a larger volume of fill on the updrift portion of the beach will allow this sand to act as feeder material for the downdrift project area as the natural littoral processes take place. Of course this is only possible when the magnitude and direction of the littoral drift are confidently known. Thus by knowing the local coastal processes that take place, a nourishment project can actually be planned to work with nature to optimize its effectiveness and lifetime.

2.1.2 Other Existing Factors

Besides wave environment, other factors can affect the design considerations of a beach nourishment project. These would include the coastal geography of the area and any structures built along the beach. These two factors can influence, or even dictate, the littoral drift to and from the project area. Examples of such situations are inlets (whether man-made or natural), groins, and seawalls. Often a beachfill must be placed where such structures are already in place. The effect that they will have on the fill project should be considered in the design process. For example, a terminal groin at the downdrift end of a nourishment project would decrease the rate at which sand leaves the project area, thus increasing its lifetime. An updrift groin would have an opposite effect. Similarly, a project placed downdrift of an inlet becomes starved of sand and its lifetime is decreased. Likewise, seawalls prevent the passage of sand from the berm to the foreshore section of the beach, again accelerating the erosion and decreasing the project’s life. So, although not directly a factor in the design of a beachfill, these things must be properly considered.
Several of these factors affect the beach nourishment project at Fenwick Island. Sediment transport to and from the north is restricted by the Indian River Inlet. Only a few miles of beach separating Fenwick Island and Indian River Inlet is available as a sand source for littoral transport southward to Fenwick Island during the winter storms. The only other source would be through an inlet sand-bypassing event. Directly south of the Indian River Inlet, the inlet’s stabilizing jetty will help establish a sand source by retaining sand transported northward from the nourishment project area. The beaches north of the inlet can only benefit from this build-up of sand south of the inlet from the Fenwick Island fill by way of the newly installed sand-bypassing system at the inlet, which is operated so as to maintain an adequate beach on the inlet’s south side in this process. The Ocean City Inlet, with its stabilizing jetties, located at the southern terminal end of the project will affect the project in the same manner, but in this instance littoral drift to and from the south will be restricted for this area.

2.2 Beach Equilibrium Profiles

The most basic assumption in exploring on-offshore sediment transport is the idea that the developing profile will eventually reach a stable condition under persistent wave attack. This stable situation implies both an equilibrium form and equilibrium position of the beach profile (Swart, 1976). This assumption is used later to help predict the lifetime of a beach nourishment project.

Dean (1983) empirically developed a usable relation between particle diameters and the beach’s equilibrium profile shape. The equation and variable definitions are given below:

\[ h(y) = Ay^m \approx By^{\frac{2}{3}}D^{\frac{1}{3}} \]  

(2.1)

where:

- \( h(y) \) = water depth
- \( y \) = distance offshore
- \( A \) = scale parameter roughly proportional to \( D^{\frac{1}{3}} \)
- \( m \) = exponent (usually \( \frac{2}{3} \))
- \( B \) = unknown factor which must be determined for each case
Dean has shown that \( h(y) = Ay^2 \) does in fact represent the profiles of beaches along the eastern coast of the United States. Notice that the only variable in the above equation is \( A \), which, as stated, is proportional to the diameter of the sand which makes up the beach. Thus, the sand size is responsible for the beach's equilibrium shape to a great extent. More discussion on the effects of sand size will be discussed in Section 2.3.

Now, using the assumption of equilibrium profiles, it can be deduced that sand of the same size as the original, placed on a beach during a nourishment project will adjust to a profile identical to the equilibrium profile which existed before the fill. Ideally, then, if the beachfill sand is placed in a fashion as to exactly match the equilibrium profile, no profile adjustment would need to take place. The beach would be widened to a point and remain that way in an equilibrium until erosion takes place. However, achieving this equilibrium profile during placement of the fill is just not very practical. It would be very difficult and expensive to place sand so as to form a profile exactly equal to the equilibrium profile predicted by Equation 2.1. Shaping the beach profile offshore, where the shape of the profile is critical, is not economical using present day beach nourishment techniques. For example, the technique used at Fenwick Island consisted basically of pumping the sand onshore from an offshore barge and then pushing the sand using heavy machinery, such as bulldozers, shaping the beach as best as possible. It is obvious that with these present techniques, there is not much that can be done about how the sand shapes itself offshore.

What is generally done, since precise shaping is not practical, is to place the sand in such a way to form a very wide but very steep beach. In this way, the entire volume of sand needed to achieve a certain beach width can be placed relatively quickly using bulldozers. Because the beach is made so steep at the offshore end, it is made much wider than is required by the original plans. Then, nature adjusts the profile to equilibrium. This is what is meant by profile adjustment during beach nourishment projects. This profile adjustment is shown in idealized form in Figure 2.4 and in actual profiles in Figure 2.5 taken at the Fenwick Island beachfill by DNREC. Notice how the fill placed at a steep
angle has moved seaward thus forming a milder-sloped beach, ideally evolving to the profile, \( h = Ay^3 \). So, while the initial placement of the fill at Fenwick Island resulted in an average of 91 ft of shoreline advancement, it is clear that a portion of the added beach width from the beachfill is lost during profile adjustment. Exact losses in beach width at the Fenwick Island project are given in Table 3.1 in Chapter 3 of this report. The average beach width loss at Fenwick during approximately the first 1-1/2 months was 28% with some profiles losing almost 60% of their added beach width. But, this process of profile readjustment was known to take place before the fill was even placed. By letting nature readjust the profiles time and, therefore, money is saved. The sand which transported offshore is not lost from the beachfill. Quite to the contrary, it is adding to the stability of the beach. The beach is becoming less steep and the offshore waters are becoming shallower thus causing waves to break further offshore, which adds protection to the beach, and increases the recreational value of the beach.

By placing the sand in a way that is as close to the equilibrium profile as possible, the readjustment process will be shortened diminishing any beach width losses that could
be deemed as project failure by a pessimistic evaluation. The readjustment process should take place rather rapidly, and the equilibrium profile should exist possibly within a year of the sand placement. Dean (1990) has noticed, however, that in most projects with standard construction techniques, the equilibrium shape is not reached until 5–6 years after initial placement. Figure 2.6 and Figure 2.7 show the equilibrium profile for one section of beach 2500 feet north of the Delaware/Maryland line. Two surveys taken during two separate years, but at the same time of the year, had to be compared to ensure the beach was at its equilibrium profile. Notice the similar shapes of both profiles (Figure 2.6 was taken November 11, 1985 and Figure 2.7 taken November 26, 1988 both by DNREC). Similar features include the flat bar approximately 50 feet wide, located about 2.5 feet deep and 50 feet offshore, giving way to a steep slope which then becomes milder, eventually to form another bar approximately 250 feet offshore. The similar shapes suggest the two profiles are equilibrium profiles for this section of beach during that particular time of year. As shown in Figure 2.7, because the sand was placed similar in shape to the equilibrium profile, very little profile adjustment needed to take place, and there was no net loss in
beach width from the time of placement.

A word here should be said as to why it was critical to compare profiles that were taken about the same time of the year. As shown by Dick and Dalrymple (1983), Delaware’s coast undergoes seasonal profile adjustments. Such adjustments are shown in Figure 2.8. The summer (or swell) profile is characterized by a wide berm, relatively steep foreshore, and a smooth offshore profile. The winter (or storm) profile, in contrast, has almost no berm. The sand moves offshore to form one or a series of sand bars paralleling the shoreline. The winter profile is developed by large storm waves that erode the berm and deposit the material offshore into sand bars, which then act as a type of natural storm protection. The gentle swell waves transport the sand back onshore, reshaping the berm into the summer profile. Theoretically, the sediment shifts seasonally from berm to bar so the volume of sand involved remains relatively constant over the profile. Thus, even though the beach width is greatly reduced during the winter, barely any sand is actually lost, except that due to longshore transport which will be discussed later. The seasonal profile adjustments, therefore, are of little concern when dealing with beachfills because they will occur whether the fill is there or not.

2.3 Beach Nourishment Project Sediment Quality

The response of beachfills to sediment quality is perhaps one of the most studied and therefore most understood areas of beach nourishment planning. In beachfill planning, the main concern when choosing a sediment is the sediment size. More specifically, how compatible is the borrow material with the native beach sediment. Presumably, if the fill material placed on the eroded beach is compatible with the energy of the coastal processes, it will be resorted along the profile but be retained within acceptable limits in the vicinity of the project area (Stauble, 1984).

The size of the beach sediment is intimately connected to the equilibrium profile of the beach calculated from $h(y) = Ay^{3/2}$. The parameter $A$ is approximately proportional to $D^{3}$, where $D$ is equal to the median sand grain diameter. Figure 2.9 (Dean, 1983, 1990) shows that, as the sediment size increases, $A$ also increases, and as $A$ increases, the equilibrium profile becomes steeper. Thus a finer sediment will be associated with a
Figure 2.6: Profile 2500 ft. North of Stateline 11/11/85 (Dalrymple and Mann, 1985)
Figure 2.7: Profile 2500 ft. North of Stateline 11/26/88 (DNREC)

Figure 2.8: Seasonal Profile Adjustments (Dick and Dalrymple, 1983)
milder sloped profile then one composed of coarse sediment. An example of this is shown in Figure 2.10 (Dean, 1983).

The stability of different sediment sizes varies; coarser sediment is more stable and it is able to withstand stronger erosional forces than finer sediment. Fine, well-sorted borrow material, such as that commonly found in bays, backshore dunes, or the bottom surface of the offshore zone, will generally respond rapidly to wave and current conditions, moving alongshore and offshore out of the project area. Thus, material that is finer than the native beach sediment is generally not suitable for use as beach fill. The erosional rate of finer material will always be greater than that of the native material. On the other hand, coarse, more poorly-sorted material, such as that found in alluvial channels, glacial outwash, and sometimes in offshore shoals, tends to provide more stable beach fills, although the resulting beach is not always ideal for recreational purposes (James, 1974). Therefore, it seems best to choose a borrow material which is at least as coarse as the native beach sand, or somewhat coarser.
Figure 2.10: Equilibrium Beach Profiles for Sand Sizes of 0.2 mm and 0.6 mm  
\[ A(D=0.2\text{mm}) = 0.1m^{\frac{3}{3}} \quad A(D=0.6\text{mm}) = 0.2m^{\frac{3}{3}} \] (Dean, 1983)

In past studies, it has been found that borrow which is very fine in comparison to the native material will migrate offshore to form a wide shoal (Silvester, 1978). This makes sense since any material smaller than existed on the beach in the first place is not now going to be able to withstand the erosional forces for some unknown reason. Therefore, shortly after beachfill placement (one or two months), it is found that a percentage of the material, sometimes up to 20%, is lost. However, after sampling the sediment, it is discovered that much of this lost sediment are fines (i.e., finer than the native material). Table 2.2 shows the initial volume losses for the Fenwick Island beachfill project. Areas were computed using a planimeter from profiles taken by DNREC. The areas were multiplied by distances between profiles to come up with volumes of sand lost. Examples of these areas are shown in Figure 2.11. The difference in area lost and area gained gives you the net change in area that took place during the first 1-1/2 months. As shown in Chapter 3 of the report, this amount of erosion is much higher (by a factor of two) than historical erosion rates for this area. Again, though, what is being done is that most of the sand being lost is very
fine in texture, and as the fill becomes properly sorted the erosion rate will decrease to a more appropriate level.

A final factor, which is not as obvious, is how the sediment size affects the width of the beachfill. Dean (1983) has shown that the coarser the nourishment material, the greater the dry beach width per unit volume of fill placed, due to the steeper profile for the coarser material. This is illustrated in Figure 2.12 (Dean, 1983, 1990). In this figure, $A_N$ and $A_F$ denote the native and fill profile sediment scale parameters, respectively, and $h_*$ is the depth of the active profile which is not important to this discussion. Figure 2.12 shows the effect of placing the same volume of three different sized sands. In Figure 2.12a, sand coarser than the native is used and a relatively wide beach $\Delta y$ is obtained. In Figure 2.12b, the same volume of sand of the same size as the native is used and the dry beach width gained is less. More of the same volume is required to fill out the milder sloped underwater profile. In Figure 2.12c, the placed sand is finer than the native and much of the sand is utilized in satisfying the milder sloped underwater profile requirements. In a limiting case,
Table 2.2: Volume Change Profiles 100–160 AD(10/88)–AD(11/26/88)

<table>
<thead>
<tr>
<th>Profile #</th>
<th>End Area (ft²)</th>
<th>Avg End Area (ft²)</th>
<th>X 500 ft (yd³)</th>
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<td>100</td>
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<td>160</td>
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<td>Total</td>
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<td>-9540</td>
</tr>
<tr>
<td>yd³</td>
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</table>
shown in Figure 2.12d, no dry beach is yielded with all the sand being used to satisfy the underwater requirements. These results are quantified in Dean (1983).

It is obvious that each size of sand chosen as the fill material has its advantages and disadvantages. From a totally engineering point of view, it makes sense to use as coarse a fill material as possible. The coarse material is more stable, and produces a wider beach for a given volume of sand. However, fills using coarse sand are poor as far as recreational purposes are concerned. The sand is rough (i.e., full of rocks, pebbles, shells, etc.) and the beach is steep, thus causing the water to become relatively deep close offshore. The deeper water, which in itself is more dangerous for swimming, also allows waves to break closer to shore thus decreasing the beach’s suitability for recreational use such as wading, bodyboarding, and surfing.

What is usually done is to choose a middle course. It is now generally felt that it is best to select a nourishment fill that is as similar as possible to the native material. Ideally, a borrow area should be chosen that supplies sand with identical properties to the sand already on the beach. By choosing similar sand, the equilibrium profile of the beach is retained. In using similar sand, and thus producing similar equilibrium profiles, it becomes much easier to predict the future behavior of the beachfill (i.e., lifetime, profile readjustment, etc.). The nourished beach can be expected to behave in a manner similar to the un-nourished beach before it. Thus, historical data, such as recession and erosion rates, as is done in Chapter 3, can be used in calculations involving this beach after equilibrium is achieved. However, it is not always possible to find a borrow area where it is economically feasible to pump sand with similar characteristics to the native material. In this case, the nourished material should be coarser than the native material and finer material should not be considered for stability reasons.

Sediment data for three of the profiles at the Fenwick Island project are given in Table 2.3, which lists mean sediment size along certain profiles prior to and after the beach nourishment (Delaware Geological Survey). Notice that the fill is coarser than the native material. On the average the fill sand is 0.13 mm larger in diameter than the native sand; an increase in size of about 30%. Station N25+00 also shows that the coarser fill sand
**Figure 2.12:** Effect of Nourishment Material Scale Parameter, $A_F$, on Width of Resulting Dry Beach. Four Examples of Decreasing $A_F$. (Dean, 1983, 1990)
has not yet spread offshore very far. At a depth of 12 feet, the sand size has remained relatively unchanged the first two months after the fill was placed. This will change as the sand is transported seaward to establish an equilibrium shape. The data also shows that the sand is generally coarser at the waterline, indicating that the fines are being transported from that region.

Since the beach fill is coarser than the native sand, the new equilibrium beach profile will be steeper. Figure 2.13 (Dalrymple and Thompson, 1976) can be used to develop a relationship between the relative change in sand size and the effect it has on the foreshore slope, as it shows the foreshore slope as a function of the dimensionless fall velocity of the sediment, or Dean number, $\Omega = H_o/wT$, where $H_o$ is a representative wave height, $T$, a representative period, and $w$, the fall velocity of the sediment. If it is assumed that the wave characteristics remain the same (before and after the fill is placed), then the influence of the coarser fill is to increase the fall velocity of the sediment, $w$ (the fall velocity is directly related to the grain size of the sediment, although empirically). Assuming that a straight line can be fit through the data in Figure 2.13, we have the beach face slope, $\theta$, defined as the acute angle formed by the beach profile and the water line, related to the fall velocity by

$$\theta = c\Omega^a$$  \hspace{1cm} (2.2)

Differentiating both sides and dividing by $\theta$ gives us an equation relating the change in $\theta$, $d\theta$, to the change in the fall velocity, $dw$.

$$\frac{d\theta}{\theta} = -a \frac{dw}{w}$$  \hspace{1cm} (2.3)

Table 2.4 shows the calculated relative change in beach slope for a given wave climate, due to the placement of the coarser fill sand. The sand size shown in the table was an average of the beach locations: berm crest, MHW, MSL, and MLW. The constant, $a$, in Equations 2.2 and 2.3 was found to be -0.725. Along profiles N5+00, N25+00, and N45+00, the average sand size was increased by 55, 18, and 21% respectively. The corresponding increase in equilibrium foreshore slope is 40, 13, and 15% respectively. The large change at N5+00 is due to the relatively small grain size initially in place.
Table 2.3: Sediment Data Prior and After Beach Nourishment (DNREC, DGS)

<table>
<thead>
<tr>
<th>Station</th>
<th>Location</th>
<th>Mean Sand Grain Diameter (mm)</th>
<th>Prior to Nourishment (9/20/88)</th>
<th>After Nourishment (12/1/88)</th>
<th>After Nourishment (1/17/89)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N5+00</td>
<td>base dune</td>
<td>.358</td>
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<td>.689</td>
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<td>.555</td>
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<td>.473</td>
<td>.547</td>
<td></td>
</tr>
<tr>
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<td>mid-beach</td>
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<td>.599</td>
<td>.514</td>
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<td></td>
<td>-12</td>
<td>.182</td>
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</table>
Figure 2.13: Beach Slope vs. Dimensionless Fall Velocity (Dalrymple, Thompson, 1976)

Table 2.4: Relative Changes in Beach Slope Due to Changing Sand Size

<table>
<thead>
<tr>
<th>STATION</th>
<th>Mean Beachface Sand Diameter, d (mm)</th>
<th>Corresponding Fall Velocity, w (cm/s)</th>
<th>Relative Change in Sand Size $dw/w$</th>
<th>Relative Change in Beach Slope $d\theta/\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP</td>
<td>BP</td>
<td>AP</td>
<td>5.8</td>
<td>9.0</td>
</tr>
<tr>
<td>AP</td>
<td>.397</td>
<td>.574</td>
<td>5.8</td>
<td>9.0</td>
</tr>
<tr>
<td>N25+00</td>
<td>N5+00</td>
<td>.475</td>
<td>.546</td>
<td>7.2</td>
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<tr>
<td>N45+00</td>
<td>N25+00</td>
<td>.451</td>
<td>.534</td>
<td>6.6</td>
</tr>
</tbody>
</table>

BP before placement 9/20/88
AP after placement 12/1/88

$H_o, T =$ constants

$a = -0.725$
2.4 Size and Shape of the Beach Nourishment Project

The size and shape of a beachfill also help determine its lifetime. The size of the beachfill is determined by the quantity of fill material. The term shape refers to the final shoreline configuration of the fill. Both of these factors influence longshore sand losses. Remember that the gradients in longshore transport are mostly responsible for "real" losses. This is why these two factors deserve consideration.

