Online Identification and Stochastic Control for Autonomous Internal Combustion Engines

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Advanced internal combustion engine technologies have afforded an increase in the number of controllable variables and the ability to optimize engine operation. Values for these variables are determined during engine calibration by means of a tabular static correlation between the controllable variables and the corresponding steady-state engine operating points to achieve desirable engine performance, for example, in fuel economy, pollutant emissions, and engine acceleration. In engine use, table values are interpolated to match actual operating points. State-of-the-art calibration methods cannot guarantee continuously the optimal engine operation for the entire operating domain, especially in transient cases encountered in the driving styles of different drivers. This article presents brief theory and algorithmic implementation that make the engine an autonomous intelligent system capable of learning the required values of controllable variables in real time while operating a vehicle. The engine controller progressively perceives the driver’s driving style and eventually learns to operate in a manner that optimizes specified performance criteria. A gasoline engine model, which learns to optimize fuel economy with respect to spark ignition timing, demonstrates the approach.

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1 Introduction

Increasing demand for improving fuel economy and reducing emissions without sacrificing performance have induced significant research and investment in advanced internal combustion engine technologies. These technologies, such as fuel injection systems, variable geometry turbocharging, variable valve actuation, and exhaust gas recirculation, have introduced a number of engine variables that can be controlled to optimize engine operation. In particular, computing the optimal values of these variables, referred to as engine calibration, has been shown to be especially critical for achieving high engine performance and fuel economy while meeting emission standards. Consequently, engine calibration is defined as a procedure that optimizes one or more engine performance criteria, e.g., fuel economy, emissions, or engine acceleration with respect to the engine controllable variables.

State-of-the-art engine calibration methods generate a static correlation between the values of the controllable variables and the corresponding steady-state operating points [1]. This correlation is incorporated into the electronic control unit (ECU) of the engine to control engine operation. Design of experiments (DoE) has been widely used as the baseline method for engine calibration. The main objective of DoE is to expedite dynamometer tests significantly using a smaller subset of tests. This subset is utilized either in implementing engine calibration experimentally or in developing mathematical models for evaluating engine output. Using these models, optimization methods can determine the static correlations between steady-state operating points and the controllable engine variables. Rask et al. [2] developed a simulation-based calibration method to generate rapidly optimized correlations for a V6 engine equipped with two-step variable valve actuation and intake cam phasing. Guerrier et al. [3] employed DoE and advanced statistical modeling to develop empirical models to enhance the powertrain control module calibration tables. Stuhler et al. [4] implemented an automated calibration environment using an online DoE to decrease calibration cost.

Calibration becomes more difficult for transient engine operation. Correlations of optimal values for controllable variables associated with transient operating points cannot be quantified explicitly; to prespecify the entire transient engine operation is impractical. Research efforts in addressing transient operation have focused on simulation-based methods to derive calibration maps for transients of particular driving cycles. Burk et al. [5] presented the necessary procedures required to utilize cosimulation techniques with regard to predicting engine drive cycle performance for a typical vehicle. Jacquelin et al. [6] utilized analytical tools to run the FTP-75 driving cycle through precomputed engine performance maps, depending on engine speed, load, intake, and exhaust cam centerline positions. Atkinson et al. [7] implemented a dynamic system to provide optimal calibration for transient engine operation of particular driving cycles. These methods utilize engine models sufficiently accurate to portray fuel economy and feed-gas emissions during transient engine operation. However, identifying all possible transients, and thus deriving optimal values of the controllable variables through calibration maps for those cases prior, remains very difficult. Alternative approaches involve use of artificial neural networks (ANNs) [8–13] to evaluate engine performance criteria with respect to controllable variables. ANNs are computationally efficient for optimization; however, the above difficulties related to transient operating points remain.

This article presents the theoretical framework and a control algorithm that makes the engine an autonomous intelligent system that can learn its optimal calibration in real time while the driver drives a vehicle. The engine is treated as a controlled stochastic system, and engine calibration is formulated as a sequential decision-making problem under uncertainty. While the engine is running the vehicle, it progressively perceives the driver’s driving style and eventually learns to operate in a manner that optimizes specified performance criteria, e.g., fuel economy, emissions, or engine acceleration. Optimal calibration is achieved for steady-state and transient engine operating points resulting from the driver’s driving style. The engine’s ability to learn its optimum calibration is not limited to a particular driving style. The engine can learn to operate optimally for different drivers if they indicate their identity before starting the vehicle. The engine can then adjust its operation to be optimal for a particular driver based on what it has learned in the past regardless of his/her driving style.

