The Asset Allocation Debate: Provocative Questions, Enduring Realities*

Executive summary. In a landmark paper published in 1986, “Determinants of Portfolio Performance,” Gary P. Brinson, L. Randolph Hood, and Gilbert L. Beebower concluded that asset allocation is the primary determinant of a portfolio’s return variability, with security selection and market-timing playing minor roles. In the past decade, several authors have revisited the Brinson study, updating or challenging it. Some research has confirmed the study’s conclusions. Others have criticized the study—or, more accurately, its interpretation by the investment industry—and raised doubts about its applicability to general investors.

Through a review of this debate, empirical analysis, and application of financial theory, we conclude that:

- Broadly diversified portfolios with limited market-timing tend to move in tandem with broad financial markets over time, resulting in high time-series $R^2$s as reported by Brinson and others. Despite this co-movement, active management creates significant performance dispersion across portfolios, resulting in low $R^2$s across funds’ actual and policy returns in a given period, as reported by William W. Jahnke (1997) and Roger G. Ibbotson and Paul D. Kaplan (2000). Brinson and Jahnke focused on different aspects of portfolio returns, and the conclusions of both are right.

- Brinson’s results are a function of the broadly diversified nature and limited active management of pension fund portfolios in the aggregate. The magnitudes of time-series and cross-sectional $R^2$s are lower for portfolios that engage in a greater degree of active management.

• The ultimate concern in the active/passive decision is whether active management can increase the returns and/or decrease the risks of a portfolio, not whether it decreases the portfolio’s R² over time or across funds. We find that, on average, active management has reduced a portfolio’s returns and increased its volatility compared with a static index implementation of the portfolio’s asset allocation policy. However, active management creates an opportunity for a portfolio to outperform appropriate market benchmarks.

• Due to the distinct return patterns of asset classes, the impact of one asset allocation choice versus another on returns is generally modest and relatively stable over time. The influence of security selection and market-timing on returns can be more significant. However, active strategies tend to have a high skill hurdle, less stable and less predictable relative returns over time, and higher costs.

• Unless there is a strong belief in the ability to select active managers who will deliver higher risk-adjusted net returns, investors’ focus should be on the asset allocation choice and its implementation using broadly diversified, low-cost portfolios with limited market-timing.

Introduction

A portfolio’s policy, or long-term, asset allocation is the primary determinant of its return variability over time. This is widely accepted among investment researchers and practitioners, but it’s also the source of a heated debate among these same researchers and practitioners. This seeming paradox reflects disagreement about the practical implications of the empirical results, not about the results themselves.

In their landmark 1986 paper, Brinson and colleagues concluded that a portfolio’s static target asset allocation explained most of the portfolio’s total return and volatility over time. Active investment decisions—security selection and/or market-timing—played minor roles. These findings were subsequently confirmed by Vanguard and other researchers (Ibbotson and Kaplan, 2000). Investment advisors have generally interpreted this research to mean that selecting an appropriate asset allocation is more important than selecting the funds used to implement the allocation.

This interpretation has provoked criticism from some practitioners, notably Jahnke (1997), who argue that Brinson’s focus on explaining return volatility over time ignores the wide dispersion of total returns among portfolios. A portfolio may end up with very different wealth levels at the end of the investment horizon depending on which fund or funds were selected. In other words, Brinson’s approach might show that the return volatility of two funds, each with a portfolio of 60% stocks/40% bonds, is explained primarily by their asset allocation. What the Brinson methodology doesn’t reveal is that these two funds can have very different total returns (as opposed to return volatility over time), reflecting the results of the active decisions made in each portfolio and the costs associated with implementing those decisions. In addition, the magnitudes of R² over time and across funds are lower for portfolios that engage in greater degrees of active management.
Regardless of the degree of a portfolio's active management, the ultimate concern is whether active management can increase the portfolio's risk-adjusted returns. Our analysis shows that, on average, active management reduces a portfolio's returns and increases its volatility compared with a static index implementation of the portfolio's asset allocation policy. This is partly due to the higher implementation and management cost hurdles of active portfolios. In addition, our comparison may be imperfect, since the average fund universe may have somewhat different style and size exposures than the indexed policy benchmark. However, active management creates an opportunity for the portfolio to outperform—along with the risk to underperform—appropriate market benchmarks.