It is easy to see how the size of a beachfill will affect its lifetime. The greater volume of sand placed, the longer it will last. However, the greater the volume placed, the higher the costs. Therefore, determining an ideal volume of sand to place per foot of beach is more a matter of economics than engineering. It becomes a matter of comparing benefit/cost ratios, when government entities decide just how much money they are willing to pay to extend the lifetime of a beach.

How the shape of a beachfill affects its lifetime is not so obvious. The planform, or plan view, evolution of a beach nourishment project is very important in determining just how much sand tends to stay in the nourished area and how much leaves by way of longshore transport. The linearized equation of beach planform evolution was first developed and applied by Pelard Consideré in 1956. This equation is developed by combining two equations: the sediment transport equation and the equation of sediment conservation which are listed below.
Sediment Transport Equation:

\[ Q = \frac{K \cdot H_b^{\frac{3}{2}} \sqrt{\frac{g}{\kappa}}}{8 \cdot (1 - p)(s - 1)} \sin \frac{2(\beta - \alpha_b)}{2} \]  \hspace{1cm} (2.4)

where

\( Q \) = volumetric flow rate of sand
\( K \) = factor proportional to sediment size (shown in Figure 2.14)
\( H_b \) = breaking wave height
\( g \) = gravity
\( \kappa \) = spilling breaking wave proportionality factor (=0.78)
\( p \) = sediment porosity (~ 0.35 - 0.40)
\( s \) = sediment specific gravity (=2.65)
\( \beta \) = azimuth of an outward normal to the shoreline
\( \alpha_b \) = azimuth of the direction from which the breaking wave originates.

Figure 2.15 is a defining sketch of selected terms

Equation of Sediment Conservation:

\[ \frac{\partial y}{\partial t} + \frac{1}{(h_\ast + B)} \frac{dQ}{dx} = 0 \]  \hspace{1cm} (2.5)

where

\( y \) = distance on-offshore
\( t \) = time
\( h_\ast \) = breaking depth
\( B \) = berm height
\( (h_\ast + B) \) = depth of closure
\( Q \) = volumetric flow rate of sand
\( X \) = distance longshore

Figure 2.15 is a defining sketch of selected terms
Figure 2.14: Plot of $K$ vs. $D$ (Dean, 1983, 1989; modified from Dean, 1978)

Figure 2.15: Definition Sketch (Dean, 1983, 1989)
Pelnard–Consideré (1956) then combined these two equations to come up with the basic tool used in determining planform evolutions of beachfill projects. In the derivation of the following equation Pelnard–Consideré assumes small shoreline orientation angles with respect to the y-axis. Referring to Figure 2.15, in using this formulation one must assume that the angle between the y-axis ($\mu + 90^\circ$) and the shore normal ($\beta$) is small. Therefore the solution is not valid for use on highly irregular shorelines. The Delaware Atlantic coast is relatively straight and thus application of this equation should be valid for this case.

Combined Equations for Beach Planform Evolution:

$$\frac{\partial y}{\partial t} = C_o \frac{\partial^2 y}{\partial x^2}$$

(2.6)

where

$$G = \frac{K H_o^2 \sqrt{\frac{a}{s}}}{8(s-1)(1-p)(h_\ast + B)}$$

(2.7)

with the parameters $K$, $p$, $s$, etc. are defined as is Equation 2.4. Alternatively, in terms of deep water conditions, assuming straight and parallel contours seaward of the effects of a beach nourishment project, as derived by Dean (1989),

$$G = \frac{K H_o^{2.4} C_o^{1.2} g^{0.4} \cos^{1.2}(\beta_\circ - \alpha_\circ) \cos 2(\beta_\circ - \alpha_\circ)}{8(s-1)(1-p)C_\ast k^{0.4}(h_\ast + B) \cos(\beta_\circ - \alpha_\circ)}$$

(2.8)

where the subscript “o” denotes deep water conditions, $C_\circ$ is the wave celerity in the water depth $h_\ast$, $\alpha_\circ$ is the angle of wave approach at the water depth $h_\ast$. The ratio $C_\ast/C_o$ is

$$\frac{C_\ast}{C_o} = \tanh \left( \frac{2\pi h_\ast}{L} \right)$$

(2.9)

and by using Snell’s Law

$$\alpha_\ast = \beta_\circ - \sin^{-1} \left[ \frac{C_\ast}{C_o} \sin(\beta_\circ - \alpha_\circ) \right]$$

(2.10)

Equation 2.6 is a single equation describing the planform evolution for a shoreline which is initially out of equilibrium. The parameter $G$ may be considered as a “shoreline diffusivity”
Table 2.5: Values of $G$ for Representative Breaking Wave Heights (Dean, 1983)

<table>
<thead>
<tr>
<th>$H_b$ (ft)</th>
<th>Value of $G$ in $\text{ft}^2/\text{s}$</th>
<th>$\text{mi}^2/\text{yr}$</th>
<th>$\text{m}^2/\text{s}$</th>
<th>$\text{km}^2/\text{yr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0214</td>
<td>0.0242</td>
<td>0.00199</td>
<td>0.0626</td>
</tr>
<tr>
<td>2</td>
<td>2.121</td>
<td>0.137</td>
<td>0.0112</td>
<td>0.354</td>
</tr>
<tr>
<td>5</td>
<td>1.194</td>
<td>1.350</td>
<td>0.111</td>
<td>3.50</td>
</tr>
<tr>
<td>10</td>
<td>6.753</td>
<td>7.638</td>
<td>0.628</td>
<td>19.79</td>
</tr>
<tr>
<td>20</td>
<td>38.2</td>
<td>43.2</td>
<td>3.55</td>
<td>111.9</td>
</tr>
</tbody>
</table>

Note: In this table the following values have been employed: $K = 0.77$, $\kappa = 0.78$, $g = 32.2 \text{ ft}/\text{s}^2$, $s = 2.65$, $p = 0.35$, $h_a + B = 27 \text{ ft}$.

with dimensions of $(\text{length})^2/\text{time}$. It is recognized that the combined equation is in the form of the heat or diffusion equation for which a number of analytical solutions are available, some of which will be explored later in this section.

Dean (1983) has developed a table giving approximate values of the shoreline diffusivity, $G$. These values are given in Table 2.5 (Dean, 1983). A plot of this relation is given in Figure 2.16. It is seen that $G$ depends strongly on $H_b$, and secondarily on $(h_a + B)$ and $K$.

To use the equation for beach planform evolution, assume the initial beachfill planform, presented in Figure 2.17 (Dean, 1983), to be rectangular with a longshore length, $\ell$, and extending into the ocean a distance, $y$. This planform is an appropriate idealized configuration for the beachfill at Fenwick Island when considered as an extension to the project in Ocean City. The vertical axis could be considered the Ocean City Inlet jetty and the extreme right end of the rectangular fill could be considered the 6000 foot mark north of the Delaware/Maryland line. The analytic solution for this initial planform can be expressed in terms of two error functions as

$$y(x, t) = \frac{Y}{2} \left\{ \text{erf} \left[ \frac{\ell}{4\sqrt{Gt}} \left( \frac{2x}{\ell} + 1 \right) \right] - \text{erf} \left[ \frac{\ell}{4\sqrt{Gt}} \left( \frac{2x}{\ell} - 1 \right) \right] \right\}$$  \hspace{1cm} (2.11)

where the error function "erf()" is defined as

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2} du$$  \hspace{1cm} (2.12)
Figure 2.16: Shoreline Diffusivity, $G$, as a Function of Breaking Wave Height
Figure 2.17: Evolution of an Initially Rectangular Beach Planform on an Otherwise Straight Beach (Dean, 1983)

and $u$ is a dummy variable of integration. The solution is examined in Figure 2.17 where it is seen that initially the ends of the beach fill spread out and as the effects from the ends move toward the center, the planform distribution becomes more like a normal probability distribution. Therefore, just from planform evolution, since Fenwick Island is at the end of a beachfill, sand will be lost from the area and be transported northward along the Delaware Coast. Thus downdrift beaches benefit at the expense of Fenwick Island. So the location along the beachfill concerned also plays an important role in lifetime determination. Ends of beachfills erode more quickly at first due to longshore planform adjustment, but the effects of this diminish with time as the originally rectangular beachfill "flattens" and "smooths" out. The end erosion rates are eased by sand being transported downdrift from other parts of the beachfill. This is where Fenwick Island benefits from sand being transported longshore downdrift from the Ocean City beachfill. This is shown by examining Figure 2.18 and Figure 2.19, which show the August 22 and September 19 N10+00 profiles (located on the stateline), respectively, both of which were taken before
placement. By examining the southern portion of the Fenwick Island project, the DNREC has shown that even the earliest pre-project surveys show the effect of northerly transport of fill from Ocean City. In some cases, a three foot vertical accretion in the berm area can be noted, as shown in the figures. In summary, the planform not only evolves to a normal distribution but also tends to move along the coast in the downdrift direction.

In examining the solution to the developed diffusion equation, it is seen that the important parameter is

$$\eta \equiv \frac{\ell}{\sqrt{Gt}}$$

(2.13)

where \(\ell\) is the length of the rectangular planform and \(G\) is the parameter in the diffusion equation as discussed earlier. Thus two beach planforms with the same \((\frac{\ell}{\sqrt{Gt}})\) quantity, also have the same planform evolution. Examining this further, if two nourishment projects are exposed to the same wave climate, which determines the shoreline diffusivity, \(G\), but have different lengths, then the project with the greater length would tend to last longer. In fact, the longevity of a project varies as the square of the length, thus if Project A with a shoreline length of one mile “loses” 50% of its material in 2 years, Project B subjected to the same wave climate but with a length of 4 miles would be expected to lose 50% of its material from the region where it was placed in a period of 32 years. Thus the project length is very significant to its performance. This makes sense since if a beachfill is very long, it would take a relatively long time for sand located at the updrift end of the project to be transported longshore out of the project area, which would be at the downdrift end of the project. In a sense, the lifetime of the project is determined, as far as longshore transport goes, by how long it takes sand to travel from one end of the project (the updrift end) to the other (the downdrift end). The basis for this is that sand moving from one area of the project to another stretch of beach still in the project area is not yet lost to the designated beachfill.

As far as the Fenwick Island project is concerned, the 6000 foot length of the project is not really relevant, as the Fenwick beachfill is an extension of the Ocean City beachfill. In this calculation, the combined length of the project in Fenwick and Ocean City must be used and then the lifetime would be for this entire area. So as far as Fenwick Island
Figure 2.18: Profile N10+00 August 22, 1988 (DNREC)

Figure 2.19: Profile N10+00 September 19, 1988 Showing Transport of Sand Northward from Ocean City (DNREC)
is concerned, sand leaves Fenwick by spreading out northward along the Delaware coast, but enters Fenwick from Ocean City. The difficulty in this calculation is that the littoral drift nodal point is sometimes located in this area, so sand may travel north, south, or both directions in any given year.

The next obvious case to consider is when two projects of the same length but located in different wave climates are compared. It is seen that the “activity” varies with the wave height to the $\frac{3}{2}$ power. Thus if Project A is located where the wave height is 4 feet and loses 50% of its material in a period of 2 years, then Project B with a similarly configured beach planform located where the wave height is 1 foot would be expected to last a period of 64 years. Thus, the life of a beachfill project is even more dependent on the wave climate than its length. As far as beachfill projects along the Delaware Coast, or even much of the U.S. Atlantic coast, for that matter, are concerned, variations in wave climate are not important. The wave climate along Delaware’s coast can be considered basically a constant for any given time of year. From October to March, wave height off the coast of Delaware averages 1.2 meters (3.9 feet), and 0.3 meters (1.0 feet) for the remainder of the year (Polis and Kupferman, 1973). Ocean waves under severe storm conditions have been estimated to be nine meters high in the surf zone (USACE, 1956). However, since the offshore wave climate is basically constant over the entire Delaware coast, a beachfill placed on an Atlantic beach in southern Delaware would be expected to last just as long as one placed on an Atlantic beach in northern Delaware, keeping all other factors constant. However, all other factors along Delaware’s Atlantic coast are not constants. A major variable is the offshore topography. The northern Delaware beaches, e.g., Rehoboth Beach, are shielded from severe wave activity by large offshore shoals which refract, reflect, dissipate much of the wave energy, thus decreasing the magnitude of the erosional forces acting upon them, via Equation 2.4. So, in general, a given beachfill will be expected to last longer in Rehoboth than it would in southern Delaware, e.g., Fenwick Island, where the nearshore wave conditions are capable of producing greater erosional forces. It is also likely that the wave climate for Delaware, will remain unchanged with time in the near future, barring any unforeseen consequences such as those from the rising
sea level due to the “greenhouse” effect. Therefore, the Fenwick Island beach nourishment project should provide a representative behavior from which other beachfill projects on the Delaware Atlantic coast could be modeled.

Figure 2.20 presents a specific example of beach evolution and Figure 2.21 presents results in terms of the proportion of sediment remaining in front of the beach segment where it was placed as a function of time (Dean, 1983). These results are presented for several examples of combinations of wave height and project length. The analytical expressions for these figures are developed in Dean (1983). Figure 2.20 shows that since Fenwick Island is at the end of a beachfill project, it loses much of the added width (about 50%) almost immediately to evolution. However, after the initial adjustments, the ends of projects become stabilized, with little change in beach width from year to year. Figure 2.21 does not really pertain to the Fenwick Island beachfill because it is just an extension of a larger project. However, the figure could be used for future projects in Delaware using a representative wave height found to be between 1-4 feet (depending on the season). Figure 2.22 shows a similar correlation of percentage of material remaining versus the parameter $1/\eta \equiv \sqrt{Gt}/\ell$ (Dean, 1983, 1989).

One final factor associated with the beachfill’s shape that may affect the project’s lifetime is the effects of the ends of the fill. One approach to retain the sand within the project boundaries as long as practical is to install retaining or stabilization structures near the end of the fill. This, however, is expensive and is known to induce erosion on the downdrift sides of such structures. A second approach is to simply set back the limits of the fill from the project boundaries with the understanding that the sand would soon “spread out.” So initially, some areas of the project would not receive any sand, knowing they would be “naturally” nourished due to the beach planform evolution. Figure 2.23 (Dean, 1983) presents results for relative end set-backs $\Delta/\ell = 0, 0.2, 0.5$. As shown, the effects of set-back are greatest early in the project life ($1/\eta \equiv \sqrt{Gt}/\ell = 0.6 - 0.8$) when the sand would redistribute itself to an area still within the project limits.

A third approach is to taper the ends of the beachfill, as shown in Figure 2.24b (Dean, 1983). Basing the longevity on the retention of sand within the placed planform,
Figure 2.20: Example of Evolution of Initially Rectangular Nourished Beach Planform. Example for Project Length, $\ell$, of 4 miles and Effective Wave Height, $H$, of 2 feet and Initial Nourished Beach Width of 100 ft (Dean, 1983, 1989)

Figure 2.21: Fraction of Material Remaining in Front of Location Placed for Several Wave Heights, $H$, and Project Lengths, $\ell$. Effect of Longshore Transport (Dean, 1983)
tapered-end planforms have a substantially greater longevity than rectangular planforms. As shown, the evolution of the planform is such that the early changes are the most extreme. The tapered end planform, which approximates the evolved rectangular planform at a later stage, has early evolution stages approximate to that of the later (and less dramatically changing) stages of a rectangular fill. Table 2.6, compiled by Dean (1983), summarizes the cumulative losses from the region placed over the first five years. It is seen that tapered end-fills have reduced the end losses by about 33%.
Figure 2.23: % of Material Remaining in Designated Area of Length, \( \ell + 2\Delta \). Rectangular Beach Fill of Length, \( \ell \) (Dean, 1983)

Table 2.6: Comparison of Cumulative Percentage Losses from Rectangular and Tapered Fill Planforms (Dean, 1983)

<table>
<thead>
<tr>
<th>Years After Placement</th>
<th>Cumulative % Losses With Rectangular Planform</th>
<th>Rectangular Planform With Triangular Fillets</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>5.7</td>
<td>2.4</td>
</tr>
<tr>
<td>2</td>
<td>9.5</td>
<td>4.6</td>
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<td>3</td>
<td>11.8</td>
<td>6.6</td>
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<tr>
<td>4</td>
<td>13.8</td>
<td>8.3</td>
</tr>
<tr>
<td>5</td>
<td>15.5</td>
<td>9.8</td>
</tr>
</tbody>
</table>

\((G = 0.02\text{ft}^2/\text{sec}, \ell = 3\text{miles}, Y = 55\text{ft})\)
Figure 2.24: Calculated Evolution of a Beach Nourishment Project (Dean, 1983)
Chapter 3

PREDICTING LIFETIMES USING HISTORICAL RECESSION RATES AND VOLUMETRIC EROSIONAL RATES

Here, the first method of computing beach nourishment project lifetimes is presented. In these calculations, it was necessary to use data from historical records. Much of these data came from three past reports: *A Coastal Engineering Assessment of Fenwick Island, Delaware* (Dalrymple and Mann, 1985), *Coastal Engineering Study of Bethany Beach, Delaware* (Dick and Dalrymple, 1983), and *Sediment Budget and Sand Bypassing System Parameters for Delaware’s Atlantic Coast* (Coastal and Offshore Engineering and Research, Inc. (COER), 1983). Some of the data in these reports was in turn compiled by the U.S. Army Corps of Engineers. The only recent data (within the past 3 years) came from the DNREC. They supplied beach profiles for every 100 ft of the 6000 ft beachfill at Fenwick Island for times immediately before the sand placement and immediately after (October 1988). The DNREC also obtained profiles for every 500 feet of the project approximately every 3 months thereafter (the most recent available survey at the time this report was produced being June 1990). The recent data obtained by the DNREC, thus, only contains information for the first $1\frac{3}{4}$ years of the life of the project. Therefore, it can be assumed that some of this data would not be indicative of the beachfill’s entire lifespan. The beachfill in all probability is still adjusting to reach an equilibrium. Additional surveys will be taken in the future by DNREC on a semi-annual basis.

Using historical records is very convenient, especially when dealing with values such as recession and volumetric erosion rates. Values like these, when averaged over many years, take into account many factors which could be overlooked or omitted when values are only calculated for one specific time, say the most recent. Historical records take into account factors such as storm frequency, changing longshore littoral drift rates,
and wave data. By averaging historical data for, say, beach recession rates, all of these other factors are also "averaged-in," and become a value that better represents what will happen to this beach in the future than would a value from only the most recent year. For instance, if only the most recent recession rate is used so that only the most up-to-date data is being used, then the calculation is done assuming that every year in the future will be identical to the one used. It would be assumed that the severity and frequency of storms remained the same, the wave magnitude and direction remained the same, etc. It would be possible to use only the most recent recession rate and not make all of these assumptions, but this would involve many more calculations. More specifically, calculations of the effects of all these factors, such as littoral drift and wave data, would have to be incorporated. So therefore, it is determined that historical data will in fact be relatively accurate in determining the beachfill's lifetime when compared with any other available methods.

However, even though historical records were used, only the most recent data was chosen. Records that were very old, say over 50 years, were avoided. The reason for this is that it is believed that the erosion rate along the Delaware coast is presently higher than it has been in the past (COER, 1983). Therefore, averaging data from long ago into our calculations would produce an erosion rate that is too low in reality, thus causing the lifetime calculations to be too high.