The remainder of the article proceeds as follows: Sec. 2 presents the theoretical framework of modeling engine operation as a controlled Markov decision process and formulates engine calibration as a sequential decision-making problem under uncertainty; the control algorithm that solves the decision-making problem in real time is also introduced. The effectiveness of the approach is demonstrated on a gasoline engine model in Sec. 3, while the engine is running the vehicle, it learns to optimize fuel economy with respect to spark ignition timing. Conclusions are presented in Sec. 4.
2 Proposed Method

Engines are streamlined syntheses of complex physical processes determining a convoluted dynamic system. They are operated with reference to engine operating points and the values of various engine controllable variables. At each operating point, these values highly influence engine performance criteria, e.g., fuel economy, emissions, or acceleration. This influence becomes more important at engine operating point transitions designated partly by the driver’s driving style and partly by the engine controllable variables. Consequently, the engine is a system whose behavior is not completely foreseeable, and its future evolution (operating point transitions) depends on the driver’s driving style.

Transient operation constitutes the largest segment of engine operation over a driving cycle compared with the steady-state one [14,15]. Emissions during transient operation are extremely complicated [15], vary significantly with each particular driving cycle [16,17], and are highly dependent on the calibration [17,18]. Engine operating points, during the transient period before their steady-state value is reached, are associated with different brake-specific fuel consumption (BSFC) values, depending on the directions from which they have been arrived, as illustrated qualitatively in Figs. 1 and 2. Pollutant emissions, such as NOx, and particulate matters demonstrate the same qualitative behavior, as shown by Hagen et al. [19]. Consequently, the optimal values of the controllable variables corresponding to steady-state operating points cannot capture efficiently the transient engine operation.

Engine operation is described in terms of its operating points, and the evaluation of performance indices is a function of various controllable variables. Here, the engine performance indices are treated as random functions, the engine is treated as a controlled stochastic system, and the engine operation is treated as a stochastic process. Engine calibration is thus reformulated as a sequential decision-making problem under uncertainty. The goal is to select values of the controllable variables for each engine operating point in real time that optimize the random functions (engine performance indices). The Markov decision process (MDP) provides the mathematical framework for modeling sequential decision-making problems under uncertainty [20]; it comprises (a) a decision maker (controller), (b) states (engine operating points), (c) control actions (engine controllable variables), (d) a transition probability matrix (driver), (e) a transition cost (or reward) matrix (engine performance criteria), and (f) an optimization criterion (e.g., maximizing fuel economy, minimizing pollutant emissions, and maximizing engine acceleration). A discrete-time, stochastic controlled MDP is defined as the tuple

$$(1) \{ S,A,(P(\cdot,\cdot)),R(\cdot,\cdot) \}$$

where $S=\{1,2,\ldots,N\}$, $N \in \mathbb{N}$, denotes a finite state space, $A = \cup_{k=1}^N \mathcal{A}(s_k)$ stands for a finite action space, $P(\cdot,\cdot)$ is the transition probability matrix, and $R(\cdot,\cdot)$ is the transition cost matrix. The decision-making process occurs at each of a sequence of decision epochs $k=0,1,2,\ldots,M$, $M \in \mathbb{N}$. At each epoch, the decision-maker observes a system’s state $s_k \in S$ and executes an action $a_k$, from the feasible set of actions $\mathcal{A}(s_k) \subseteq A$ at this state. At the next epoch, the system transits to the state $s_{k+1}=j \in S$ imposed by the conditional probabilities $p(s_{k+1}=j|s_k=i,a_k)$, designated by the transition probability matrix $P(\cdot,\cdot)$. These conditional probabilities of $P(\cdot,\cdot)$, $p:S \times A \rightarrow [0,1]$, satisfy the constraint

$$(2) \sum_{j=1}^{N} p(s_{k+1}=j|s_k=i,a_k) = 1$$

Following this state transition, the decision maker receives a cost associated with the action $a_k$, $R(s_{k+1}=j|s_k=i,a_k), R:S \times A \rightarrow \mathbb{R}$, as imposed by the transition cost matrix $R(\cdot,\cdot)$.