This paper reviews the different aspects of the asset allocation debate. We start with the most widely discussed disagreement: the differences between the variation in returns over time (the focus of Brinson's 1986 study) and the variation in returns across portfolios (the heart of Jahnke's 1997 critique of Brinson). We explore the impact of the sample used in the Brinson study on the results and the study's implications for an investor with a broader set of investment options. Finally, we report on the historical "success" of active management in increasing a portfolio's returns and/or decreasing its volatility.

We find that an investor's allocation to stocks, bonds, and cash investments is the most important determinant of the return variability and long-term total return level of broadly diversified portfolios with limited market-timing. As a portfolio assumes a higher degree of firm-specific (and, in theory, uncompensated) risk or market-timing risk, the impact of asset allocation on the portfolio's returns declines.

Despite the large potential influence of security-selection and market-timing strategies on a portfolio's returns, the amount of skill required to justify active management is very high (Kritzman and Page, 2003). Active returns tend to be unstable and unpredictable over time (Carhart, 1997). On the other hand, the impact of one asset allocation choice versus another on returns is relatively stable or “controllable” over time because of the distinct return patterns of asset classes. Therefore, investors should focus on the more controllable asset allocation choice and hold broadly diversified portfolios with limited market-timing. Because the cost spectrum among competing investment products with similar mandates can be large, cost-conscious implementation of the portfolio is crucial.

**Time-series or cross-sectional R²s: What do they mean to investors?**

The 1986 Brinson study represents a time-series analysis of the effect of asset allocation on performance. The methodology compared the performance of a policy, or long-term, asset allocation represented by appropriate market indexes with the actual performance of a portfolio over time. The findings indicated that, on average, most of a portfolio's return variability over time was attributed to its policy asset allocation return variability. Active investment decisions—market-timing and security selection—had relatively little impact on return variation over time.

This statement is not controversial, at least not in a universe of broadly diversified pension funds with limited market-timing. All broadly diversified portfolios are exposed to the systematic (undiversifiable) risk factors of financial markets, such as business cycles and interest rates. An assessment of what drives the performance of a diversified portfolio over time is likely to find a strong relationship between the

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1 According to the Capital Asset Pricing Model, investors are only compensated for bearing systematic risk since firm-specific risk can be diversified away.  
2 The difference between the lowest and the highest expense ratios of Morningstar large-capitalization core equity index funds was 247 basis points as of December 2006.
performance of a static portfolio made up of market benchmarks and the performance of an actual portfolio made up of asset-class exposures similar to those represented by the benchmarks. Brinson and colleagues found that pension funds were exposed to a high level of systematic market risk, resulting in high R²s between the funds' actual returns and the returns of their policy portfolios over time. Ibbotson and Kaplan (2000) found similar results for the balanced mutual fund universe.

Even so, the returns of the policy portfolio and the actual portfolio are not the same. As illustrated in Figure 1, idiosyncratic risks and differential exposure to systematic risk factors (factor or tactical overweights) can create significant performance variation across portfolios, resulting in a low R² across funds' actual returns and their policy returns in a given period, such as a month or several years.

Table 1 displays the results of our study of “balanced” mutual funds, which include asset allocation funds, total return funds, and traditional balanced funds. The first column presents the R² between the actual average returns of balanced funds and the average returns of their policy portfolios over time. The first column shows that, on average, fund returns tend to move in tandem with the markets.

The second column displays much lower R². These figures are at the heart of the “cross-sectional” critique of the 1986 Brinson study. Jahnke (1997) argues that the volatility of portfolio returns over time is unimportant to investors. Investors care about actual returns and the range of possible investment outcomes at the end of their time horizons. Jahnke’s approach is to examine the cross-sectional dispersion of total returns—that is, the range of returns produced by a group of portfolios over a particular time period. He finds that the differences in asset allocation among funds cannot explain the variation in total returns among funds.

Table 1. The role of asset allocation policy: Return variation and return dispersion of balanced funds, 1966–2006

<table>
<thead>
<tr>
<th></th>
<th>% of actual return variation explained by policy return variation</th>
<th>% of actual return dispersion explained by policy return dispersion</th>
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<tbody>
<tr>
<td>Average</td>
<td>82.1%</td>
<td>20.2%</td>
</tr>
<tr>
<td>Median</td>
<td>85.5</td>
<td>15.0</td>
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Notes: The sample included 189 balanced funds. Calculations were based on monthly returns, but results were similar for three-year return dispersion. The performance data shown represent past performance, which is not a guarantee of future results. Sources: University of Chicago Center for Research in Security Prices (CRSP) Survivor Bias-Free US Mutual Fund Database; Vanguard calculations.