3.1 Lifetime Based on Historical Shoreline Recession Rates

In this section, the lifetime of the beachfill is estimated on the beach recession rates (i.e., how fast the width of the beach decreases with time). As stated in Chapter 2, the beach recession immediately after placement is ignored. The recession at this time is for the most part related to the beach changing shape towards its equilibrium profile. These initial beach recessions are shown in Table 3.1, which shows how far the beach accreted (widened) when the fill was first placed (BP-AP), how far the beach receded in the first month after placement (AP-Nov. 26), how much wider the beach was as of November 26 than before the beach fill was placed (BP-Nov 26) and, finally, the percentage of the added beach width that was lost in the first month (AP-Nov 26). The profile numbers are
Table 3.1: Initial Beach Recessions for the Profiles of the Beachfill at Fenwick Island, 1988

<table>
<thead>
<tr>
<th>Profile</th>
<th>Beach Accretion Due to Sand Placement (BP-AP) (ft)</th>
<th>Beach Recession Due to Initial Profile Adjustment (AP - Nov. 26) (ft)</th>
<th>Net Change (BP - Nov 26) ft</th>
<th>% Width Loss Due to Adjustment (AP - Nov 26)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>70.21</td>
<td>-39.06</td>
<td>31.15</td>
<td>-55.6%</td>
</tr>
<tr>
<td>105</td>
<td>99.67</td>
<td>-60.94</td>
<td>38.73</td>
<td>-61.1%</td>
</tr>
<tr>
<td>110</td>
<td>102.16</td>
<td>-48.44</td>
<td>53.72</td>
<td>-47.4%</td>
</tr>
<tr>
<td>115</td>
<td>91.94</td>
<td>-21.88</td>
<td>70.06</td>
<td>-23.8%</td>
</tr>
<tr>
<td>120</td>
<td>98.67</td>
<td>-34.38</td>
<td>64.29</td>
<td>-34.8%</td>
</tr>
<tr>
<td>125</td>
<td>42.92</td>
<td>3.13</td>
<td>46.05</td>
<td>7.3%</td>
</tr>
<tr>
<td>130</td>
<td>96.57</td>
<td>-31.25</td>
<td>65.32</td>
<td>-32.4%</td>
</tr>
<tr>
<td>135</td>
<td>112.00</td>
<td>-37.50</td>
<td>74.50</td>
<td>-33.5%</td>
</tr>
<tr>
<td>140</td>
<td>106.29</td>
<td>-28.13</td>
<td>78.16</td>
<td>-26.5%</td>
</tr>
<tr>
<td>145</td>
<td>67.33</td>
<td>12.50</td>
<td>79.83</td>
<td>18.6%</td>
</tr>
<tr>
<td>150</td>
<td>107.00</td>
<td>-17.19</td>
<td>89.81</td>
<td>-16.1%</td>
</tr>
<tr>
<td>155</td>
<td>114.53</td>
<td>-43.75</td>
<td>70.78</td>
<td>-38.2%</td>
</tr>
<tr>
<td>160</td>
<td>83.25</td>
<td>-18.75</td>
<td>64.50</td>
<td>-22.5%</td>
</tr>
<tr>
<td>AVG</td>
<td>92.33</td>
<td>-35.82</td>
<td>63.61</td>
<td>-28.2%</td>
</tr>
</tbody>
</table>

BP = before placement of fill Sept-Oct 1988
AP = immediately after placement of fill Oct 1988
Nov. 26, 1988 = first post-construction survey of the fill.

for the beach profiles every 500 ft, with Profile 100 at the DE/MD line, Profile 105 500 ft north of the DE/MD line, ..., up to Profile 160 which is the beach profile 6000 ft north of the DE/MD line. The changes in beach width were measured from the profiles provided by the DNREC. Example distances are shown in Figure 3.4. Note that the 333,500 yd$^3$ of fill added, on average, 91.33 ft to the width of the beach. This is in very good accordance with Dalrymple and Mann (1985) which approximates that 341,000 yd$^3$ placed on this 6000 ft of beach would widen the beach 100 ft, in the profiles' design stage.

Also note that Table 3.1 shows that on the average, the beach narrowed by 35.82 ft the first $1\frac{1}{2}$ months, or 28.2% of the added width was lost from the beachfill. This number, -35.82 ft/month (430 ft/yr), is obviously not a value that is indicative of the
transformation of the beach throughout its life. Most calculations of the recession rate for Fenwick Island tend to be between 1.5–3.0 ft/yr. This shows that historical data must be used. Using the recession rate of the first 1\frac{1}{2} months would produce a beachfill lifetime of only a few months which is definitely not the case. The fact that the large recession rates are due to the profile readjusting is also shown in Fig 3.4. Note that the sand is not lost but has been transported by the waves from the beach berm to a position slightly offshore. Volumetric calculations in fact show that only 3% of the fill left the nourishment area during this time. Refer to Appendix A and Appendix B, for the Fenwick Island post-project data of shoreline change, and volume changes, respectively, which have provided through the DNREC. This data is presented in graphical form in Figures 3.1,3.2 and 3.3, respectively. It is important to note the vertical exaggeration introduced by the choice of the axes scales in Figures 3.1 and 3.2. The shorelines in actuality appear much straighter than those shown in the figure. The different scales were chosen so as to make the changes in shoreline position that are taking place more clear.

Table 3.2 lists historical recession rates determined in past studies. Notice that the estimated recession rates from the studies vary greatly. Some values are almost three times the value found in other studies. In fact, notice that the standard deviation is greater than the value itself in some of the studies. Dalrymple and Mann (1985) found that between May 1977 and June 1979 the shoreline was eroding at 31.5 ft/yr (standard deviation = 18.6 ft/yr)! However, most values seem to be in the neighborhood of a recession rate between 1.5 and 4.0 ft/yr. An average value of 2.36 ft/yr seems reasonable. It may be more realistic, however, to use a slightly larger number say, 2.75–3.0 ft/yr, since it is known that the erosion rate of Delaware’s coast has accelerated recently (approximately the last 27 years) (Dalrymple and Mann, 1985).

3.1.1 Calculations Assuming Identical Pre- and Post-Fill Equilibrium Profiles

To determine the lifetime of a beachfill the concept of an equilibrium profile will be used. The general idea is to assume that the beach will maintain its equilibrium profile, throughout the lifetime of the fill, no matter how much erosion or accretion takes place. The method will assume that the equilibrium profiles both prior to and after the fill is
Figure 3.1: Fenwick Island Beach Nourishment Project: Progression of Shoreline Position with Time; BF 10/88→4/89 (DNREC)
**Figure 3.2:** Fenwick Island Beach Nourishment Project: Progression of Shoreline Position with Time; 4/89→6/90 (DNREC)
Figure 3.3: Fenwick Island Beach Nourishment Project: Volume of Fill Sand Remaining as a Function of Time (DNREC)
Table 3.2: Historical Recession Rates for Fenwick Island, Delaware

<table>
<thead>
<tr>
<th>Source</th>
<th>Means of Deriving Rate</th>
<th>Years Averaged</th>
<th>Area Averaged</th>
<th>Recession Rate (ft/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mann</td>
<td>Aerial photography</td>
<td>1938-1979</td>
<td>1880 ft south of DE/MD line to 8695 ft North of DE/MD Line</td>
<td>1.7 Standard Deviation (S.D. = 0.8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hayden</td>
<td></td>
<td></td>
<td></td>
<td>1.9 (S.D. = 3.9)</td>
</tr>
<tr>
<td>Mann</td>
<td>Profiles (Depth of Closure $(d_e=30$ ft)</td>
<td></td>
<td></td>
<td>3.2 (S.D. = 3.3)</td>
</tr>
<tr>
<td>Mann</td>
<td>Profiles $(d_e=25$ ft)</td>
<td></td>
<td></td>
<td>3.8</td>
</tr>
<tr>
<td>Mann</td>
<td>Conservative</td>
<td></td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>Mann</td>
<td>Average of Methods</td>
<td></td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>Dick</td>
<td></td>
<td>1954-1964</td>
<td>Indian River Inlet to DE/MD Line</td>
<td>2.0</td>
</tr>
<tr>
<td>COER</td>
<td>Profiles $(d_e=35$ ft)</td>
<td>1964-1983</td>
<td>$1 \frac{1}{4}$ mi north of DE/MD Line</td>
<td>1.43</td>
</tr>
<tr>
<td>USACE</td>
<td></td>
<td>1853-1983</td>
<td>South Delaware Average $= 1.64-2.95$</td>
<td>2.36</td>
</tr>
</tbody>
</table>
Figure 3.4: Profile Showing Beach Recession and Accretion During First 1 1/2 Months placed are identical, thus assuming that the pre- and post-project sand sizes are identical (see Section 2.2 and 2.3). As indicated in Table 2.3 this is not the case. However, these assumptions enable a quick, easy method of determining how much added beach width can be expected once the beach has reached equilibrium. Later, equations developed by Dean (1990) will be used to estimate the added equilibrium beach width which does, in fact, consider the differing pre- and post-project equilibrium profiles. For now, in this section, by using such an assumption, the beach recession rate is related to the volumetric loss of sand as shown in Figure 3.5.

The definitions of the three terms are as follows:

\[ B = \text{average berm height after the fill has been placed} \]
\[ d_c = \text{depth of closure} \]
\[ R = \text{beach recession rate} \]

If the beach maintains its equilibrium profile throughout the erosion process, which
is a fairly good assumption, the two shaded regions in Figure 3.5 are equal in area. Therefore, if the entire beach of average berm height, $B$, and depth of closure, $d_c$, recedes a uniform amount, $R$, then each two-dimensional profile, as shown in Figure 3.5, is in essence losing area equal to either one of the two shaded regions. Then by multiplying this average area loss by the length of the beachfill, a volume is attained, which represents the amount of sand lost over the entire beachfill.

Now an outline of the calculation procedure will be presented:

Step (1): Determine: $V = \text{total volume of fill placed (yd}^3\text{)}$

$$L = \text{length of beachfill (ft)}$$

Step (2): Calculate: $A_F = \frac{V}{L}$

$= \text{area of fill per unit length of beach (yd}^3/\text{ft)}$
Step (3): Calculate: \[ A_E = (B + d_c) \times 1 \text{ft} \]

\[ = \text{area of fill lost (or gained)} \]

\[ \text{due to 1 foot recession (or accretion) (ft}^2) \]

Step (4): Calculate: \[ R_{TOTAL} = A_F/A_E \]

\[ = \text{total possible shoreline advancement due to the fill,} \]

\[ \text{assuming total profile readjustment (ft)} \]

Step (5): Calculate: Lifetime of Beachfill = \[ R_{TOTAL}/R_{HIST} \] (yr)

where \[ R_{HIST} \] is the annual shoreline recession

Values from the Fenwick Island beach nourishment project for Step (1) were supplied by the DNREC. The total volume of sand placed was calculated by comparing the profiles immediately before and immediately after placement of the beachfill. The net area added to each profile by the beachfill was calculated using an ISPR format computer program. The program basically calculated the areas of all the cut/fill cells along the length of the profile. Then all the cut/fill cell areas are added to obtain a net area change for that profile. Figure 3.6 shows an example of the cut/fill cells, of which the areas were computed. Actually, the areas are thought of as volume of sand placed per foot of beach, i.e., yd^3/ft, which has units identical to that of an area. The volume of sand placed between two adjacent profiles is then computed by averaging the two net changes of area due to the beachfill of the two profiles and multiplying it by the total distance between them. Finally, the volumes of sand placed between all adjacent profiles are summed to obtain the total volume of fill placed. This survey data was provided by the DNREC.

Now, for the Fenwick Island beach nourishment project:

\[ L = 6000 \text{ ft} \]

\[ V = 333,479 \text{ yd}^3 = 9,003,933 \text{ ft}^3 \]
Figure 3.6: Cut/Fill Cell Areas Along a Profile

In Step (2), $A_F$, the area of fill was computed. $A_F$ is the volume of sand placed by the beachfill per foot of beach.

Then, for the Fenwick Island beach nourishment project:

$$A_F = \frac{V}{L} = \frac{333,479 \text{ yd}^3}{6000 \text{ ft}} = \frac{1501 \text{ ft}^3}{\text{ft of beach}} = 55.58 \text{ yd}^3/\text{ft of beach}$$

So for every foot of beach in the beachfill area, an average of 55.58 yd$^3$ of sand was placed.

In Step (3), $A_E$, the area of fill lost due to one foot of beach recession, was calculated. This calculation makes use of the assumption that the beach maintains its equilibrium profile, as stated before. This assumption greatly simplifies the calculations, since the area of beach lost due to erosion, which actually looks something like the curved shaded region in Figure 3.5, is now equal to the much more easily computed rectangular region shown also in the figure. With this assumption, one can think of one foot of recession as
picking the entire beach up, while retaining its shape, and moving it landward one foot. Therefore, $A_E$ is the amount of area lost along a profile when the beach retreats one foot.

Note that $A_E$ is dependent on the depth of closure $d_c$ of the beach. The depth of closure is the depth at which there is no active sediment transport. At this depth, all beach profiles taken at different times at a site should coalesce. Looking at Figure 3.7 it is obvious that the DNREC did not extend its profiles to the depth of closure. The separate profile lines, which as shown extend to a depth of approximately 15 feet, do not join and become one. Therefore, in determining the volume of sand between the two profile lines (i.e., areas), the best that can be done is to estimate the profile lines out to a predicted depth of closure. The DNREC calculations did not do this but simply cut off the profiles at a point 480 feet from a predetermined reference point. Noting that on most of the profiles, the lines are only slightly separated at this distance, and will probably only get closer the deeper the surveys go. These calculations are probably within a reasonable error of the actual values. Later profiles surveyed by the DNREC extended further offshore.
Table 3.3: Depth of Closure Values for the Delaware Coast

<table>
<thead>
<tr>
<th>Source</th>
<th>Equation</th>
<th>Depth of Closure $d_c$ (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hallermeier (1978)</td>
<td>$d_c = 2.28H_e(H_e^2/gT_e^2)$</td>
<td>19.8</td>
</tr>
<tr>
<td></td>
<td>$H_e =$ extreme wave height</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_e =$ period of extreme wave</td>
<td></td>
</tr>
<tr>
<td>Hallermeier (1983)</td>
<td>$d_c = 2.9H(s - 1)^{-0.5}$</td>
<td>23.0</td>
</tr>
<tr>
<td></td>
<td>$H =$ wave height of representative wave</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s =$ specific gravity of the sand</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(H=10$ ft, $s=2.58)$</td>
<td></td>
</tr>
<tr>
<td>Weggel (1979)</td>
<td>$d_c e^{-\alpha x} = (h - h_o)$</td>
<td>35.0</td>
</tr>
<tr>
<td></td>
<td>$x =$ horizontal coordinate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h =$ vertical coordinate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_o =$ datum adjustment factor</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(H=15$ ft $S=2.58)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha =$ empirical constant</td>
<td>40-50</td>
</tr>
</tbody>
</table>

The problem with taking surveys out to the depth of closure is determining exactly what the depth of closure is. In Dalrymple and Mann (1985), several depths of closure for the Delaware Coast are computed using various methods. Several of these values are listed in Table 3.3. As shown, the values of $d_c$ are quite varied. However, both of the Hallermeier equations suggest that $d_c$ could not be 35 feet. Also, Weggel's formula is dismissed since the profile data does not seem to be described by an exponential fit. Therefore, Dalrymple and Mann (1985) suggest using a depth of closure of 28 feet.

In Step (4), $R_{TOTAL}$ is calculated. $R_{TOTAL}$ represents the total possible shoreline advancement, assuming total profile readjustment to the equilibrium profile. In other words, this step determines how far the beach will be expected to widen given a certain volume of fill. So once at equilibrium, the nourished beach will be expected to have a width increase of $R_{TOTAL}$. In this calculation, $R$ is treated as an accretion and not a recession. In a sense, the reverse of Step (3), where area/volume losses were estimated from beach recession, is carried out. Here, beach widening is estimated from area/volume gains (i.e., beachfill). At this step, a check can be made to determine if the beach profiles have reached equilibrium. It is expected that after only $1\frac{1}{2}$ months since the beachfill
was placed, the profiles would not yet have reached their equilibrium state. Therefore, all of the net changes in beach width from before placement on November 26 listed in Table 3.1 should be greater than the $R_{TOTAL}$ values listed in Table 3.4, if the calculations are accurate. By examining these two tables, it is found that this is indeed the case except profiles 100, 105, 125, where the net change is only slightly under the $R_{TOTAL}$ values. It then can be assumed that as of November 26 the beachfill had not yet reached its equilibrium profile. Using the average estimate of $d_c = 28$ ft, $R_{TOTAL}$ is calculated to be 40.83 ft. This value compares very well with the equilibrium shoreline advancement that seems to be developing at Fenwick Island as shown in Figure 3.12.

Step (5), the final step, is where the actual lifetime of the beachfill is computed. The lifetime is estimated by simply dividing the total width added by the beachfill, which was calculated in Step (4), by the expected recession rate for the beach. Table 3.4, lists the lifetimes of beachfills for various recession rates and depths of closure. This data is also plotted in Figure 3.8. These calculations were done using the following values which were calculated earlier:

\[
V = 333,479 \text{ yd}^3
\]

\[
L = 6000 \text{ ft}
\]

\[
A_F = 55.58 \text{ yd}^3/\text{ft}
\]

\[
B_{avg} = 8.75 \text{ ft}
\]

Using Figure 3.8, the lifetime of the beachfill, using any of several estimates of the recession rate and depth of closure, can be determined. Using the $d_c$ values obtained by studies listed in Table 3.3 and $R_{HIST}$ values obtained in Table 3.2, lifetimes could vary anywhere between 9.03-36.66 years.

Using the average values of $d_c = 28$ ft and $R_{HIST} = 2.75$ ft/yr, a probable lifetime of this beachfill using this method would be approximately 14.8 years.

