TIME DOMAIN BOUNDARY INTEGRAL EQUATIONS FOR
SCATTERING OF ACOUSTIC WAVES BY MULTIPLE LAYERED OBSTACLES

{TIANYU QIU, FRANCISCO JAVIER SAYAS}  DEPT. OF MATHEMATICAL SCIENCES, UNIVERSITY OF DELAWARE

ABSTRACT

The problem of interest is the scattering of an acoustic wave by a bounded obstacle of different material properties, immersed in a uniform environment. We derive a system of time domain boundary integral equations which we approximate by a Galerkin boundary element method in space and convolution quadrature in time. With the aid of evolution equation theory, we derive stability and error estimates. Finally, some numerical examples are given.

TIME DOMAIN BOUNDARY INTEGRAL SYSTEM

The coefficients $\omega_{ij}(n)$ are defined through expansion

$$\sum_{n=0}^{\infty} \omega_{ij}(n) \phi(t-n) = \sum_{n=0}^{\infty} \omega_{ij}(n) \phi(t-n)$$

For space discretization, we use a boundary element Galerkin method, with the discontinuous space $X_h$ approximating $H^{1/2}$, the continuous space $Y_h$ for $H^{1/2}$, $\Gamma_h$ is a partition on the boundary of size $h$.

$$X_h = \{ \cdot : \Gamma \rightarrow \mathbb{R} : \mathbf{p} \in \mathbb{P}, \forall r \in \mathbb{R} \}$$

The error is measured at the final time step $T = 5$. Lower right is the convergence study of maximum error on several observation points in each subdomain. Lower left is the relative $L^1$ error of the unknowns on the boundary.

DISCRETIZATION IN TIME AND SPACE

Numerical Simulation

For time discretization, we use BDF-2 Convolution Quadrature. It steps on a uniform grid in time: $\phi(t_n) = \phi(m\Delta t)$, $m = 0, \ldots, n$. The convolution equation is approximated by its discrete analogue as an example,

$$(V \ast \phi)(t) = \sum_{n=0}^{\infty} \omega_{ij}(n) \phi(t-n)$$

The error in the numerical experiment is displayed on the left. An exact plane wave solution is constructed. Traces and normal derivatives are then fed into the Galerkin solver to see whether the exact solution can be recovered.

CONVERGENCE STUDY

Semidiscrete Stability and Error Estimate

Theorem 1. If $u^h$ is the semidiscrete acoustic field for a smooth enough incident wave $u_{\text{inc}} \in W^{1}_0(R; H^{1/2}(\Gamma_1), \gamma \in C_0^\infty(R; H^{1/2}(\Gamma_1)), \gamma \in C_0^\infty(R; H^{1/2}(\Gamma_1)))$, then $u^h \in \mathcal{C}^0_\text{db}(\Gamma_1; \Gamma_2)$, $\gamma u^h \in \mathcal{C}^0_\text{db}(\Gamma_1; \Gamma_2)$, $\gamma u^h \in \mathcal{C}^0_\text{db}(\Gamma_1; \Gamma_2)$. Moreover, $\|u^h\|_{L^2} + \|\gamma u^h\|_{L^2}$

$$\| \phi (t) - u(t) \|_{L^2} \leq C \sum_{j=0}^{\infty} j \left( \| \phi (t) - u(t) \|_{L^2} + \| \gamma u^h \|_{L^2} \right)$$

Theorem 2. If $\lambda \in W^{1}_0(R; H^{1/2}(\Gamma_1), \phi \in W^{1}_0(R; H^{1/2}(\Gamma_1), i = 1, 2$, then

$$\| \phi (t) - u(t) \|_{L^2} \leq C \sum_{j=0}^{\infty} j \left( \| \phi (t) - u(t) \|_{L^2} + \| \gamma u^h \|_{L^2} \right)$$

REFERENCES