We reach the same conclusion in our analysis of balanced mutual funds. Table 1 shows that the differences in return produced by funds’ policy allocations can explain around 20% of the actual dispersion of monthly returns. These actual returns reflect each fund’s idiosyncratic risks, risk factor exposures, costs, luck, and investment decisions. Although balanced fund returns move in tandem with broad markets over time, actual returns can vary.

3 We derived the policy allocations from the funds’ actual allocations on a five-year rolling basis. This approach allowed us to account for long-term policy shifts that reflected changes in a fund’s risk tolerance or assessment of long-term changes in risk premiums. Any short-term deviation from the five-year policy was considered active management. See the Appendix for details.

4 The cross-sectional R² for five-year returns, which is not reported here, is less than 30%. See the Appendix for details.
The impact of the sample population on time-series and cross-sectional $R^2$

The magnitudes of time-series and cross-sectional $R^2$s depend on the behavior of the portfolios analyzed. Consider a balanced portfolio that holds one stock and one bond. Changes in the price of each security would be influenced by the general movements of the stock and bond markets, producing a relatively high time-series $R^2$ between the variation in return of the one stock/one bond portfolio and the variation in return of a policy portfolio represented by stock and bond market indexes.

It’s likely, however, that the total return produced by the broad stock and bond markets and the total return of the two-security portfolio would be very different, leading to a low $R^2$ between the total returns of a sample of portfolios and their policy allocations for any given time period. On the other hand, if funds engaged in no active management, simply implementing their static policy allocations with index funds with the same cost, both the time-series and cross-sectional $R^2$s would theoretically be 100% (policy performance would explain all performance variation across funds as well as over time).

The high time-series $R^2$ of the 1986 Brinson study is a result of the broadly diversified nature and limited active management of pension fund portfolios. For instance, in the study, the lowest time-series $R^2$ was 75.5%, indicating that pension funds closely followed their indexed static asset allocation policies. Updates of the study (Ibbotson and Kaplan, 2000; Tokat and Sheay, 2007) found that while balanced funds are also typically broadly diversified, their management tends to be more active than pension funds, leading to lower time-series and cross-sectional $R^2$s. For instance, the fifth percentile time-series $R^2$ was 46.9% for Ibbotson and Kaplan’s balanced fund sample. In our sample, which includes total return funds, asset allocation funds, and traditional balanced funds, we find that the lowest time-series $R^2$ was 33.6%. These results suggest that the magnitudes of time-series and cross-sectional $R^2$s are a factor of the degree of active management in the portfolio.

What has been overlooked in this debate is that the ultimate concern of an investor is not the time-series or cross-sectional $R^2$ but whether active management can increase a portfolio’s return without increasing the portfolio’s risk. The 1986 Brinson study provided a framework for addressing this issue.

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Data

We analyzed balanced funds in the University of Chicago CRSP Survivor Bias-Free US Mutual Fund Database. The data include monthly net returns, annual allocations to asset classes, and fund characteristics such as expense ratios and turnover rates. Multiple share classes of the same fund were aggregated by market cap, weighting returns and expenses. To ensure reliability, we only analyzed funds with at least 36 months of return history.

Balanced mutual funds were selected using several filters. We defined a “balanced” fund as one with average long-run equity and bond allocations of more than 20% over its lifetime. Among these funds, we selected total return funds, income funds, asset allocation funds, and traditional balanced funds based on CRSP fund categorizations. If a fund return for a single month was missing, that month was excluded from the analysis. Funds with more than 5% of their assets devoted to an asset class other than stocks, bonds, and cash investments over their lifetimes were excluded from the analysis.

We found that the fifth percentile time-series $R^2$ was 59.4% in our sample.

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What matters most to investors: Return and risk

The most important contribution of Brinson and colleagues (1986) was the attribution of a portfolio’s total return to indexed static asset allocation policy, security selection, and market-timing components. They showed that, on average, pension funds have not been able to add value above their static indexed policy returns through market-timing or security selection. This result is consistent with the observation that indexing outperforms a significant portion of active portfolios in equity and bond markets (see, for example, Mark Carhart (1997)).