Figure 3.8, shows, as is obvious, that higher recession rates create shorter beachfill lifetimes. Also notice that higher $d_c$ values at a given recession rate decrease the calculated lifetime. In viewing Figure 3.5, this becomes clear since the rectangular shaded region,
Table 3.4: Lifetime Estimates of the Penwick Island Beachfill for Various Recession Rates and Depths of Closure

<table>
<thead>
<tr>
<th>$R_{HIST}$ (ft/yr)</th>
<th>$d_c = 20$ ft</th>
<th></th>
<th></th>
<th>$d_c = 25$ ft</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_E$ (yd³/ft)</td>
<td>$R_{TOTAL}$ (ft)</td>
<td>Life (yrs)</td>
<td>$A_E$ (yd³/ft)</td>
<td>$R_{TOTAL}$ (ft)</td>
<td>Life (yrs)</td>
</tr>
<tr>
<td>1.0</td>
<td>1.06</td>
<td>52.43</td>
<td>52.43</td>
<td>1.25</td>
<td>44.46</td>
<td>44.46</td>
</tr>
<tr>
<td>1.43</td>
<td>1.06</td>
<td>52.43</td>
<td>36.66</td>
<td>1.25</td>
<td>44.46</td>
<td>31.09</td>
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<td>1.64</td>
<td>1.06</td>
<td>52.43</td>
<td>31.97</td>
<td>1.25</td>
<td>44.46</td>
<td>27.11</td>
</tr>
<tr>
<td>1.7</td>
<td>1.06</td>
<td>52.43</td>
<td>30.84</td>
<td>1.25</td>
<td>44.46</td>
<td>26.15</td>
</tr>
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<td>1.06</td>
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<td>27.59</td>
<td>1.25</td>
<td>44.46</td>
<td>23.40</td>
</tr>
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<td>1.06</td>
<td>52.43</td>
<td>26.22</td>
<td>1.25</td>
<td>44.46</td>
<td>22.23</td>
</tr>
<tr>
<td>2.95</td>
<td>1.06</td>
<td>52.43</td>
<td>17.77</td>
<td>1.25</td>
<td>44.46</td>
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<td>44.46</td>
<td>13.89</td>
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<td>13.80</td>
<td>1.25</td>
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<td>1.25</td>
<td>44.46</td>
<td>8.89</td>
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<td>52.43</td>
<td>8.74</td>
<td>1.25</td>
<td>44.46</td>
<td>7.41</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>$R_{HIST}$ (ft/yr)</th>
<th>$d_c = 30$ ft</th>
<th></th>
<th></th>
<th>$d_c = 35$ ft</th>
<th></th>
<th></th>
</tr>
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<tr>
<td></td>
<td>$A_E$ (yd³/ft)</td>
<td>$R_{TOTAL}$ (ft)</td>
<td>Life (yrs)</td>
<td>$A_E$ (yd³/ft)</td>
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<td>Life (yrs)</td>
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<td>1.0</td>
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<td>38.60</td>
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<td>1.44</td>
<td>38.60</td>
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<td>34.31</td>
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<td>1.62</td>
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<td>17.16</td>
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<td>2.95</td>
<td>1.44</td>
<td>38.60</td>
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<td>34.31</td>
<td>11.63</td>
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<td>11.44</td>
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<td>1.62</td>
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<td>10.16</td>
<td>1.62</td>
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<td>9.03</td>
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<td>1.62</td>
<td>34.31</td>
<td>6.86</td>
</tr>
<tr>
<td>6.0</td>
<td>1.44</td>
<td>38.60</td>
<td>6.43</td>
<td>1.62</td>
<td>34.31</td>
<td>5.72</td>
</tr>
</tbody>
</table>
Figure 3.8: Beachfill Lifetime vs. Recession Rate ($d_e = 20, 25, 30, 35$ ft)
which corresponds to the eroded area, becomes taller when \( d_c \) is increased, thus increasing the area for a given \( R \). Also note that increasing \( B \) would serve the same purpose. The effect of changing \( d_c \) (or \( B \)) on the calculated lifetime of the beachfill decreases with increasing \( R \). Thus, as you increase the recession rate, \( R \), you will reach a point where the \((B + d_c)\) term no longer dominates the lifetime expectancy of the beachfill, but instead is dominated by the recession rate approximation. Therefore, in using a relatively low recession rate approximation, say, less than 1.5 ft/yr, the lifetime expectancy is very critical of the \( d_c \) value chosen. On the other hand, if a relatively high recession rate is chosen, say greater than 4.0 ft/yr, the lifetime expectancy varies relatively little with changing \( d_c \) values. Therefore, the recession rate \( R \) dictates how accurate the \( d_c \) value used should be.

3.1.2 Calculations Considering Differing Pre- and Post-Fill Equilibrium Profiles

Dean (1990) has developed a method to calculate the added beach width that can be expected after a beach nourishment project has reached its equilibrium profile. Thus, as the method discussed in the last section was limited in its accuracy by assuming identical pre- and post-project equilibrium profiles, as shown in Figure 2.12b, Dean’s formulas are applicable to all four cases shown in Figure 2.12. For the case of the Fenwick Island beach nourishment project, the fill material was coarser than the natural material, i.e., Figure 2.12a \((A_F > A_N)\). This method goes as follows for the case of \( A_F > A_N \):

Step (1): Find critical volume associated with intersecting/non-intersecting profiles which is

\[
(V')_{c1} = \left(1 + \frac{3}{5B'}\right) \left[1 - \left(\frac{A_N}{A_F}\right)^{\frac{3}{2}}\right]
\]

(3.1)

where

\[(V')_{c1} = \frac{V_1}{BW}\]

\(V_1\) critical volume

\(BW\) volume placed per unit shoreline length
\[ W_* = \left( \frac{h_*}{A_N} \right)^{\frac{3}{2}} \quad \text{reference offshore distance associated} \]
\[ B' = \frac{B}{h_*} \quad \text{non-dimensional berm height} \]

According to this equation, the Fenwick Island project will have non-intersecting profiles, since the critical volume is not reached.

Step(2): Find the shoreline advancement, \( \Delta y \), associated with \( V_1 \). The equation for the case of non-intersecting but emergent profiles is as follows:

\[ V'_2 = \Delta y' + \frac{3}{5B'} \left\{ \left[ \Delta y' + \left( \frac{A_N}{A_F} \right)^{\frac{3}{2}} \right]^{\frac{3}{2}} - \left( \frac{A_N}{A_F} \right)^{\frac{3}{2}} \right\} \]  \hspace{1cm} (3.2)

where

\[ \Delta y' = \frac{\Delta y}{W_*} \quad \text{non-dimensional shoreline advancement} \]

Solving this equation for \( \Delta y' \) is difficult, so Dean presented the equations in graphical form, as shown in Figure 3.9 and Figure 3.10. The data values used in estimating the shoreline advancement after the fill at Fenwick Island has reached equilibrium are as follows:

Using Figure 2.9: \( D_N = 0.441 \text{ mm} \rightarrow A_N = 0.14 \text{ m}^3 \)
\[ D_F = 0.551 \text{ mm} \rightarrow A_F = 0.16 \text{ m}^3 \]
\[ A' = \frac{A_F}{A_N} = 1.14 \]
\[ h_* = 28 \text{ ft} = 8.5344 \text{ m} \]
\[ W_* = 476 \text{ m} = 1561 \text{ ft} \]
\[ B = 8.75 \text{ ft} \]
\[ V_2 = 55.58 \text{ yd}^3/\text{ft} \]
\[ V' = 0.1099 \]
\[ \frac{h_*}{B} = 3.2 \]
Figure 3.9: Variation of Non-Dimensional Shoreline Advancement $\frac{\Delta y}{W}$, with $A'$ and $V'$. Results Shown for $\frac{h}{B} = 2.0$ (Dean, 1990)
Figure 3.10: Variation of Non-Dimensional Shoreline Advancement $\frac{\Delta y}{W_*}$, with $A'$ and $V'$. Results shown for $\frac{h}{B} = 4.0$ (Dean, 1990)
Finally, interpolating between Figure 3.9 and Figure 3.10, a shoreline advancement, $\Delta y$, of 91.78 ft is determined.

With this $\Delta y$ and once again using the average historical recession rate, $R_{\text{HIST}} = 2.75 \text{ ft/yr}$, an expected lifetime of 33.4 years is estimated for the Fenwick Island project. This estimate is probably too high. The shoreline advancement from this beach fill will surely be less than 91.78 ft. As shown in Appendix A the shoreline increased on the average only 91.33 feet immediately after placement. This value will likely only decrease during the period of shoreline adjustment towards equilibrium, as discussed in Section 2.2.

At this point there are two estimates of the "lifetime" of the Fenwick Island beach nourishment project, 14.8 and 33.4 yrs. The discrepancy between the two arises from the difficulty in calculating the expected equilibrium shoreline advancement, $\Delta y$, from the placement of a given volume density of fill. The first estimate assumed identical pre- and post-equilibrium profiles, which will not be the case, due to the varying pre- and post-sediment sizes. It will be expected that the nourished beach will be steeper. This means that $\Delta y$ using this method is too small, since steeper nourished beaches allow a greater shoreline advancement for a given volume of fill placed. It was difficult to apply this method to the Fenwick Island beachfill because it is dependent on an accurate estimate of the pre- and post-nourished beach sand grain sizes. The measurements of the sand sizes by the DGS may or may not be indicative of the entire volume of fill sand. The grain sizes used in the above calculations are an average of samples from only three locations along the beach, taken immediately after placement. For more accurate results, a more in depth sampling, in both time and space, needs to be conducted. This would allow processes, such as sorting and depletion of the fine material, to be more fully developed. Even with an accurate estimate of the sand grain diameters, error is likely introduced in reading $A$ values from Figure 2.9, since it is a log-log plot. Then, once the $A$ values are determined, difficulty is encountered in using Figure 3.9 and Figure 3.10. Small deviations in the $A'$ estimate can produce large variations in the dimensionless shoreline advancement, $\frac{\Delta y}{W}$, which itself is again dependent on the pre-nourishment sand size, since $W$ is a function of $A_N$. 
It is concluded that the lifetime estimate of 14.8 years is likely too low, and 33.4 years is almost definitely too high. The respective estimates of equilibrium shoreline advancement, $\Delta y$, are probably off in the opposite sense. However, a value of $\Delta y$ is needed for future calculations in this paper. Thus, an average of the two, 66.3 feet, will be used. This estimate, is reasonable for the following two reasons: (1) the design specifications called for an increase of 74 feet of shoreline advancement after the fill was placed, and (2) the average shoreline advancement after one year, as shown in Appendix A, was 46.82 feet, which when accounting for the 2.75 feet of erosion during the first year is 49.57 feet. The estimate of 66.3 feet falls acceptably between the two.

3.2 Lifetime Based on Historical Volumetric Erosional Rates (i.e., Sediment Budget Analysis)

In this section, the lifetime of the beachfill is estimated by using historical volumetric erosional rates for the Delaware coast. This method is a somewhat simplified version of the method using recession rates. The idea behind both is essentially the same: estimate how long it will take the beachfill sand to leave the area using known recession rates of the past. The equation used in this section is as follows:

$$\text{Beachfill Lifetime} = \frac{V}{EL}$$

(3.3)

where

$V = \text{total volume of fill (yd}^3\text{)}$

$E = \text{volumetric erosional rate per foot of beach (yd}^3/\text{ft/yr)}$

$L = \text{length of beachfill}$

Here, as shown, the idea is to simply calculate how much sand historically leaves the area and then determine how long the volume of sand in the fill can be expected to last. As shown in the previous section $V=333,479$ yd$^3$ and $L = 6000$ ft. Historical volumetric erosional rates for the Delaware coast, especially in the Fenwick Island area, estimated in several past studies are listed in Table 3.5.

Many of these values listed in Table 3.5 can be discounted. The USACE study of 1929-1954 took place too far in the past. 1.77 yd$^3$/ft/yr is too low, remembering that it is
Table 3.5: Historical Volumetric Erosional Rates for Fenwick Island, Delaware

<table>
<thead>
<tr>
<th>Source</th>
<th>Years of Study</th>
<th>Area of Study</th>
<th>Erosional Rate (yd³/ft/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USACE</td>
<td>1929-1954</td>
<td>South of Bethany to DE/MD State line</td>
<td>1.77</td>
</tr>
<tr>
<td>USACE</td>
<td>1954-1964</td>
<td>South of Bethany to DE/MD line</td>
<td>8.21</td>
</tr>
<tr>
<td>COER*</td>
<td>1964-1982</td>
<td>DE/MD line to 1/2 mi north</td>
<td>6.78</td>
</tr>
<tr>
<td>COER*</td>
<td>1964-1982</td>
<td>1/2 mi north of line to 1 1/2 mi north of line</td>
<td>6.30</td>
</tr>
<tr>
<td>COER*</td>
<td>1964-1982</td>
<td>DE/MD line to 5.76 mi north of line</td>
<td>5.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>60.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(using profile extensions)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(depth adjusted profiles)</td>
</tr>
<tr>
<td>COER*</td>
<td>1964-1982</td>
<td>DE/MD line to 1/2 mi north</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(depth adjusted profiles)</td>
</tr>
<tr>
<td>COER*</td>
<td>1964-1982</td>
<td>1/2 mi north of line to 1-1/2 mi north of line</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(depth adjusted profiles)</td>
</tr>
<tr>
<td>Dalrymple Mann</td>
<td>1983</td>
<td>Fenwick Island</td>
<td>3.6</td>
</tr>
</tbody>
</table>

* = COER studies assume: MD/DE line is a nodal point  
No sediment transport across the MD/DE line
believed that the erosional rate in Delaware's southern coast has accelerated in the past 10-15 years (Dalrymple and Mann, 1985). The USACE study 1954-1964 which estimates the erosional rate at 8.21 yd$^3$/ft/yr is probably too high. The reason for this is that this study includes the entire area south of Bethany to the DE/MD line. Therefore, the York Beach area, which has a higher erosional rate than Fenwick Island would be averaged in (COER, 1983). Because this value includes areas with higher erosional rates than those concerned, it would not be accurate to use this value.

The COER studies are more appropriate since erosional rates are given for more specific areas along the Delaware coast. From the COER study, erosional rates more indicative of the Fenwick Island area are available. The COER study is also very recent, 1964-1982, so these values are still relevant. COER felt that the values obtained using profile extensions were grossly too high, and that the erosion rates obtained using depth adjusted profiles were too low. The reason these two techniques were used in the first place was that the profiles provided to COER by the Corps of Engineers were not sufficiently accurate. COER suggests that the most accurate erosional rates are probably the values obtained without using either technique. Thus, considering all factors, such as time of the study, location of the study, and recommendations of the study groups, the 6.78 and 6.30 yd$^3$/ft/yr erosional rates determined by COER are probably the most accurate. Because both of these erosional rates were computed using neighboring profile lines in the vicinity of Fenwick Island, an appropriate volumetric erosional rate for Fenwick Island would be an average of the two equal to 6.54 yd$^3$/ft/yr.

The erosional rate estimated by Dalrymple and Mann (1985) of 3.6 yd$^3$/ft/yr may also be appropriate. In this study, the erosional rate was calculated using littoral drift values calculated from the WIS data. The study is recent, 1985, and limited to the Fenwick Island area, and therefore should be considered as a possible value.

Table 3.6 lists several beachfill lifetimes along with their corresponding volumetric erosional rate estimates. A graph of these same values is shown in Figure 3.11. From this data, it is clear that as the volumetric erosional rate increases, the corresponding change in expected beachfill lifetime decreases. The lower the erosional rate estimate is,
Table 3.6: Beachfill Lifetimes for Various Volumetric Erosional Rates

<table>
<thead>
<tr>
<th>Volumetric Erosion Rate $E$ (yd³/ft/yr)</th>
<th>Beachfill Lifetime (yrs)</th>
<th>Volumetric Erosion Rate $E$ (yd³/ft/yr)</th>
<th>Beachfill Lifetime (yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>54.3</td>
<td>6.0</td>
<td>9.0</td>
</tr>
<tr>
<td>1.5</td>
<td>36.2</td>
<td>6.5</td>
<td>8.3</td>
</tr>
<tr>
<td>2.0</td>
<td>27.1</td>
<td>7.0</td>
<td>7.8</td>
</tr>
<tr>
<td>2.5</td>
<td>21.7</td>
<td>7.5</td>
<td>7.2</td>
</tr>
<tr>
<td>3.0</td>
<td>18.1</td>
<td>8.0</td>
<td>6.8</td>
</tr>
<tr>
<td>3.5</td>
<td>15.5</td>
<td>8.5</td>
<td>6.4</td>
</tr>
<tr>
<td>4.0</td>
<td>13.6</td>
<td>9.0</td>
<td>6.1</td>
</tr>
<tr>
<td>4.5</td>
<td>12.1</td>
<td>9.5</td>
<td>5.7</td>
</tr>
<tr>
<td>5.0</td>
<td>10.9</td>
<td>10.0</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Beachfill Lifetime = $V/EL$

$V = 333,479$ yd³

$L = 6000$ ft

the more accurate it should be. Only slight differences in the erosional rate estimate below a certain value, say 2.5 yd³/ft/yr, will cause large differences in the calculated lifetime. On the other hand, varying larger erosional rates cause relatively small variations in the calculated beachfill lifetime.

The erosional rates that were deemed most probable presently for the Fenwick Island area, of 6.54 yd³/ft/yr and 3.6 yd³/ft/yr, are both in the "safe region," i.e., where lifetime expectancies vary little with slight differences in erosion rate estimates. From Figure 3.11, it is seen that 3.6 yd³/ft/yr and 6.54 yd³/ft/yr correspond to beachfill lifetimes of approximately 15.4 years and 8.5 years, respectively. Note that the calculated 15.4 years lifetime calculation compares very favorably to the 14.8 years estimated previously using beach recession rates. Because of this fact, greater reliability may be assumed in using 3.6 yd³/ft/yr as an erosional rate. However, the entire space 3.6–6.54 yd³/ft/yr should be considered as possible. As stated before, 6.54 yd³/ft/yr was obtained using profiles without reaching a depth of closure, and as shown in Table 3.5, by trying to compensate for this by depth adjusting the profiles, lower erosional rates were estimated. Therefore,
Figure 3.11: Fenwick Island Project: Beachfill Lifetime vs. Volumetric Erosional Rate

TOTAL VOLUME OF FILL = 333,479 YD³
the lower portion of this space, 3.6–5.0 yd$^3$/ft/yr, is probably most accurate, and the best estimate for this beachfill lifetime using this method is 10–15 years.

As listed in Appendix B, the beachfill in its first 1$\frac{1}{2}$ months (10/88–11/26/88), lost 12,612 yd$^3$, or 4% of the entire volume of fill placed. But as stated, this corresponds to factors such as loss of fines and the transport of material to the north of the project site. Losing 12,612 yd$^3$ in 1$\frac{1}{2}$ months corresponds to an erosional rate of 16.82 yd$^3$/ft/yr which is entirely too high to be considered as the actual erosional rate for Fenwick Island. Using this erosional rate, the beachfill would be expected to last only four or five years, which is much too short a time. This fact tends to support the idea the beachfill is obviously not yet at its equilibrium shape. The beach is still in a process of redistributing the sand in a manner to reach the equilibrium that existed before the fill was placed.

3.3 Comparison of Results to DNREC’s Field Data from the Fenwick Island Project

As stated, at this early stage of the Fenwick Island beach nourishment project, using the DNREC’s field survey data to compute recessional and volumetric erosional rates to compare to historical values is ill-advised, because the beach has not been given ample time to reach its equilibrium. At this point, most of the changes can be attributed to profile adjustment, loss of fines, and “spreading-out” losses, while historical losses are probably far less influential.