Our analysis produced a similar conclusion. Table 2 shows that, from 1966 to 2006, balanced mutual funds, on average, detracted from their performance and increased their volatility relative to their indexed static asset allocation policies. Figure 2 illustrates that, for the same period, the average median net excess return versus funds’ indexed static policy benchmarks was negative. This is partly due to the higher implementation and management cost hurdles of active portfolios. In addition, our comparison may be imperfect since the average fund universe may have somewhat different style and size exposures than the indexed policy benchmark. The results over shorter time frames are similar.

However, when funds were ranked based on their rolling five-year net excess returns, active management created meaningful cross-sectional variation in performance (see Figure 2). Confirming Jahnke’s (1997) criticism, the return difference between funds in the top and bottom 25th percentiles was as high as 34.9%, with an average of 18.5%.

Table 2. The role of asset allocation policy: Returns and volatility of balanced funds, 1966–2006

<table>
<thead>
<tr>
<th>Policy return as % of actual return</th>
<th>Policy volatility as % of actual volatility</th>
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<tbody>
<tr>
<td>Average 106.8</td>
<td>89.3%</td>
</tr>
<tr>
<td>Median 106.8</td>
<td>92.0</td>
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</table>

Note: The sample included 189 balanced funds. Calculations were based on monthly returns. The policy portfolio was assumed to have a cost of 2 basis points each month (approximately 25 basis points annually). The performance data shown represent past performance, which is not a guarantee of future results. Source: University of Chicago CRSP Survivor Bias-Free US Mutual Fund Database; Vanguard calculations.

Figure 2. Rolling five-year cumulative net excess returns against estimated benchmark, 1968–2006

Note: Includes data for total return funds, income funds, asset allocation funds, and traditional balanced funds. Multiple share classes of the same fund are aggregated by fund asset size, weighting returns. Balanced-fund policy benchmarks are assigned using style analysis over five-year rolling periods (requiring a minimum of three years of data). The performance data shown represent past performance, which is not a guarantee of future results. Sources: University of Chicago CRSP Survivor Bias-Free US Mutual Fund Database; Vanguard calculations.

6 Actively managed portfolios tend to have smaller market caps than their respective benchmarks, which may create benchmark problems with broad market indexes, such as the Dow Jones Wilshire 5000 Composite Index, which we used in this study.
Although active management can create significant performance variation, the degree of skill required to justify active management is very high (Kritzman and Page, 2003). As illustrated in Figure 3, 56% of balanced funds underperformed their policy portfolios on an annual basis over a fifteen-year period. About 58% underperformed their estimated policy portfolios over three and five years. Since actively managed funds tend to have smaller market capitalizations than their respective benchmarks, the percentages reported in Figure 3 may vary, in part due to the benchmark comparison issues.7

Although a greater degree of active management reduces both time-series and cross-sectional R²s, it does not necessarily increase performance. Financial theory and empirical evidence show that exposure to systematic risk is compensated over time. Active management risk is not compensated on average (Sharpe, 1991); however, it is compensated if skill overcomes the higher cost hurdle of active management.

7 For instance, dollar-weighted average market cap was $57.2 billion for the Lipper Average General Equity category as of June 30, 2007, compared with $83.0 billion for the Dow Jones Wilshire 5000 Index as of December 31, 2006.
Characteristics of funds with positive and negative alpha

Our results confirm that the average actively managed fund reduces returns and increases return variability versus a passive policy benchmark. Of course, our analysis also reveals that some actively managed balanced funds have significantly outperformed their policy benchmarks over time. What are the general characteristics of these “winning” funds? And how do they compare with the broader universe of active balanced funds?

Table 4 sorts our universe of balanced funds into three cohorts: (1) funds that posted a statistically significant positive “excess return,” or alpha, over their estimated policy benchmarks (30 of the 189 balanced funds, or about 16% of the sample), (2) those funds that have significantly trailed the performance of their policy allocations (36% of the funds), and (3) the remainder of the funds, whose average excess return is calculated at approximately zero (48% of the funds).