For the record, however, these erosional rates for the first twenty months of the project, as dictated by the DNREC’s data will be presented. From values listed in Appendix A, the shoreline recessional rate (discounting the profile adjustment period) for the Fenwick Island project is on average 25.9 ft/yr. Assuming the expected equilibrium shoreline advancement for the project to be 66.3 ft, the present shoreline recession rate is, 10.8 ft/yr. Although 10.8 ft/yr is more in line with historical values, both rates are much too high to be expected to continue into the future. Figure 3.12 shows how the retreat of the shoreline has already dramatically slowed down. Figures 3.1 and 3.2 shows the shoreline location for the various surveys performed by the DNREC throughout the monitoring program.
Figure 3.12: Fenwick Island Nourishment Project: Average Shoreline Advancement Beyond Pre-Project Position
From values listed in Appendix B, the volumetric loss of fill sand (discounting “spreading losses” and loss of fines) for the Fenwick Island project is on average -3.92 yd$^3$/ft/yr. The volume of fill remaining within the project limits is shown in Figure 3.3. This value is surprisingly well within the expected historical volumetric erosional rates. The expected difference from the historical values could have been limited if there were only a small percentage of “fines” in the fill material, and if sand loss due to “spreading-out” losses was compensated by a transport of sand northward from the Ocean City portion of the project. Because of the good correlation, an increased confidence can be placed on the 10–15 year lifetime prediction, calculated in Section 3.2.
Chapter 4

USING A ONE-LINE NUMERICAL MODEL TO PREDICT THE PERFORMANCE OF A BEACH NOURISHMENT PROJECT

This chapter will use a one-line numerical model developed by R.G. Dean to predict how a beach nourishment project will perform. The model is discussed in much greater depth in Dean’s report: *Development of Methodology for Thirty-Year Shoreline Projections in the Vicinity of Beach Nourishment Projects* (1989).

4.1 Outline of the One-Line Numerical Model

As with most shoreline evolution models, this model predicts the position of one contour, such as the NGVD contour, MLW, MHW, or MSL contour. Again, it is assumed that the nourished profile has reached equilibrium, and moves unchanged in form in a landward (or seaward, for an accreting beach) direction as the beach erodes. As will be the case for the model used in Chapter 5, the shoreline changes due to planform spreading and those due to intact recessional rates are treated as being additive changes. This model uses an explicit scheme in which the sediment transport (Equation 2.4) and continuity (Equation 4.1) equations are solved sequentially.

\[ \text{Continuity } : \frac{\partial V}{\partial t} = - \frac{\partial Q}{\partial x} \]  

(4.1)

where,

- \( V \) = sediment volume per unit length of beach
- \( t \) = time
- \( Q \) = volumetric sediment transport in the alongshore direction
  (positive to the right facing the ocean)
- \( x \) = alongshore distance (positive to the right facing the ocean)
In particular, referring to Figure 4.1, the shoreline positions are held constant for a time step, $\Delta t$, while the sediment transport is computed. Then the sediment transport is held constant for a time step, and the equation of continuity is applied to these transport values to update the shoreline positions. This explicit model has a stability criterion which limits the maximum time step, $\Delta t$, given by the following equation:

$$\left(\Delta t\right)_{\text{max}} = \frac{1}{2} \frac{\Delta x^2}{G} \quad (4.2)$$

where $G$ is defined by Equation 2.8.

As with any numerical model, boundary conditions are needed. Dean’s model calls for one of the following two types of boundary conditions: (1) specify a shoreline position (e.g., shoreline is fixed at its initial value) at one or both ends of the computational domain, or (2) specify the sediment transport rate at the boundaries (e.g., no transport past a littoral barrier). For this report, the boundary condition imposed at either end
of the domain is the transport condition with the background transport as the imposed values.

A primary advantage of this numerical model over the model to be introduced in Chapter 5, is its flexibility in prescribing initial conditions. Any arbitrary initial shoreline configuration can be specified. In addition, littoral barriers of arbitrary length can be placed at any location along the beach, and with minor modifications to the program, renourishments can be represented.

4.2 Application of the Numerical Model to the Fenwick Island Beach Nourishment Project

Once again, the procedure is developed and presented in greater depth in Dean (1989). The process of using this model involves developing an input data file (DNRBS.INP), which includes all the necessary nourishment project and wave environment parameters. This file is then used as input for the program DNRBS.FOR which creates the output file DNRBS.OUT. The output file contains shoreline position and area of dry beach remaining at any given time throughout the beach nourishment project's history. The process of applying this program went as follows:

Step 1: Specify Beach Nourishment Project Characteristics

Much of this data determination was previously discussed in Chapter 3. First, the project length was specified. The total length of the project (Fenwick Island + Ocean City) is 9.3 miles. Thus the required project characteristics are as follows:

$$\text{Project Length } \ell = 9.3 \text{ miles} = 49104 \text{ ft}$$

For calculations:

$$\Delta x = 500 \text{ ft}$$  
$$\text{Fill Sand Diameter } D_F = 0.551 \text{ mm}$$  
$$\text{Transport Factor (see Figure 2.14) } K = 0.656$$  
$$\text{Berm Height } B = 8.75 \text{ ft}$$
Depth of Closure \( h_* = 28 \text{ ft} \)

Longshore Axis Orientation (see Figure 2.15) \( \mu = 357^\circ \) (WIS Station 66)

Deep Water Contour Orientation (see Figure 2.15) \( \beta_o = 87^\circ \) (WIS Station 66)

Step 2: Determine Equilibration Project Width, \( \Delta y_o \)

This step has already been completed in Sections 3.1.1 and 3.1.2. To model an initially rectangular beachfill project, which can then be used in comparison to the other prediction models, the average value of \( \Delta y_o = 66.28 \text{ ft} \) was used. Thus it was assumed that an equal initial added beach width was placed everywhere along the project, from Ocean City Inlet to the DNREC survey station ST0+00 in Fenwick Island.

Due to the flexibility of this model, a second input file was created using the actual \( w_{net} \) values as of November 1988 (listed in Appendix A) extended from the DNREC's baseline as initial shoreline positions for the Fenwick Island portion of the project. The initial shoreline position for the remainder of the project in Ocean City was set at the average beachfill width of that survey, \( \Delta y_o = 63.13 \text{ ft} \). The reason that November survey positions were chosen over the October surveys, which are actually the ones taken initially after placement, was that this would allow for much of the initial profile readjustment to take place. Hopefully, this would help compensate for the assumption in this model that the profiles move unchanged in their equilibrium state, while still allowing a justified comparison of the model output to the field data which is only available for approximately the first two years of the project at this time.

Step 3: Specify the Wave Climate

As discussed in Section 2.2.1, the wave data used in this report was taken from the WIS data for Station 66. Dean, has introduced the idea of "effective" values to be used as input into this model. As mentioned in Section 2.1.1, the effect of wave direction is relatively unimportant in this model (Dean, 1989), so the assumed average wave direction for this study was normal to the beach, i.e., \( \alpha_o = \beta_o = 87^\circ \) (see Figure 2.15). This choice was made arbitrarily, but in fact is essentially the mean wave direction (see Section 2.1.1)
The effective wave period was taken as the average wave period over the twenty years of WIS data, $T = 5.43$ seconds.

The determination of effective wave height is more involved. As will be shown in Chapter 5, simply using the average wave height is inappropriate. Using the mean wave height as a constant severely underestimates the beach erosion when compared to using the actual wave time series. At least for WIS Station 66, the average wave height, $H_{\text{mean}} = 1.65$ ft., is relatively small. Equation 2.4 shows that the transport of sand is proportional to $H^{3/2}$. Thus, much of the sand will be transported during periods of high wave heights, such as storms, and relatively little will be moved during calm periods, as may be indicative of a wave height of only 1.65 ft. Therefore, averaging is ill-advised since the periods of high sediment transport (storm conditions) are lost from the input data.

Dean (1989) uses the fact that there exist a wave height, the effective wave height, $H_{\text{eff}}$, that when applied as a constant for every time step of the calculations, will yield identical shoreline position results as those calculated using the actual time series of varying wave heights. This greatly simplifies the input conditions. This representative wave height, $H_{\text{eff}}$, of a given time series to be used in calculating the shoreline diffusivity, $G_o$, through Equation 2.8, is determined as follows:

$$H_{\text{eff}} = \left[ \int_0^\infty H^{2.4} p(H) dH \right]^{1/2.4}$$

(4.3)

where, all wave heights are in deep water, and $p(H)$ is the probability distribution of the wave height data set.

Dean (1989) has given further simplifications for the case when the wave heights follow a Rayleigh distribution, which would be easily calculated for such a data set using the following formula (Dean and Dalrymple, 1984):

$$p(H) = \frac{2H}{H_{\text{rms}}^2} e^{-\frac{H^2}{H_{\text{rms}}^2}}$$

(4.4)

This was found not to be the case for the WIS data set at Station 66. Figure 4.2 shows that the data does in fact appear similar to a Rayleigh distribution except for the excess of very small wave heights. Further checking by way of the IMSL subroutine PROBP, indicates that the data in fact has no particular “well-known” probability distribution. The
subroutine indicates, as expected, that there are proportionately too many wave heights on the low end of the scale (especially calm periods, i.e., $H=0$) for the distribution to be Rayleigh. Further checks of a partial data set provided little progress toward achieving a Rayleigh distributed data set. Checks were performed on partial data sets such as those ignoring calm times, ignoring the smallest 1% of the waves, ignoring the smallest 5% of the waves, etc. Since a particular probability distribution could not be fitted to the WIS data, one had to be developed.

First, the WIS data, which as stated in Section 2.1.1, contains wave data at a depth of 10 m, had to be transformed back to deep water using Snell’s Law and the Conservation of Energy equations given below:

$$\frac{\sin \theta_{WIS}}{C_{WIS}} = \frac{\sin \theta_o}{C_o} \quad (4.5)$$

$$H_o = \frac{H_{WIS}}{K_s K_r} \quad (4.6)$$

where,

$$K_s = \text{shoaling coefficient} = \sqrt{\frac{C_o}{2C_{o,WIS}}}$$

$$K_r = \text{refraction coefficient} = \sqrt{\frac{\cos \theta_o}{\cos \theta_{WIS}}}$$

Then having a data set of deep water values, its probability density function was determined by IMSL subroutine DESPL, shown in Figure 4.2. Now with the probability density function, $p(H)$, calculated, the effective wave height representative of this data set could be determined using Equation 4.3. The results are as follows:

$$(H_{eff})_{20yr} = 2.19 \text{ft}$$

$$(T_{mean})_{20yr} = 5.43 \text{s}$$

A similar effective value for the ratio $\frac{C_{G0}}{C_s}$ to be used in Equation 2.8 is developed (Dean, 1989) below:

$$\left(\frac{C_{G0}}{C_s}\right)_{eff} = \frac{1}{N} \sum_{n=1}^{N} \frac{C_{G0,n}}{C_{s,n}} \quad (4.7)$$
Figure 4.2: Probability Density Function Estimate of the Deep Water Wave Heights, $H_o$
for WIS Station 66
The result of such a calculation for this WIS time series is:

\[
\left( \frac{C_{\text{in}}}{C_{\text{out}}} \right)_{\text{eff}} = 1.14 \left( \frac{t}{T} \right)^{0.2}
\]

At this point, enough of a background has been set to show the accuracy and inaccuracy of using effective and mean wave conditions, respectively, as input to the model. Of course, the advantage of using one wave condition and treating it as a constant throughout the simulation process is that in using Equation 2.11 only one calculation is necessary. Given one \( G \) (calculated using either mean or effective wave conditions for input to Equation 2.8), solve Equation 2.11 for \( y(x,t) \) at the desired location \( x \) and time \( t = 20 \) years. This direct calculation takes only seconds on any computer. Applying a complete time series (one value every 3 hours for 20 years), on the other hand, on an IBM mainframe computer takes between 7–8 minutes (22 minutes on the Vax mainframe). Of course, a portion of the time advantage gained by using constant parameters is lost in the process of calculating the suitable set of representative conditions themselves, especially in the determination of effective values in which extensive calculations are required, as discussed in this section. The check for using effective and mean wave parameters will be the 20-year WIS data time series. If the effective or mean parameters produce planform results identical to those simulated with the complete time series, their use will be assumed correct. In effect, the check is to see if either, or both, of the effective and mean wave parameters satisfactorily represent the input of the WIS time series as a whole.

The results of applying the “effective \( G \)” as a constant for 20 years, as compared to applying the WIS data file correctly (discussed in Section 5.1) as a time series is shown in Figure 4.3. The planform using effective values as a constant and the planform using the WIS data time series are almost indistinguishable. For persuasive purposes, the ineffectiveness of applying mean wave conditions as constants for input to Equation 2.11 is shown in Figure 4.4. Applying the mean values as constants significantly underestimates the erosion of the beachfill planform. Again, this is due to the loss of the storm conditions during the averaging process. The storm conditions, which are few and far between, but
nonetheless account for a substantial portion of the sand loss, fail to influence the mean wave conditions enough. Thus effective values produce the correct results while mean values do not.

Step 4: Develop Background Erosion Data as Piecewise Linear Segments

Here, the background erosion rate was chosen as in Chapter 3, to be $R_{HIST} = -2.75$ ft/yr. This rate was applied as a constant to the entire length of the project. Differing rates could have been applied along the beach, but adequate data was not available for much of the project area to make such a specification appropriate.

At this point, a structure was also introduced into the model. A 100 ft littoral barrier was placed at the southern most point of the project to simulate the Ocean City Inlet. At this location, there will effectively be no longshore transport of sand unless there is a bypass of the barrier which would occur if the beach would happen to extend further than 100 ft at that point.

Step 5: Develop Input File

The data is placed in the input file (DNRBS.INP) in a prescribed format. The time increment, $\Delta t$, was chosen as 1 day $= 86400$ seconds. This is actually eight times larger than the three hour time increment between data points in the WIS data set. The total number of time steps is arbitrary, but was set at 7300 (20 years), expecting the shoreline to have retracted to its pre-nourished position within this timespan.

Step 6: Run the Program

On the Sun workstation, program DNRBS.FOR took approximately 30–45 minutes to simulate 20 years of shoreline movement.

Step 7: Examine Output in File DNRBS.OUT

This file contains both the shoreline position and the percentage of added dry beach remaining from the fill within the project limits, for any given time period after
Figure 4.3: Comparison of 1 Year Beachfill Planforms: Effective Wave Conditions as Constant vs. WIS Data Applied as a Time Series
Figure 4.4: Comparison of 1 Year Beachfill Planforms: Mean Wave Conditions as Constant vs. WIS Data Applied as a Time Series
placement of the project. Two cases were run: (1) A rectangular fill with initial added width, \( \Delta y_o = 66.28 \text{ ft} \), and (2) actual survey data to determine the post-placement shoreline location to use as initial conditions. The results for Cases (1) and (2) are shown in Figures 4.5 and 4.6, respectively. For the x-axis of these figures, and similar ones to follow, 0 represents the DNREC station ST0+00, -1000 represents ST10+00, ..., and -6000 represents ST60+00, the end of the project.

Case(1), which assumes an initially rectangular planform, is actually making an unnecessary simplification. In using this numerical model, there is no need to simplify the available field data to a rectangular planform. However, this case is run so that it may be compared with the results of the modelling technique used in Chapter 5, in which an initial rectangular planform of the beach nourishment project must be assumed.

Case(2), which makes use of actual field survey data, is the one most appropriate for comparison to the DNREC's survey data listed in Appendix A. To apply the survey data to this model, the location of the pre-project shoreline had to be considered, since the actual shoreline configuration, and not just the net change in shoreline configuration is important for this model. Thus the \( w_{net} \) values \( (= w_{after \; placement} - w_{before \; placement}) \) for November are not the shoreline positions used. These values had to be added to the initial pre-project shoreline locations, to arrive at what the actual nourished shoreline planform would be, \( w_{planform} \). This calculation is an attempt to compensate for the fact that the beachfill planform is not transposed on an originally straight beach, which is a requirement for using Equation 2.11. It is assumed that the beach initially (before placement) parallels the DNREC's straight survey baseline, at an average pre-project width. The planform widths are calculated by finding the net change in beach width from this average pre-project location. In equation form:

\[
 w_{planform} = w_{post-project} - (w_{pre-project})_{avg}
\]

Figure 4.7 shows the results for the first two years of simulation by DNRBS.FOR, along with the shoreline survey data of Appendix A (modified as stated). Figure 4.8 is a close-up view of the Fenwick Island project limits for Figure 4.7. As shown, this one-line numerical model does very well (keeping in mind the vertical exaggeration introduced by the axis
Figure 4.5: One-Line Numerical Model Simulation Assuming Initially Rectangular Planform ($\Delta y_o=66.28$ ft)
Figure 4.6: One-Line Numerical Model Simulation Using the DNREC Survey Data As Initial Conditions
scales). The field data is generally centered around the simulated shoreline positions. The largest discrepancies are at locations 0 and -2000 where the field data repeatedly shows a much narrower beach than is predicted. The cause for this is not certain, but a possible reason may be that these two locations are downdrift of littoral barriers, thus being continually starved of sand.

This good correlation of the model output with the field data, allows a greater confidence in predicting the beach nourishment project's lifetime using this one-line numerical model. Using this model, as visible in Figure 4.6, allows for two different estimates of the lifetime that may fit the definition presented in Chapter 1. In one case, the lifetime could be interpreted as the time it takes for the entire planform within the project limits, to be further landward than existed before the beachfill was placed. This would be the case if the definition of "lifetime" presented in Chapter 1 is strictly adhered to. An example of this situation is shown as the twenty year planform in the figure. Using this criterion, a lifetime of just over 17 years for the Fenwick Island portion of the beach nourishment project is predicted. The 17 and 18 year planforms are shown in Figure 4.9. The problem with using such a criterion is that while some of the beach still has a positive net beach width when compared with pre-project widths (≡ "living"), other locations (necessarily those closer to the ends of a project) are becoming narrower than they were before placement of the fill (≡ "dead"). This phenomenon is shown in this figure, and is due to the "spreading" losses of the beach nourishment project, which as discussed in Section 2.4, are very important, especially for the ends of a project, as is the case for Fenwick Island. This lifetime definition will generally be inappropriate since usually the reason a beachfill is placed is to widen a beach which was considered too narrow to function effectively either for recreation, or storm protection, or both. It is likely that as a nourished beach becomes more narrow than the pre-project beach at any location, the lifetime of the project would be complete, and renourishment should soon be considered.

The second case states that as soon as any location along the shoreline reaches a position further landward than existed before the project, the beachfill lifetime would be considered over. This is the lifetime criterion that will be deemed most desirable. At
Figure 4.7: Comparison of the First Two Years of Shoreline Simulation Using the One-Line Numerical Model with the DNREC Field Survey Data (X=0→-15000)
Figure 4.8: Comparison of the First Two Years of Shoreline Simulation Using the One-Line Numerical Model with the DNREC Field Survey Data (X=0→6000)
Figure 4.9: One-Line Numerical Model: Beach Planform Which is Entirely Landward of its Pre-Project Location
this point it should be noted that this criterion will not be appropriate for field survey data. If a given survey station at a given time of year unexpectedly yields a beach which is narrower than the the pre-nourished beach, the project should obviously not be considered “dead.” In all likelihood, this beach width deficit is a temporary fluctuation from its long-term position, due to factors such as seasonal or storm adjustments as mentioned in Chapter 2. Such losses are shown in Figure 3.1, where the shoreline of March 1989 eroded past the pre-nourished shoreline only to fully recover by July of that same year. Now, back to predicting the lifetime of the nourishment project, using this criterion, as shown in Figure 4.10, the result for the Fenwick Island project is 12 years. As expected this criterion will always produce a shorter lifetime expectancy.

Note that the lifetime estimates based on the two criteria differ very little. The reason for this is that, by this time in the beachfill’s evolution, the planform is becoming relatively flat. Spreading losses, which are directly related to the planform shape, are becoming less important than they were during the first few years of the project. At these later stages, most of the losses are attributed to the recessional losses. Thus, at this time in the evolution of the shoreline, it will generally retreat as a whole, with only minor changes in its planform shape. Therefore, the time to progress from lifetime criterion (2) to (1) is relatively short, at least for the case of Fenwick Island.
Figure 4.10: One-Line Numerical Model: Beach Planform Which is Only Partially Landward of its Pre-Project Location
Chapter 5

PROBABILISTIC PREDICTION OF BEACH NOURISHMENT PROJECTS' LIFETIMES USING AN ANALYTIC SOLUTION

Predicting a beachfill lifetime as a definitive number of years is perhaps, at the very least, questionable. Too little is known about the coastal processes that govern the shoreline evolution to make a model which produces results that can be thought to be 100% accurate. In general, assumptions must be made, either in formulating the model itself, or in prescribing the input conditions. The prediction methods used in Chapter 3 assume that the future erosional rates will be identical to the average historical rates. Dean's numerical model, used in Chapter 4, assumes that the wave environment that the beach nourishment project will encounter will be identical to the WIS data set. With the limited erosional and wave data available for the Fenwick Island Atlantic coast, the input values used in these models (average annual erosional rates and effective wave conditions) are the best available. However, having to decide on one value to be used as the input into the model will obviously produce only one result, which will necessarily be taken as the best possible estimate using that model.

Coastal communities which have placed a beach nourishment project are often caught by surprise by its unexpectedly poor or good performance. Either the beach erodes more or less than had been predicted. If all other factors, such as sediment size, equilibrium profiles, and littoral barriers (discussed in Chapter 2), are correctly accounted for, the unexpected performance is most likely due to unexpectedly extreme wave conditions (stormy or calm). For "poorly" performing projects, the waves it encountered were probably stormier than past conditions; for better than predicted performances, the wave conditions were probably calmer (fewer storms) than expected from the past history of that beach. The WIS data, the only extensive wave data set available for Fenwick Island,
in the manner it was compiled does not account for extreme conditions, such as tropical storms. The deletion of such storm conditions classifies this data set as at least questionable for use in predicting shoreline erosion. Storm conditions are when the most erosion takes place. In fact, one study has shown that the “top 5% of the days [wave condition-wise] in a year account for 50% of the total erosional losses” (Dalrymple and Mann, 1985, p. 11). Disregarding these days would greatly underestimate any erosional calculations, and thus tend to overestimate any beach nourishment project lifetime estimates.

This chapter presents a model which will take into account the possibility of varying wave conditions including the prospect for storm conditions. The key to such an attempt is to acquire many different suitable sets of input wave conditions. Then each run of the model, with its own unique input wave time series, will produce a different realization of the shoreline evolution. With enough realizations calculated (in this study the model was run 100 times) a probability can be incorporated into the results. For a given length of time following the placement of a beachfill, several possible locations of the beach planform will be calculated (each with its own set of input wave conditions). Then having many realizations, the probability that the beach at some location along the planform will be at least a given width after a given time can be computed.

5.1 Outline of the Probabilistic Prediction Model

Equation 2.11, the analytic solution to the evolution of an initially rectangular beach planform, discussed in Section 2.4 is used in this model to determine the shoreline evolution. For convenience, the equation will be stated again,

\[ y(x, t) = \frac{Y}{2} \left\{ \text{erf} \left[ \frac{\ell}{4\sqrt{Gt}} \left( \frac{2x}{\ell} + 1 \right) \right] - \text{erf} \left[ \frac{\ell}{4\sqrt{Gt}} \left( \frac{2x}{\ell} - 1 \right) \right] \right\} \]

where

\[ G = \frac{K H_o^{2.4} C_{1.2}^{0.4} \cos 1.2(\beta_o - \alpha_o) \cos 2(\beta_o - \alpha_s)}{8(s - 1)(1 - p) C_s R^{0.4}(h_s + B) \cos(\beta_o - \alpha_s)} \]

and \( y(x, t) \) is the beach width at time \( t \) along the beach at location \( x \). In addition to the “spreading-out losses” which are modelled using the above equation, an average historical recession rate is considered. This recession rate is applied as a constant such that every location along the beach planform erodes an equal distant. Therefore, the effect
of applying the recession rate is to simply "pick-up" the entire beach planform and move it, unchanged in shape, landward a distance, \( \Delta y \). All the while, the planform shape is being manipulated by the spreading losses. During the actual simulation, the entire time span was run considering only spreading losses using Equation 2.11. Then after these spreading losses for the given time period were calculated, the historical recession rate losses were applied. Of course, in nature the recession rate loss is a continuous process, but the above method will produce results identical to a continuous process. This method, however, greatly simplifies the calculations.

In modelling a given beach nourishment project, remember that the important parameter is

\[
\eta \equiv \frac{\ell}{\sqrt{Gt}}
\]

Note that all of the wave input conditions \((H_0, C_{Go}, C_\ast, \alpha_0, \alpha_\ast)\) are contained in \(\eta\) through \(G\), as shown above. All of the other parameters are determinant design characteristics of the project itself \((Y, h_\ast, B, \ell, s, p)\). The difficulty in determining the depth of closure, \(h_\ast\), and the initial added equilibrium beach width from a beachfill, \(Y\), has already been discussed. However, once these parameters are satisfactorily determined, the initial conditions are then almost fully resolved. The only remaining variable is \(G\), a function of the wave conditions. The variables \(x\) and \(t\) are wholly an arbitrary choice of the modeller, as to where along the project and at what time in the project's future the results are desired.

The different realizations of this model were possible by generating 100 sets of input wave data. This process of generating the wave time series in discussed in the next section, Section 5.2. The wave times series then enables a time series of “shoreline diffusivity” parameters, \(G\), to be computed via Equation 2.8. This time series consists of one \(G\) value every three hours for twenty years, which is consistent with the temporal spacing in the WIS data \((G_{3hrs}, G_{6hrs}, G_{9hrs}, \ldots, G_{1law}, G_{1day}, G_{3day}, \ldots, G_{19yrs}, G_{364days}, G_{21hrs}, G_{20yrs})\). The length of the time series was terminated at twenty years, suspecting that within this time frame, no matter how "calm" the generated time series was, the project will have evolved landward of the pre-placement shoreline and thus been considered “dead.” As
expected, and shown in Section 5.3, input time series with high \( G \) values cause quickly eroding beaches, while those with generally low \( G \) values simulate slowly eroding beaches.

The process of applying a time series of \( G \) values to Equation 2.11 is not as straightforward as it may first appear. It is not possible to simply take the next \( G \) value in the time series, increase \( t \) by the 3 hour time increment, and apply them to the model. The reason for this is the basic assumption by Equation 2.11 of an initially rectangular planform. By using the above stated procedure, Equation 2.11 infers that every time \( t \) is increased and a new value of \( G \) is used, the beach planform is once again assumed back to its original post-placement rectangular state. The effects of all the previous time steps on the shoreline's evolution are erased. The results of this procedure are to find the beachfill planform for the situation in which the wave conditions inherent to the present \( G \) value in the time series have been a constant for the entire time in the simulation (up to that point in the time series). For example, the final beach planform using this method would be identical to one simulated by simply applying the last \( G \) in the time series along with \( t = 20 \) years.

The trick to accomplishing the application of a time series to Equation 2.11 is to change the initial conditions for each new time step during the simulation. The new initial conditions must effectively take into account all of the beach transformations that have taken place in all of the previous time steps of the simulation. Each time step in the simulation should go as follows: 1) determine the planform resulting from the previous time step, \( y(x, t) \), and 2) determine how this new planform will change in 3 simulated hours of attack by the wave conditions of the present time step. The technique of finding appropriate initial conditions for each time step is to treat the variables \( G \) and \( t \), in Equation 2.13 as a single parameter \((Gt)\). The planform shape of the beachfill \( y(x, t) \) is uniquely determined for a given \((Gt)\) holding all other parameters constant for the entire simulation, as will always be the case in modelling a single project. Only the first time step, with its corresponding \( G \), will make use of the assumed initially rectangular planform and will use the actual simulated time span in its calculations. Thus in finding the planform shape 3 hours after placement of the project, the initial conditions will be \( y(x, 0) = \Delta y_0 = Y \)
and the input will be $G = G_1$ and $t = 3$ hours. No other time step in the simulation will be performed in quite this same manner. The initial conditions (planform shape) from the second time step on are inherited from the previous time step. An “equivalent time,” $t_{\text{equiv}}$, is then calculated which will represent the time it would have taken the new wave conditions (expressed through $G$) to evolve the initially rectangular planform to the present step’s initial planform condition. This is done by solving Equation 2.11 iteratively for $t_n$ given $G_n$ and $y(x, t_{n-1})$ using a modified Newton-Raphson technique. Finally, the 3 hour time increment is added to this “equivalent time” and the planform $y(x, t_n)$ is solved with $t_n = t_{\text{equiv},n} + 3\text{hrs}$ and $G = G_n$. Note that the actual simulated time in the history of the evolution is never actually used after the first time step, except in adding $\Delta t$ to the “equivalent time,” which remains at the actual simulated time span of 3 hours. The modelled time period in the simulation process is simply increased 3 hours every time step, and has no correlation with the “equivalent time.” Therefore, in presenting the results, the actual simulated times, $t_n$, are associated with the planforms and the the $t_{\text{equiv}}$’s are discarded after they are used in the intermediate calculation step.

The flow of the program is as follows:

1) Define the Initial Planform Conditions as:
   $$y_{n-1}(x, t_{n-1}) = \text{function of } (x, t_{\text{equiv},n-1} + 3\text{hrs}, G_{n-1})$$

2) Get the New Wave Conditions:
   $$G_n$$

3) Iteratively Solve for the Equivalent Time via Equation 2.11:
   $$t_{\text{equiv},n} = \text{function of } (y_{n-1}(x, t_{\text{equiv},n-1} + 3\text{hrs}, G_{n-1}), G_n)$$

4) Solve for the New Planform via Equation 2.11:
   $$y_n(x, t_n) = y_n(x, t_{\text{equiv},n} + 3\text{hrs}, G_n)$$
   where $t_n = t_{n-1} + 3\text{hrs}$
For clarity, an illustrative example is presented. Say the first time step is complete, using \( \Delta y(x, 0) = \Delta y_0 \), \( G = G_1 \), and \( t = 3 \) hours. The result is \( y(x, t_1 = 3\text{hrs})_{G=G_1} \). Now the next \( G \) value in the time series, \( G_2 \), is taken and the calculation of the planform shape after 6 hours is begun. First, the equivalent time must be calculated. Say \( G_2 < G_1 \), thus signifying that the wave conditions are less erosive during the second three hour period than they were the first. It will obviously take the \( G_2 \) waves longer to reach the planform established in the first time step than it did the \( G_1 \) waves. Thus expect \( t_{\text{equiv,2}} > t_{\text{equiv,1}} = 3 \) hrs. Now the second time step can be completed using \( \Delta y(x, 0) = \Delta y_o \), \( G = G_2 \) and \( t = t_{\text{equiv,2}} + 3\text{hrs} \). This resultant planform, \( y(x, t_2 = 6\text{hrs}) \), is actually associated with the 6 hour point in the beach nourishment project’s history even though the time used in the calculations for this case was \( t_{\text{equiv,2}} > 6\text{hrs} \). Obviously the opposite scenario works in the opposite sense (i.e., \( G_n > G_{n-1} \rightarrow t_{\text{equiv,n}} < t_{\text{equiv,n-1}} + 3\text{hrs} \rightarrow y(x, t_n)_{G_n} = t_{\text{equiv,n}} + 3\text{hrs} \)).

The process of applying different wave conditions every three hours from a measured or generated time series produces the realistic effect of the beach eroding at varying rates throughout its history. Fast erosion takes place in short episodes of stormy conditions; the erosion is slower and more steady when the wave conditions are calm. Figure 5.1 shows the unsteady nature of such erosion when one month of the WIS time series is applied to the Fenwick Island beach nourishment project at DNREC station ST50+00. In contrast, Figure 5.2 shows how the erosion of the shoreline is smooth and steady when a constant wave condition is used. Most shoreline erosion is known to occur in episodes (i.e., storms) rather than steadily with time, therefore, the modelling technique depicted in Figure 5.1 is favored at least for depicting short-term beach changes.

Applying 100 generated time series to the model, produced 100 realizations of possible beach planforms of the Fenwick Island beach nourishment project. The next step is to associate a probability with the results. The desired outcome is to generate several plots of cumulative distribution functions (CDF) for the beachfill width at given alongshore locations of the project at given time periods in its lifetime. Section 5.3 presents and discusses
Figure 5.1: Fenwick Island Beach Nourishment Project ST50+00: Shoreline Change When Exposed to the First Month of the WIS Data Time Series
Figure 5.2: Fenwick Island Beach Nourishment Project ST50+00: Shoreline Change When Exposed to Constant Wave Conditions
such functions calculated for the Fenwick Island beach nourishment project. The probability theory used for this study came from Ang and Tang (1975). The theory is as follows:

**Definition:** CDF $\equiv F_X(x) = P(X \leq x)$ for all $x$

where $P()$ = cumulative probability

$X$ = continuous random variable

Thus

$$F_X(x) = P(X \leq x) = \sum_{all \; x_i \leq x} P(X = x_i)$$

With properties:

a) $F_X(-\infty) = 0 \quad F_X(+\infty) = 1.0$

b) $F_X(x) \geq 0$ and is non-decreasing with $x$

c) It is continuous with $x$

d) $P(a < X \leq b) = F_X(b) - F_X(a)$

In relating the above theory to this study, $X$ will represent a certain percentage of the original beachfill width remaining chosen arbitrarily at distinct intervals.

The CDF calculations for this study are as follows:

1) The planform data points (i.e., alongshore location versus time) are divided into the desired categories. In this study it was decided to model the percentage of original added beachfill width remaining at certain DNREC survey stations at certain time periods in the project's history, so the data was grouped as such. For example, all 100 realizations of the percentage of remaining beach width at, say, survey station ST30+00 after, say, 10 years comprise one group.
2) Each group is then arranged in decreasing order (100% remaining being the highest possible value).

3) Each group was then sectioned into 20 intervals; each section includes all the realizations falling between two limits, say, 25–30%.

4) Find the probability, \( P(a < X \leq b) \), of each section of realizations occurring. For example, if 5 out of the 100 realizations fell between 75–77% beach width remaining then \( P(.75 < X \leq .77) = 0.05 \). There is a 5% chance that there will be between 75–77% of the original beachfill width remaining at ST30+00 after 10 years.

5) Finally, the cumulative distribution function, CDF, for that data group is determined by cumulatively adding the probabilities of each occurring section in the group. For example, if the probability of the smallest subsection \( P(x_{\text{min}} < X \leq x_{1}) = 0.05 \) and for the next smallest subsection \( P(x_{1} < X \leq x_{2}) = 0.07 \) then the CDF at \( x_{\text{min}}, x_{1}, \) and, \( x_{2} \) are 0, 0.05, and 0.12, respectively. In the context of this report, there is a 12% chance that there will be less than \( x_{2}\% \) remaining of the original added beachfill width.

### 5.2 Generating Wave Time Series

Section 5.1 explained the need for multiple input wave condition values to enable probability to be incorporated into the result presentation. The goal is to generate a time series of wave conditions that resembles the WIS data set as closely as possible. The WIS data is chosen as the guideline because it is the best (and maybe the only) site specific long-term set of wave data available. The WIS data covers a 20 year time period (1956–1975), and therefore will likely span the lifetime of the Fenwick Island beach nourishment project. The 20 year time span is also desirable since the frequency and severity of a wider variety of wave conditions is included, when compared with say only a 1 year time series repeated 20 times. Also, the benefit of the WIS data being in the form of a time series was discussed in the previous section.
The check to determine how well the two wave time series generation models to follow perform will be a comparison of the probability distributions of wave conditions from the model and from the WIS data itself. Actually, before any comparisons are made, the wave conditions (wave height, period and direction) are combined into the single parameter, the "shoreline diffusivity," $G$, with Equation 2.8. $G$ is chosen since it is the primary parameter in the model (see Equation 2.11), and in the end actually simplifies the model since only one time series is needed ($G$) and not three ($H_o, T, \theta_o$). The effectiveness of the generation model is then checked with the one criterion: the ability to produce a 20-year time series of wave conditions that duplicates the distribution of $G$ parameters produced by the 20-year WIS data set, shown in Figure 5.5. Note that it is not important for the exact WIS time series to be duplicated, but only its 20 year $G$ distribution. In fact, duplicating the actual time series would be undesirable since then the results of every simulation would be identical. No stipulations are made as to how extreme (within physically explained reason) the wave conditions can get, how frequent the storms will be, when the storm conditions will occur in the time series, etc. As long as at the end of the 20 year simulation the generated time series as whole closely resembles the WIS data set distribution, the wave generation model is regarded as acceptable. This distribution comparison inherently not only checks for obvious requisites such as similar means, but also the relative frequency of all other $G$'s (wave conditions).

5.2.1 AR(2) Model

The first wave record generation model studied was an AR(2) model, autoregressive of the order 2. The theory used in this section follows Pandit and Wu (1983). AR($n$) are the most popular models of time series simulation and forecasting used presently in hydrology; thus, the transition to wave studies seems justified. AR($n$) models are intended for modelling a time series whose components are Gaussian distributed. The models chosen for this study assume the time series to be stationary, i.e., no general trends or seasonality in the first two moments, mean and covariance, respectively. For the case of modelling a natural wave time series, assuming no general trend with time is probably an accurate one, barring any long-term effects such as sea level rise. In fact, this same
assumption was the basis for the historical-based prediction techniques used in Chapter 3. Obviously seasonality is significant in a natural wave time series. However, to account for seasonality would require a much more complicated and in-depth AR(n) model. For long-term calculations, such as lifetime predictions, any advantage gained by considering seasonality would not be worth the expanded calculation procedure. If accuracy in short-term changes was needed, an AR(n) model could be roughly modified by simply developing a separate AR(n) model for each time period with similar characteristics, say one for each month of the year, and then interpolating between them for the desired time in the series.

The general form of an AR(n) model is:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_n x_{t-n} + a_t$$

(5.1)

where,

- $x_t = \text{data value calculated at time } t$
- $\phi_n = \text{correlation constants } (|\phi_n| < 1)$
- $a_t = \text{sequence of uncorrelated variables}$

Since the model expresses the dependence, or regression, of $x_t$ on its own past values, it is called an autoregressive model. If the dependence, or relation, between $x_t$ and $x_{t-n}$ is strong, $\phi_n$ will be large in magnitude, and if weak, $\phi_n$ will be small in magnitude. By assuming a stationary time series it is required that

$$|\phi_n| < 1$$

This will prohibit $x_t$ from increasing or decreasing without bound.