Table 4 reveals that the “winning” funds outperformed their policy benchmark returns by 24.0 basis points per month, or 2.88 percentage points per year, on average. The funds that consistently underperformed trailed their policy benchmarks by an average of 16.8 basis points per month, or 2.02 percentage points per year. As is shown in the table, outperforming funds achieve higher returns than their policy allocations (72.7% policy-to-actual return ratio) by incurring more active-management risk (84.7% policy-to-actual volatility ratio). Conversely, underperforming funds earn a lower return than their policy allocations (129.5% policy-to-actual return ratio) while still incurring more active-management risk than their benchmarks (90.6% policy-to-actual volatility ratio).

Although manager skill certainly plays a role in distinguishing positive-alpha from negative-alpha funds, other differences shown in the table are noteworthy. In general, we find that “winning” active funds had lower expenses, lower portfolio turnover, and more assets under management than the consistently underperforming funds.

<table>
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<tr>
<td><strong>Average risk and return statistics</strong></td>
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<tr>
<td><strong>Average monthly alpha (basis points)</strong></td>
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<tr>
<td>All funds</td>
</tr>
<tr>
<td>3.1</td>
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<tr>
<td>Policy return as percentage of actual return</td>
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<tr>
<td>Policy volatility as percentage of actual volatility</td>
</tr>
<tr>
<td>Return variability explained by policy variability</td>
</tr>
<tr>
<td><strong>Average fund characteristics</strong></td>
</tr>
<tr>
<td>Expense ratio</td>
</tr>
<tr>
<td>Net assets (millions)</td>
</tr>
<tr>
<td>Turnover</td>
</tr>
<tr>
<td>Number of funds</td>
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</tbody>
</table>

Sources: University of Chicago Center for Research in Security Prices (CRSP) Survivor Bias-Free US Mutual Fund Database; Vanguard calculations.
**Should an asset allocation policy be static or dynamic?**

The recent asset allocation debate has also called into question the wisdom of establishing a static long-term policy allocation. Investors determine their asset allocation policy based on their risk tolerance, their financial goals, their time horizon, their nonfinancial wealth (such as income), and the risk premiums of the asset classes. Any one of these variables can change, potentially prompting a change in an investor’s asset allocation policy. Some changes—for example, in time horizon or financial goals—are easy to gauge, allowing for a relatively simple adjustment to the policy allocation. Other changes—for example, in expected returns and risk premiums—are harder to detect.

Jahnke (1997) was the first to point out that the investment industry’s interpretation of the 1986 Brinson study, namely, its conclusion that an indexed static asset allocation policy is the optimal approach for investors, has been misinterpreted. In the industry, the conclusions of Brinson and his colleagues were typically used to focus on getting the asset allocation right without much regard for funds’ performances or costs. Jahnke noted that static allocations rarely related directly to a client’s specific circumstances or long-term financial goals. It is clear from financial theory and practical experience that investors’ asset allocation choices should be linked with their specific circumstances or long-term financial goals.

More recently, several authors have issued a more profound challenge to the concept of a static policy asset allocation. These researchers are asking whether investors should change their asset allocation policies dynamically in response to changing expected returns and capital market opportunities (Jahnke, 1997; Bernstein, 2003; Foley, 2004). Expected returns are not static, at least over shorter time frames, so the logic of a static asset allocation is suspect.

**Table 3. Historical returns from market-timing and security selection**

<table>
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<tbody>
<tr>
<td>Market-timing</td>
<td>–0.66%</td>
<td>–0.26%</td>
</tr>
<tr>
<td>Security selection</td>
<td>–0.36</td>
<td>+0.26</td>
</tr>
<tr>
<td>Other</td>
<td>–0.07</td>
<td>–0.07</td>
</tr>
<tr>
<td>Total active return</td>
<td>–1.10%</td>
<td>–0.08%</td>
</tr>
</tbody>
</table>

Note: The sample included 227 balanced funds. Calculations were based on monthly returns, but results were similar for three-year return dispersion. The performance data shown represent past performance, which is not a guarantee of future results.

Sources: Brinson et al. (1986, 1991).

Although these authors’ premise is sound, the implementation of dynamic asset allocation is problematic. Only if investors have the ability to predict expected returns in financial markets can dynamic, or tactical, asset allocation enhance portfolio performance. Asset-return predictability studies (for instance, Goyal and Welch, 2004; Campbell and Thompson, 2004) show that the in-sample predictive ability of financial and economic variables strongly deteriorates in out-of-sample forecasts. What works in historical studies has been far less successful in other time periods.