The crucial assumption of an AR(n) model is that the $a_t$'s at different $t$'s are independent. For convenience, as is the norm, in the studied models $a_t$ was assumed to be distributed normally so that:

$$a_t \sim \text{NID}(0, \sigma_a^2)$$
where NID means Normally Independently Distributed. Other assumed distributions of \( a_t \) were examined, such as Rayleigh, but failed to yield improved results.

Note that an AR\((n)\) model will generate a time series for only a single variable, \( x \). The obvious parameter time series to generate for the shoreline evolution model at hand is one for \( G \) (Equation 2.8). Separate AR\((n)\) models could be developed for \( H_0 \), \( T \), and \( \theta_0 \) (see Equation 2.8) but there would be no means to correlate the three generated time series. The three generated time series would all be independent of one another. By combining the three parameters into one before formulating the AR\((n)\) model, the appropriate correlation between the three as it exists in the WIS data set is consequently taken into account. Figures 5.3 and 5.4 shows the definite dependence of \( G \) with the value it held in the previous two time steps. This is what would be expected of a natural wave time series with a temporal spacing of only 3 hours between values. The wave conditions from one 3 hour period to the next will generally be similar, with changes with time being brought about gradually rather than abruptly. For example, it is unlikely that the wave heights would increase from, say, an average of 1 foot to 15 feet in the time period of 3 hours. You would expect a progressive increase with minimal changes between time increments. Figures 5.3 and 5.4 suggests that an AR\((2)\) model will be appropriate for generating a time series of \( G \) values. The AR\((2)\) model is inherently at least as accurate as an AR\((1)\) model (in fact more accurate if \( \phi_2 \neq 0 \)). Favorably, the calculations for an AR\((2)\) model are only moderately more involved than those in developing an AR\((1)\) model. Higher AR\((n)\) models, say \( n = 3 \) or 4, require much more involved data analysis in determining \( \phi_3 \), \( \phi_4 \), and \( a_t \) and at the same time generally make little improvement in the generated outcome when compared with that of an AR\((2)\) model.

In developing an AR\((2)\) model to simulate a WIS-like time series, the first problem encountered was the fact that the \( G \) value time series generated from the WIS data set is not Gaussian distributed, and thus cannot itself be simulated using an AR\((n)\) model. The shoreline diffusivity had to first be transformed into a Gaussian distributed data set using a power transformation technique described in Box and Cox (1964), Draper and Cox (1969), Hinkley (1975), and Hinkley (1977). The technique is to first shift all the values
Figure 5.3: Correlation of Consecutive $G$ Values (Time Lag of 3 Hours) in the WIS Data Set

Figure 5.4: Correlation of $G$ Values with a Time Lag of 6 Hours in the WIS Data Set
in the data set a constant value so that they all are positive. Then raise all the values in the shifted non-Gaussian data set to a power that will produce a normally distributed data set. The new “normalized” data set can then be used to develop an AR(2) model. The resultant time series from the AR(2) is then transformed back to a non-Gaussian time series by applying the reverse transformation, i.e., taking the generated values to the reciprocal power and subtracting any necessary shift. The transformation procedure is as follows, where \( \ast \) will denote transformed data which ideally is normally distributed:

1) \( \hat{G}_{WIS} = (G_{WIS} + \text{shift})^n \)

2) \( \hat{G}_{WIS} \rightarrow \text{AR}(2) \text{ model} \rightarrow \hat{G}_{\text{generated}} \text{ time series} \)

3) \( G_{\text{generated}} = (\hat{G}_{\text{generated}})^{\frac{1}{n}} - \text{shift} \)

The AR(2) model for the intermediate transformed data is then:

\[
\hat{G}_t = \phi_1 \hat{G}_{t-1} + \phi_2 \hat{G}_{t-2} + a_t
\]  

(5.2)

where,

\( \hat{G}_t \) = transformed shoreline diffusivity at time \( t \)

\( \hat{G}_{t-1} \) = transformed shoreline diffusivity at time \( (t-3 \text{ hrs}) \)

\( \hat{G}_{t-2} \) = transformed shoreline diffusivity at time \( (t-6 \text{ hrs}) \)

\( \phi_1, \phi_2 \) = correlation constants

\( a_t \) = normally distributed random variable

The model is completely specified only when \( \phi_1, \phi_2, \) and \( \sigma_a^2 \) are given. A model similar to the above procedure has been used to generate hourly wind speed time series by Brown, Katz, and Murphy (1984).
For the WIS data $G$ time series ($G_{WIS,n}$), a power transformation using shift = 0.1 (arbitrarily larger than $G_{min} = | - 0.027|$) and $n=1/5$ worked best. Observing Figure 5.5 shows that taking the data of this curve to the $1/5$ power, will keep the very small $G$ values small, but significantly decrease the large $G$ values upon such a transformation. Thus, the left side of the curve will tend to retain its shape, while the tail to the right will be pushed much further left towards smaller $G$ values, thus producing a more Gaussian-shaped distribution. The distribution of the tranformed data of Figure 5.5 is shown in Figure 5.6. Although the distribution of Figure 5.6 is obviously still not Gaussian, the relative length of the tail is decreased. In attempting to model the WIS time series with an AR($n$) model, the dilemma was in generating these values in the tail which are far from the mean. The generation technique would not allow the time series to stray this far from the mean. Therefore, the goal was to decrease the tail to a point where all its values could be reached with an AR($n$) model.

Now the normalized data set is used to develop the AR(2) model of the form given in Equation 5.2. Without derivation (see Pandit and Wu (1983)) the equations used to calculate the needed parameters are listed below:

\[
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix} = \begin{bmatrix}
\frac{CD-BE}{AC-B^2} \\
\frac{AE-BD}{AC-B^2}
\end{bmatrix}
\] (5.3)

where,

\[
A = \sum_{t=2}^{N-1} \hat{G}_t^2
\] (5.4)

\[
B = \sum_{t=2}^{N-1} \hat{G}_t^2 \hat{G}_{t-1}
\] (5.5)

\[
C = \sum_{t=1}^{N-2} \hat{G}_t^2
\] (5.6)

\[
D = \sum_{t=3}^{N} \hat{G}_t^2 \hat{G}_{t-1}
\] (5.7)

\[
E = \sum_{t=3}^{N} \hat{G}_t^2 \hat{G}_{t-2}
\] (5.8)
Figure 5.5: Probability Distribution Function of $G$ Values Derived from the WIS Data at Station 66
Figure 5.6: Probability Distribution Function of the Power Transformed $G$ Values Using shift = 0.1 and n=1/5
and
\[
\sigma_a^2 = \frac{1}{N-2} \sum_{t=3}^{N} (\hat{G}_t - \phi_1 \hat{G}_{t-1} - \phi_2 \hat{G}_{t-2})^2
\]  \hspace{1cm} (5.9)

The \( \hat{G}_t \)'s are the power transformed shoreline diffusivity parameters calculated from the WIS time series and \( N \) is the total number of points in the given data set (for the case of the WIS data set \( N = 58,440 \)).

For WIS Station 66 the results were as follows: \( \phi_1 = 0.878831, \phi_2 = 0.051921 \), and \( \sigma_a^2 = 0.001077 \). Note that \( \phi_2 \ll \phi_1 \) (\( \phi_2 \approx 6\% \phi_1 \)), which indicates that the importance of the time lags is decreasing sharply and therefore little increase of accuracy would be expected by increasing the order of the AR\( (n) \) model past \( n=2 \). This particular model states that a generated value will be 88% of the value in the previous time step, added to 5% of the value two time steps previously and then either adding or subtracting a random part with mean of 0 and variance equal to 0.001. In order to start using the AR\( (2) \) model, Equation 5.2, to generate a time series, \( \hat{G}_1 \) and \( \hat{G}_2 \) as seeding values are needed. \( \hat{G}_1 \) was determined using a Monte Carlo method assuming the input transformed data set was Gaussian, and \( \hat{G}_2 \) was then determined from \( \hat{G}_1 \) using an AR\( (1) \) model with parameters of its own calculated from the power transformed data set in a manner similar to the construction of the AR\( (2) \) model. The reverse power transformation was then applied to this generated data set to finally arrive at a simulated 20 year time series of shoreline diffusivity parameters for the Fenwick Island Atlantic coast.

A check of the AR\( (2) \) model is to solve Equation 5.2 for \( a_t \) given the generated time series of \( \hat{G} \) values. If the \( a_t \)'s do in fact appear to be random and independent, the AR\( (2) \) model is found to be adequate, and as shown by Figure 5.7 this is the case for the derived model. The values of \( a_t \) do in fact appear to be independent of one another.

A comparison of the distributions of the simulated \( G \) time series calculated directly from the WIS data is shown in Figure 5.8. The AR\( (2) \) model combined with the power transformation does a poor job of simulating the WIS time series. The AR\( (2) \) model does a fair job of generating a time series with a mean value close to that of the WIS data, and the maximum and minimum generated \( G \) values are reasonably close to those found in the WIS time series. As shown, however, the most serious problem is that the AR\( (2) \)
Figure 5.7: Check of Adequacy of the AR(2) Model Developed for Modelling WIS Station 66 Data by Establishing the Independence of the Individual Random Components
model does not simulate the desired peakedness which exist in the WIS data. The AR(2) model generates values which are too spread-out around the median value (which is very close to zero). Even with similar means, the AR(2) model, which in general produces a much higher percentage of “extreme” wave conditions than that given in the WIS data, will generate a time series of $G$’s that will simulate a much higher beach erosion rate and thus tend to underpredict a beach nourishment project’s lifetime.

Most likely, the difficulty the AR(2) model has with simulating the WIS time series arises from the fact that the WIS data does not produce a normally distributed shoreline diffusivity time series. Remember, AR(n) models will always generate a normally distributed data set. In order to compensate for this phenomenon, a power transformation was used to act as a bridge between a normal distribution and a distribution similar to the WIS data time series. Obviously, this effort was inadequate. The distribution of the WIS calculated $G$ time series differed further from Gaussian than could be offset by the rather simple power transformation used. The transformed $\check{G}_{WIS}$ time series, shown in Figure 5.6, was not exactly Gaussian, and therefore was not justifiably capable of being simulated by an AR(2) model. The degree to which this criterion was not met results in the large difference in the outcome, Figure 5.8. The AR(2) model generated a normally distributed data set, and again the power transformation was inadequate in making the conversion from normal to the distribution shown in Figure 5.5.

If future improvements are possible in simulating a non-Gaussian time series using an AR(n) model, its use would be well worthwhile. Once the parameters ($\phi$’s and $\sigma^2$) of the model are computed, the generation of the time series is trivial. Simply seed the simulation with the required number of values and use this single equation to generate a time series of any desired length. Say, for each WIS station, to generate a characteristic time series for that location would require only that the appropriate $\phi$ and $\sigma^2$ values be calculated from the available WIS data set. Since the entire simulation procedure is condensed into a single equation, a year long time series could be generated in seconds by even the most basic computer.
Figure 5.8: Comparison of the PDF of the Shoreline Diffusivity Parameters: WIS Calculated $G$ Values and AR(2) Model Generated $G$ Values
5.2.2 Alternative Numerical Procedure

The method described by this section was developed by Borgman and Scheffner in *The Simulation of Time Sequences of Wave Height, Period, and Direction* (1990). Their report describes the theory and implementation of a numerical procedure for simulating arbitrarily long time sequences of wave height, period, and direction which are statistically similar to those of an existing finite length time series (i.e., WIS data). Thus, the objective of their study was identical to that of Section 5.2.1. Similar to the overlying purpose of this thesis (i.e., predict beach nourishment project lifetimes) it is their hope to use the simulations to evaluate long-term fate and stability of a dredged material mound. As with this thesis, they hoped to avoid the use of a discrete or repeated historical events for design conditions.

There will be no effort here to elaborate in any depth on the theory used by Borgman and Scheffner (1990) in developing their procedure. Here, only a brief outline will be presented and further explanation will be left for reference to their report. Their procedure is based on the calculation of the statistical parameters describing the intercorrelations among the wave field parameters (wave height, period, and direction) of the existing data. Generation of the intercorrelation matrix from the WIS data base takes approximately 30 minutes on an IBM mainframe, but this is only a one time procedure. Once the intercorrelation matrix has been computed, the original data (WIS data files) is no longer required in the wave generation procedure, as is the case for the AR(2) model once $\phi_1$, $\phi_2$, and $\sigma^2$ are known. The matrix is then used to compute a simulated time series (a 20 year time series takes on the average 7 minutes 18 seconds to generate on an IBM mainframe) which reflects the primary (determined as such by the researchers) statistical properties of the entire original data base: 1) the univariate probability law for the three wave parameters of height, period and direction, 2) the temporal and spatial correlations within and between the wave parameters, and 3) nonstationarity such as seasonal variations. It is obvious that in achieving these goals, this numerical simulation method is much more intensive than a simple AR(2) model. Many more features are incorporated into this method. For one, the three individual parameters' time series
can be simulated simultaneously; this removes the need to combine the three parameters into a single shoreline diffusivity parameter, $G$, prior to generating the time series as was required by the nature of the AR(2) model of Section 5.2.1. Even more importantly, this numerical procedure is tolerant of any distribution of the original time series which is to be modelled. Borgman and Scheffner also used the probability distribution of the results as a check of applicability against the WIS data distributions, which they also chose as their “standard.”

This numerical simulation was run and the three parameters, wave height, period, and direction, were combined (after running the model) into the single parameter, shoreline diffusivity, $G$, by way of Equation 2.8. Figure 5.9 shows the comparison of the simulated and WIS data distributions. As shown, the correlation between the two is excellent. The curves are nearly alike, with the most apparent separation occurring to the immediate left of the peak where the simulation slightly over-estimates the percentage in that range of $G$ values. Of course, these small differences between the simulated and WIS curves would eventually be eliminated by averaging over many simulations. Most importantly, the peak, which is the obvious dominating feature of the distribution of $G$ values in the time series, is well-modelled. Thus, even with small fluctuations in the number of extreme values represented by the tails of the distribution in Figure 5.9, the criterion for accuracy (i.e., similarity in probability distributions) will continually be met with varying simulations. The numerical simulation model will inherently counterbalance the generation of very high values by generating low values to maintain the required probability distribution.

5.3 Application of the Model to the Fenwick Island Beach Nourishment Project

The model described in Section 5.1 is applied to the beach nourishment project at Fenwick Island with project parameters determined in the previous chapters ($w_0 =$ initial planform width $= 66.28$ ft, $\ell =$ equivalent planform length $= 18.6$ miles, etc.). The Borgman-Scheffner model was chosen over the AR(2) model to generate wave time series simulations as input to the beach planform evolution model because of the much higher degree of accuracy even though the computation time is longer (approximately 5
Figure 5.9: Comparison of the Probability Distributions of Shoreline Diffusivity Parameters, $G$, in the Time Series: WIS Data Set and the Borgman, Scheffner Model Simulation
minutes longer for a 20-year simulation). The planform evolution model was run 100 times with 100 different 20-year (one value every 3 hours) diffusivity time series simulations as input. One hundred different time series was guaranteed by using a different seeding value for each simulation. As discussed in Section 5.1 the results of the 100 simulations were then presented, as cumulative distribution functions for all the possible percentages of the original added beach width ($w_{total}/w_o$) remaining at varying locations along the project length at various times in the project's life. Every 2000 feet of the project, along with one location 1000 feet beyond the project limits, were chosen as the locations to model. These locations could then be compared to the DNREC field survey stations of ST0+00, ST20+00, ST40+00, ST60+00, and ST70+00, respectively, to check for model accuracy, at least for the first two available years of data for the beach nourishment project's life. The time periods checked were 1, 5, 10, 15, and 20 years after the initial placement of the project.

An example of one of the CDF plots is given in Figure 5.10. Here the results of 100 simulations for location ST20+00 (2000 feet north of the MD/DE stateline), 5 years after the initial placement are presented. By example, Figure 5.10 will be used to show how these plots are to be interpreted. First note that in using 100 different series of possible wave conditions the narrowest beach yielded 39% of the original planform width remaining after 5 years at ST20+00, and the widest beach yielded 69.3% remaining. Stated another way, at least for these 100 simulations, there is a 100% chance that there will be at least 39% of the original planform width remaining, and there is 0% chance that there will be more than 69.3% remaining after 5 years at ST20+00. The widths in between the extremes are read as "at least" or "more than" widths for the given probability of occurrence. For example, in Figure 5.10 there is a 90% chance that there will be "at least" 52.5% of the original planform width remaining, 50% chance that there will be "at least" 64% remaining, but only a 20% chance that there will be "more than" 66.2% remaining.

The remainder of the CDF plots for the various locations and time periods are in Appendix C. A few general features of the plots common to the locations within the project limits (i.e., excluding ST70+00) are revealed. Interpretation of the CDF plots for
Figure 5.10: Cumulative Distribution Function of Percentage of Original Planform Width Remaining (ST20+00 after 5 years)
ST70+00 will be made later in this section. First, is the obvious difference between the CDF of ST60+00 and all of the others. ST60+00 is located at the terminal end of the beachfill planform. As defined by Equation 2.11, the behavior of the endpoints is peculiar. Theoretically, Equation 2.11 predicts that the width of a rectangular beach planform at the endpoints will decrease to 50% instantaneously after the beachfill is placed and remain that width throughout the evolution of the shoreline until the original “rectangular perturbation” is smoothed-out to once again form a straight beach. Therefore, the beach width at the endpoint is identically determined (i.e., at 50% of the original planform width) regardless of the wave input conditions as far as “spreading losses” are concerned, which are those losses predicted by Equation 2.11. The movement of the beach width with time at ST60+00 is due entirely to the historical background beach recession rates. Since this recession rate was assumed a constant \( R_{HIST} = -2.75 \text{ ft/yr} \) again regardless of the input wave conditions, the expected beach width at ST60+00 will be the same for all 100 simulations at a given time period. The uniqueness of the expected percentage of remaining planform width is indicated by the single lines extending to 100%.

All of the other CDF plots have the same general features. As expected, at a given location, the maximum simulated beach width decreases with time. The maximum simulated beach width, is a simulation in which there are few if any severe storms, and therefore the shoreline erosion is dominated by the historical recession rate. For example, at ST0+00, the maximum simulated remaining width was 95.75% after 1 year and decreases to 75.6% after 5 years. Spreading losses affect a rectangular planform as described in Section 2.4, causing the simulated beach widths to decrease the further the location is from the center of the project (i.e., the Ocean City Inlet) for any given time period. The maximum predicted remaining beach width for ST0+00 after 1 year is 95.75% but only 89.1% for ST40+00.