The 1986 Brinson study raises additional doubts about the wisdom of dynamic asset allocation. If we assume that pension funds in the study changed their asset allocation policies in response to changing market conditions (rather than in response to funding concerns), Table 3 indicates that, even before management costs are factored in, active asset allocation, on average, has detracted from the performance of pension funds from 1974 to 1987. This finding underscores the difficulty of timing markets. However, it is important to recognize that some pension funds have done better and others have done worse than their policy performance.
Vanguard’s assessment

The goal of active management is to increase the risk-adjusted returns of a portfolio. Active management around the static index implementation of an asset allocation policy has, on average, reduced returns and increased volatility. However, active management creates an opportunity for the portfolio to outperform appropriate market benchmarks. While the variability of returns can be explained largely by asset allocation policy, the range of total returns produced over a given time period can vary greatly. Since the impact of active management tends to be less stable and less predictable than the impact of an asset allocation choice, we believe it is preferable to select asset allocations appropriate to investors’ unique circumstances and to construct broadly diversified portfolios with limited market-timing. To the extent that active management plays a role in a portfolio, investors should consider selecting active funds where the hurdles that must be overcome by skill—for example, costs—are lower. Asset allocation remains the primary determinant of returns in portfolios made up of index or broadly diversified funds with limited market-timing.

References


Some key terms

**Alpha.** A risk-adjusted measure of the “excess return” provided by an investment compared with a benchmark. Alpha can be positive, negative, or zero.

**Expense ratio.** A mutual fund’s annual operating costs expressed as a percentage of average net assets.

**Net assets.** The closing market value of a fund’s assets minus its liabilities.

**R-squared.** A measure of how much of a portfolio’s performance can be explained by the returns from the overall market (or a benchmark index).

**Regression.** Regression analysis may be used to explain the nature and strength of the relationship between one dependent variable (Y) and one or more other independent variables.

**Turnover.** An indication of a fund’s trading activity. Turnover represents the lesser of aggregate purchases or sales of securities divided by average net assets.
Appendix

Empirical methodology
To determine the relative performance of asset allocation policy and active management, we distinguished between a portfolio’s policy return—what it would have earned if it simply recreated its policy allocation with unmanaged index funds—and its actual return—the real-world return that reflects a fund’s execution of active strategies. We calculated a fund’s policy return through indirect empirical methods because, in a universe of actively managed funds, the policy return is, by definition, not observed in the actual returns.

Our empirical and quantitative analysis included six primary steps: (1) style analysis, which allowed us to infer the funds’ policy allocations; (2) simple calculation of policy returns using asset-class benchmarks and policy weights inferred from style analysis; (3) time-series analysis—a regression of the funds’ actual returns against their policy returns over time—which gave us the R²; (4) calculation of the ratio of a fund’s actual return to the return of its policy allocation; (5) calculation of the ratio of a fund’s actual volatility to the volatility of its policy allocation; (6) cross-sectional analysis—a regression of the funds’ actual returns against their policy returns in a given period—which gave us the cross-sectional R².

The details of each calculation appear below.

1. Estimation of policy allocation using style analysis
The policy weightings, or asset allocation, for each fund were estimated by performing returns-based style analysis over the rolling five-year history of the fund. Style analysis (Sharpe, 1988) is a statistical method for inferring a fund’s effective asset mix by comparing the fund’s returns with returns of asset-class benchmarks. Style analysis is a popular attribution technique because it does not require tabulating the actual asset allocation of each fund for each month over time. Rather, style analysis facilitates return attribution by regressing the return of the fund against the returns of asset-class benchmarks. The following regression is estimated:

$$r_{\text{fund}} = \alpha + w_{\text{Stock}} r_{t}^{S} + w_{\text{Bond}} r_{t}^{B} + w_{\text{Cash}} r_{t}^{C} + \varepsilon_{t}$$

where

- $w_{\text{Stock}}$ is the policy allocation to stocks,
- $w_{\text{Bond}}$ is the policy allocation to bonds,
- $w_{\text{Cash}}$ is the policy allocation to cash,
- $r_{t}^{S}$ is the return on the equity benchmark in period $t$,
- $r_{t}^{B}$ is the return on the bond benchmark in period $t$,
- $r_{t}^{C}$ is the return on the cash benchmark in period $t$,
- $\alpha$ is the excess return of the fund that cannot be attributed to the returns of benchmarks, and
- $\varepsilon_{t}$ is the residual that cannot be explained by the asset-class returns.

For our purposes, style analysis requires not only that asset-class weight parameters sum to 1, but also that each asset-class weight is positive (no short sales).

2. Calculation of policy return
The policy return of a fund is calculated from the policy weights and returns of asset-class benchmarks

$$r_{t}^{\text{Policy}} = w_{\text{Stock}} r_{t}^{S} + w_{\text{Bond}} r_{t}^{B} + w_{\text{Cash}} r_{t}^{C} - \text{cost}$$

where

- $w_{\text{Stock}}$ is the policy allocation to stocks,
- $w_{\text{Bond}}$ is the policy allocation to bonds,
- $w_{\text{Cash}}$ is the policy allocation to cash,
- $r_{t}^{S}$ is the return on the equity benchmark in period $t$,
- $r_{t}^{B}$ is the return on the bond benchmark in period $t$,
- $r_{t}^{C}$ is the return on the cash benchmark in period $t$, and
- cost is the approximate cost, as a percentage of assets, of replicating the policy mix using indexed mutual funds. The cost is assumed to be 2 basis points each month (approximately 25 basis points annually).
3. Time-series regression of actual returns against policy returns
To compare variation in the policy and actual returns, we calculated an $R^2$ for each fund by regressing its actual return against its policy return:

$$r_{t, \text{fund}} = \alpha + \beta r_{t, \text{policy}} + \epsilon_t,$$

where

- $\alpha$ is the excess return of the fund that cannot be attributed to the policy return,
- $\beta$ is the sensitivity of changes in the fund return to changes in the policy return, and
- $\epsilon_t$ is the residual that cannot be explained by the policy return.

4. The ratio of the average policy return to the average actual return
The policy return as a percentage of the average actual return of each fund is the ratio of its average policy return to its average actual return:

$$\frac{1}{T} \sum_{t=1}^{T} r_{t, \text{policy}} \div \left( \frac{1}{T} \sum_{t=1}^{T} r_{t, \text{fund}} \right)$$

When the average policy return is greater than the average actual return, this ratio is greater than 100%.

5. The ratio of policy volatility to actual volatility
The policy volatility as a percentage of the actual return volatility of each fund is the ratio of the standard deviation of the policy return to the standard deviation of the actual return:

$$\sqrt{ \frac{1}{T-1} \sum_{t=1}^{T} (r_{t, \text{policy}} - \text{avg}(r_{\text{policy}}))^2 } \div \sqrt{ \frac{1}{T-1} \sum_{t=1}^{T} (r_{t, \text{fund}} - \text{avg}(r_{\text{fund}}))^2 }$$

When policy volatility is smaller than actual return volatility, this ratio is less than 100%.

6. Cross-sectional regression of actual returns against policy returns
To compare variation in the policy and actual returns across different funds, we calculated an $R^2$ in a given month by regressing the actual returns against the policy returns for all funds in that month:

$$r_{t, \text{fund}} = \alpha + \beta r_{t, \text{policy}} + \epsilon_t,$$

where

- $\alpha$ is the excess return of the fund that cannot be attributed to the policy return,
- $\beta$ is the sensitivity of changes in the fund return to changes in the policy return, and
- $\epsilon_t$ is the residual that cannot be explained by the policy return.

Benchmarks
For stock market returns, we used the Standard & Poor’s 500 Index from 1962 to 1970 and the Dow Jones Wilshire 5000 Index from 1971 to 2006. For the bond market returns, we used the S&P High Grade Corporate Index from 1962 to 1968, the Citigroup High Grade Corporate Index from 1969 to 1975, and the Lehman Brothers U.S. Aggregate Index from 1976 to 2006. For the returns on cash investments, we used the 3-month Treasury bill rate from 1962 to 1977 and the Citigroup 3-Month U.S. Treasury Bill Index from 1978 to 2006.
For more information about Vanguard® funds, visit www.vanguard.com, or call 800-662-2739, to obtain a prospectus. Investment objectives, risks, charges, expenses, and other important information about a fund are contained in the prospectus; read and consider it carefully before investing.

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