Next, notice how the shape of the CDF plots of a given location follows a general trend: after 1 year the curves start off with a sharp bend and eventually with time flatten out closer to a straight diagonal between the two endpoints of the curve. This trend with time can best be explained by realizing that the general abundance of a range of
simulated beach widths can be interpreted from the CDF plots. Remember that the CDF is a measure of the cumulative area under the probability density function from the smallest to the largest, so the relative change in the CDF curve over a given range of beach widths is directly proportional to the probability of the range (read from the pdf curve). For example, when the probability of a given beach width is high, a relatively large area will be added for a given range of simulated beach widths. Therefore, a mild slope of the CDF curves between two beach width values indicates that there were few simulations which yielded results falling in this range. On a mildly sloping portion of the curve, the probability of increasing beach widths changes very little, indicating only few simulation results are in this range (i.e., only minimal changes the probability are made). Conversely, segments of the CDF curves where the slope is steep, there is a high number of simulated beach widths; enough to significantly change the probability of that range of widths occurring.

As an example, the 1 year curves' shapes indicate that the predicted beachfill widths at this time vary only slightly. At ST0+00 there is a 95% chance that there will be more than 94% of the original fill width remaining, while there is no chance that there will be more than 95.7% remaining. This states that at ST0+00 after 1 year this model predicted that 95 of the 100 simulated beach widths fell between the narrow range of 94 and 95.7% of the initial planform width remaining (equivalent to a spread of only 1.1 feet). At ST0+00 there is only a 5% chance that the beach width will fall below this range after 1 year of erosion. For later times at ST0+00 this 95% probability of occurrence will cover a much wider range of values; for example, after only 5 years this same highest 95% of simulated beach widths varies between 46.7 and 75.8% remaining (a spread of 29.1% = 19.3 feet).

A reason for the changing curve slopes is the varying degree of wave conditions that are possible as input to the planform evolution model. The year to year distribution of wave conditions are allowed to vary as long as the distribution of the entire 20 year time series matches that of the standard (i.e., the WIS data). For argument's sake, say there were 20 periods of severe storm conditions in the original WIS data. Now, the wave time series generation model can theoretically cause these 20 storms to occur at any time throughout
the simulation process: 20 storms in the first year and none the remaining 19 years; 20 storms the first month and none the remaining 19 years and 11 months; no storms the first year, 20 the second, and none the remaining 18; one storm each of the 20 years; two storms every other year, etc. The wave generation model could even generate 22 severe storms as long as it was compensated by an excess of calm periods, and still satisfactorily mimic the WIS distribution of wave conditions. Obviously, the probabilities of the above situations occurring vary greatly. There would be only a minimal probability of say all 20 severe storms occurring in the first year, but a much better likelihood of one storm occurring each year. The shorter the generated time series is, the smaller the possible number of different combinations of wave conditions will be. Therefore, after 1 year, the range of possible input wave conditions would be expected to vary relatively little through 100 simulations, when compared to what may be expected in 100 simulations of 5 years worth of wave conditions. For example, without actual probabilistic calculations of the occurrence of storm conditions along the Fenwick Island coast, it can be surmised that the probability of one severe storm occurring is fairly likely in any given year. The likelihood of two severe storms occurring that same year are probably less, the likelihood of three even less, and so on, until say there is no, or infinitesimal, probability of more than six severe storms occurring that year. Applying these same probabilities for one year over a five year period will obviously quickly accumulate a much greater number of possible permutations of input wave conditions.

This effect is made evident by the fact that the change in slope of the CDF curves become less and less abrupt the longer the simulation time for a given location becomes. The chances of severe storms occurring increase as the simulation time increases, thus increasing the number of more narrower simulated beaches, thus increasing the slope in the low end of the beach width scale. At the same time, the chance of no storms occurring is decreasing as time passes, thus decreasing the number of minimally eroded beaches at the high end of the beach width scale, causing the slope in this region to become milder.

Variation in the wave conditions with time also has the effect of broadening the range of simulated beach widths for a given location. This will cause the probability of the
resultant shoreline widths to be spread over a wider range of values as the length of the simulated series in increased, as opposed to only a few simulations straying from the large band of values as is the case for a 1 year simulation with its limited degree of variability in its input. For example, ST0+00 after 1 year only varies between 78.2 and 95.75%, a spread of only 17.55% or 11.6 feet of beach. After 5 years, the beach width at ST0+00 varies between 43.8 and 75.6%, a spread of 31.8% or 21 feet of beach.

Note that some of the generated wave conditions may yield identical shoreline positions even though they themselves are different. Thus this effect of broadening ranges with time is expected to eventually diminish. This "broadening" effect can be thought of as a progression of the probability distribution of the generated input wave conditions towards the target distribution of the WIS values. The longer the time series becomes, the more likely it will be that its distribution will resemble the WIS distribution. Identical probability distributions of wave input conditions will produce identical results for this model as indicated by the analysis of "effective" wave conditions as input, which utilize this very fact. After only 1 year's worth of data is simulated, it is very possible that the distribution will not resemble that of the WIS data set. There are simply not enough values to model a 20 year time series adequately. Eventually, however, a time series length will be reached which is acceptably representative of an entire 20 year long series (distribution-wise). At this point, all simulations will likely produce very similar results and longer time series will only cause small fluctuations about this long-term "average" result. This diminishing effect is already seen in the 20 year plots in Appendix C in which the range of simulated beach widths is less than the simulated range of 5 year old beach widths. The time series has become long enough that its probability distribution has become well-developed and is relatively unaffected by further series length.

A word here should be said about the CDF curves for ST70+00 which is outside the project limits. Here, once again, percentages of original fill are a percentage of the average equilibrium beachfill width (66.28 ft.) within the project limits as was the case with the other locations, even though the width which existed at ST70+00 after placement actually assumed to be the base width of zero. The beachfill values at ST70+00 will not
be included in the lifetime predictions because this location is not within the project limits, but was included as a point of interest. Note that because ST70+00 is outside the project limits, the shape of its curve is different from the other locations. Again, a review of spreading losses in Section 2.4 is suggested. Outside the project limits, the beach will accrete not erode when gains from spreading losses from within the project are greater than the historical recession rate for that location. Thus, severe storms will cause the beach to widen rapidly. This will cause the region of wider beaches to have a mild slope (i.e., relatively rare stormy conditions) and narrow beaches simulated with more common mild conditions to have a steep slope. Note that the range of possible beachfill widths actually decreases with time. This is due to the counterbalancing of losses due to background recession and gains from spreading losses of the project area.

Now, since a firm base has been laid as to how the CDF plots can be interpreted, they will be used to simulate a probable evolution of the Fenwick Island beach nourishment project and finally predict a lifetime of the project. Here, a crucial decision needs to be made: How safe of a prediction is needed? Is a shoreline location that is 50% probable of at least a predicted width acceptable? Or would a minimum beach width that is 95% probable needed? Of course the predicted minimum beach width and its probability of being exceeded (i.e., even narrower) are inversely related. For example, after 10 years at ST0+00 a prediction of a minimum remaining beachfill width of 43.3% is only 50% likely, while there is a 90% likelihood that there will be at least 32.1% remaining. So there is a trade-off: maximizing the predicted beachfill width or maximizing the reliability of the prediction.

Of course, the safest prediction would result from using all of the minimum predicted beach widths. Using these 100 simulations as a basis, there is no probability of the beachfill widths being less than these values. Conversely, choosing the maximum predicted beach widths sets a maximum limit on the simulated beachfill planform. In a sense, these two conditions will set an envelope enclosing all the possible locations of the profile predicted with 100 simulations. The simulated envelopes for 1, 5, and 15 year planforms for the Fenwick Island beach nourishment project are shown in Figure 5.11, 5.12, and 5.13,
respectively. In Figure 5.11 the DNREC field survey data collected between 1 and 2 years after placement is superimposed to show how well they correspond with the model results. In large, the field data falls within the calculated envelope. It is expected that with a larger number of simulations the range between the maximum and minimum calculated widths along the beach would increase thus perhaps allowing the envelope to encompass all of the data. Figures 5.14 and 5.15 show the fraction of the original volume of the fill \( V_o = 333479 \text{yd}^3 \) remaining within the project limits \( (ST0+00 \rightarrow ST60+00) \) over the life of the project for various probabilities of occurrence. For example, the 5% line denotes the volume of fill that has only a 5% chance of being exceeded. Likewise, the 95% line denotes the volumes that have only a 5% chance of being lower. Figure 5.14 accounts for “negative” volumes by subtracting them from the total volume left in the project limits thus showing the net change of volume of sand within the project limits (i.e., \( V_{\text{total}} = (\sum \text{“positive” remaining volume}) - (\sum \text{“negative” volumes}) \)). Here “negative” volumes refer to lengths of the beach that have a net loss of beach width from the time of placement of the project thus creating areas of net volume loss. Figure 5.15, on the other hand, disregards the “negative” volumes and thus shows the total “positive” volume of original fill left within the project (i.e., \( V_{\text{total}} = \sum \text{“positive” remaining volume} \)). Figure 5.16 is a portion of Figure 5.14 enlarged with DNREC field data (Appendix B) superimposed. The field data is scattered evenly around the probabilistic model results. The fluctuations of the field data about the model results are expected, due to the known seasonal variations of the sediment transport at Fenwick Island. The data closest to the one year mark do in fact fall within the range of modelled results, as should be the case, since the seasonal variations are eliminated by comparing beaches during the same given time period of the year as discussed in Chapter 2 (Time \( t = 0 \) years = November 1988 and \( t = 1 \) year = November 1989). The trend denoted in Figure 5.16 does seem to indicate that \( t = 2 \) years will also fall within the modelled results.

Choosing a probability of less than 50% is ill-advised. Predictions based on such low probability will more than likely be exceeded. For example, there is a 70% chance that a planform constructed of 30% probability of occurrence beach widths will in fact be
Figure 5.11: Envelope of Possible Shoreline Locations after 1 Year for the Fenwick Island Beach Nourishment Project through 100 Simulations
Figure 5.12: Envelope of Possible Shoreline Locations after 5 Years for the Fenwick Island Beach Nourishment Project through 100 Simulations
Figure 5.13: Envelope of Possible Shoreline Locations after 15 Years for the Fenwick Island Beach Nourishment Project through 100 Simulations
Figure 5.14: Fraction of Original Beachfill Volume Remaining within the Project Limits with Varying Probability of Occurrence for the Fenwick Island Beach Nourishment Project (Subtracting “Negative” Volumes)
Figure 5.15: Fraction of Original Beachfill Volume Remaining within the Project Limits with Varying Probability of Occurrence for the Fenwick Island Beach Nourishment Project (Disregarding "Negative" Volumes)
Figure 5.16: Fraction of Original Beachfill Volume Remaining within the Project Limits with Varying Probability of Occurrence for the Fenwick Island Beach Nourishment Project with DNREC Field Survey Data (denoted with O's)
narrower than expected. Choosing the 50% probability beach widths may be appropriate. With this planform, there is a 50% chance the beach may be wider than predicted, and a 50% chance the beach may be narrower, but always within the envelope shown in Figures 5.11, 5.12, and 5.13, depending on the time period for which the prediction is required. This can be thought of as a “median” planform. A community requiring a beach nourishment project will more than likely need an estimate more reliable than 50% probability to justify funding the project. The community will be concerned with the chance of the project failing (i.e., eroding faster than predicted) which is the function of the CDF plots in Appendix C. A project that performs better than expected can be treated as a “bonus”; it costs the community no extra money, since the above par performance was due to the milder than average wave conditions over the project’s history. What will cost the community is raising the reliability of a planform prediction not falling below a required minimum beach width, after a given post-placement time. Say after 15 years, the community wants the beachfill width to be at least 25 feet everywhere within the project limits. Maybe with 50% probability, placing 50 yd$^2$ of sand per foot of beach would accomplish this purpose. However, to increase the reliability to 75% probability, 75 yd$^3$ of sand per foot of beach may be required. Of course, the increased sand requirement increases the money required.

A 75% probable planform will be chosen as a middle of the road. There is only a 25% chance that the project will perform worse than this prediction, and at the same time the costs are kept in check by not over-insuring the performance of the project. This 75% probable planform is given in Figures 5.17 through 5.21. To get a realistic planform shape the 25% probable planform locations for ST70+00 are used. Remember, quickly growing downdrift beach locations outside the project limits (i.e., ST70+00) are associated with quickly eroding beaches within the project (see Section 2.4).

As in Chapter 4, the lifetime of the Fenwick Island beach nourishment project will be considered over when any location along the project erodes to a state landward of the shoreline which existed before the beachfill was placed. Again, as in Chapter 4, with this definition of project lifetime, the important location to monitor is the terminal end of
Figure 5.17: Simulated Shoreline Planform after 1 Year for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width
Figure 5.18: Simulated Shoreline Planform after 5 Years for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width
Figure 5.19: Simulated Shoreline Planform after 10 Years for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width
Figure 5.20: Simulated Shoreline Planform after 15 Years for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width.
Figure 5.21: Simulated Shoreline Planform after 20 Years for the Fenwick Island Beach Nourishment Project that is 75% Probable of being the Minimum Possible Width
the project, which in our model is ST60+00, which will always erode the fastest. The lifetime is then determined by interpolating between Figure 5.19 and 5.20 between which the project shoreline becomes negative (i.e., landward of its pre-project location). This yields the following result: it is 75% probable that the Fenwick Island beach nourishment project will last 12.1 years [Note: it would take 17.7 years for the entire project planform to erode landward of its pre-project location, but by this time beach width at ST60+00 would already be 15.5 feet narrower than the pre-project beach at that location].
Chapter 6

SUMMARY AND CONCLUSIONS

Beach nourishment is recognized as a viable, and in many instances, the most desirable option in combating shoreline erosion. With the continued use of beachfills, it becomes important to understand the underlying principles controlling these projects. In particular, predicting the longevity of a beach nourishment project has come under much debate. Premature, and often illegitimate, judgments declaring beach nourishment projects as failures against erosion are presently engrained in the beliefs of many coastal communities. These remarks are generally made with little theoretical or historical backing. Instead of completely dismissing beachfills as an option, there is a need to understand how nourishment projects function and how to reasonably evaluate their effectiveness. The purpose of this thesis is to present such a background with an emphasis on predicting how long a given beachfill could be expected to last.

First, several factors that affect the lifetime of beach nourishment projects were discussed. In general, a qualitative discussion of the mode by which these factors act on a beachfill was presented, and when possible was supported by theoretical derivation. Brief statements of how these factors should be chosen so as to maximize a projects’ longevity were also made.

Next, various modelling techniques were used to predict how long a given beach nourishment project would last. The simplest models extrapolated historical recessional and volumetric erosional rates to predict such lifetimes. The second type of prediction method used a one-line numerical model developed by Dean (1989). Finally, the analytic solution, developed by Pelnard-Considere (1956), to the evolution of an initially rectangular beachfill planform was used.
Probabilistic predictions of projects' lifetimes were made possible by accumulating many different realizations of the analytic model for a given set of project parameters. Generating many different time series of wave parameters as input enabled the multiple realizations to be made. Both an AR(2) and a numerical simulation technique developed by Borgman and Scheffner (1990) were tested on their ability to simulate a set of data with similar properties to a WIS time series of wave conditions. Borgman and Scheffner's model was the most effective.

All of the models were used to predict the lifetime of the beach nourishment project placed at Fenwick Island, DE in the summer of 1988. Lifetimes ranging from 12–17 years were predicted for the Fenwick Island project. The application of the models were also compared to actual field survey data, made available for the first two years of the project by the Delaware DNREC. All models compared reasonably well to the field data.

Future emphasis should be placed on further development of probabilistic prediction models. A probabilistic prediction of a beachfill lifetime is more appropriate given the present state of uncertainty in estimating beachfill project evolution and performance, and perhaps even more useful in determining the economic feasibility of such projects.

The following are suggestions for areas of future work that may be helpful in predicting beachfill lifetimes: 1) possibility of developing an accurate AR(n) wave time series generation model which would significantly decrease the computation time of running multiple simulations for a probabilistic model over Borgman and Scheffner's generation model, 2) developing an "effective" wave parameter generation model which would make the probabilistic lifetime predictions trivial, 3) applying multiple time series to Dean's numerical model in order to achieve probabilistic outcomes, and 4) continued monitoring of the Fenwick Island and other nearby beach nourishment projects which will provide a data base against which the various models may be compared thus detecting required alterations.
Appendix A

DNREC SHORELINE POSITION DATA FOR FENWICK ISLAND BEACH NOURISHMENT PROJECT
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Δw = change in beach width (MSL) between consecutive surveys (ft)

w_{net} = beach width (ft) with respect to its width before the placement of the fill (9/88)
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$\Delta w =$ change in beach width between consecutive surveys (ft)

$w_{net} =$ beach width (MSL) with respect to its width before the placement of the fill (9/88) (ft)
Appendix B

DNREC VOLUME CHANGE DATA FOR FENWICK ISLAND BEACH NOURISHMENT PROJECT
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\( \Delta v \) = \text{change in area of fill between consecutive surveys (yd}^3/\text{ft})

\( \Delta v_{total} \) = \text{change in volume of fill between survey lines (yd}^3); \text{(average end area x 500 ft)}

\( V_{TOTAL} \) = \text{change in volume of sand for entire length of project}

* This total volume was calculated using profile lines for every 100 ft of beach.
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Δv = change in area of fill between consecutive surveys (yd²/ft)

Δv_{total} = change in volume of fill between survey lines (yd³); (average end area×500 ft)

V_{TOTAL} = change in volume of sand for entire length of project

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Appendix C

CUMULATIVE DISTRIBUTION FUNCTIONS OF THE PERCENTAGE OF ORIGINAL BEACHFILL PLANFORM WIDTH REMAINING AT VARIOUS TIMES: ESTIMATED FOR THE FENWICK ISLAND BEACH NOURISHMENT PROJECT
STO+00 AFTER 5 YEARS

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING \((W_o = 66.28)\)
STO+00 AFTER 20 YEARS

Probability of occurrence

Percentage of original fill width remaining (W₀ = 66.28°)
ST10+00 AFTER 1 YEAR

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING \(W_o=66.28\)
ST10+00 AFTER 20 YEARS

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING ($w_0=66.28'$)
ST20+00 AFTER 1 YEAR

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING ($W_o=66.28'$)
Probability of occurrence

ST20+00 after 5 years

Percentage of original fill width remaining (W_o = 66.28)
ST20+00 AFTER 10 YEARS

Probability of occurrence vs. percentage of original fill width remaining ($w_0=66.28'$).
ST40+00 AFTER 5 YEARS

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING ($W_o=66.28'$)
ST40+00 AFTER 10 YEARS

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING ($w_o=66.28'$)
ST40+00 AFTER 20 YEARS

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING (W_o = 66.28°)
ST60+00 AFTER 1, 5, 10, 15, AND 20 YEARS

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING ($w_o=66.28'$)
ST70+00 AFTER 5 YEARS

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING ($w_0=66.28'$)
ST70+00 AFTER 10 YEARS

PROBABILITY OF OCCURRENCE

PERCENTAGE OF ORIGINAL FILL WIDTH REMAINING ($W_o=66.28''$)
REFERENCES


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