The states of a MDP possess the Markov property, stating that the conditional probability distribution of future states of the process, given the present state and all past states, depends only on the current state and not on any past states; i.e., it is conditionally independent of the past states (the path of the process) given the present state. Mathematically, the Markov property requires that

$$(3) p(s_{k+1}|s_k,s_{k-1},\ldots,s_0) = p(s_{k+1}|s_k)$$

2.1 The Cost of a Markov Control Policy. The solution to a MDP can be expressed as an admissible control policy so that a given performance criterion is optimized over all admissible policies $\Pi$. An admissible policy consists of a sequence of functions

$$(4) \pi = \{ \mu_0, \mu_1, \ldots, \mu_{M-1} \}$$

where $\mu_k$ maps states $s_k$ into actions $a_k=\mu(s_k)$ and is such that $\mu_k(s_k) \in \mathcal{A}(s_k)$, $\forall s_k \in S$. A Markov policy $\pi$ determines the probability distribution of state process $\{s_k,k \geq 0\}$ and the control process $\{a_k,k \geq 0\}$. Different policies will lead to different probability distributions. In optimal control problems, the objective is to derive the optimal control policy that minimizes (maximizes) the accumulated cost (reward) incurred at each state transition per decision epoch. If a policy $\pi$ is fixed, the cost incurred by $\pi$ when the process starts from an initial state $s_0$ and up to the time horizon $M$ is

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where the expectation is taken with respect to the probability distribution of \( \{ s_k, a_k \} \) and \( \{ a_k, k \geq 0 \} \) determined by the Markov policy \( \pi \). The optimal policy \( \pi^* = \{ \mu_0, \mu_1, \ldots, \mu_M \} \) can be derived by

\[
\pi^* = \arg \min_{\pi \in \Pi} J^\pi(s_0)
\]

(6)

A large class of sequential decision-making problems under uncertainty can be solved using classical dynamic programming, originally proposed by Bellman [21]. Algorithms, such as value iteration, policy iteration, and linear programming, are employed to find optimal solution of MDPs. However, the computational cost of these algorithms in some occasions may be prohibitive and can grow intractably as the size of the problem increases. In addition, the cost of these algorithms in some occasions may be prohibitive and can grow intractably as the size of the problem increases. In addition, the cost of these algorithms in some occasions may be prohibitive and can grow intractably as the size of the problem increases. In addition, the cost of these algorithms in some occasions may be prohibitive and can grow intractably as the size of the problem increases.

2.2 Online Identification and Stochastic Control. The stochastic formulation of the engine calibration problem involves two major subproblems: (a) the engine identification problem and (b) the stochastic control problem. The first is exploitation of the information acquired from the engine output to identify its behavior; that is, how an engine representation can be built by observing engine operating point transitions and associated costs designated by the driver’s driving style, i.e., the transition probability matrix \( P(\cdot, \cdot) \) and the transition cost matrix \( R(\cdot, \cdot) \). The second forms the stochastic control subproblem, which is an assessment of the engine output with respect to alternative control policies, and selecting those that optimize specified engine performance criteria, e.g., fuel economy, pollutant emissions, or engine acceleration, that is, solving Eq. (6).

In our approach, a self-learning controller (decision maker) is faced with the problem of influencing engine operation as it evolves over time by selecting values of the controllable variables. The goal of the controller is to learn the sequences of engine operating point transitions corresponding to the driver’s driving style and select the control policy (values of the controllable variables) that cause the engine to perform optimally with respect to some predetermined performance criterion (cost function). A key aspect of the stochastic control problem is that decisions are not viewed in isolation. Consequently, the self-learning controller should select those values that balance the desire to minimize the cost function of the next engine operating transition against the desire to avoid future operating point transitions where high cost is inevitable. To this end, the predictive optimal decision-making (POD) computational learning model [22] is employed. The model embedded in the self-learning controller aims to address the state estimation and system identification problem for a completely unknown system by learning in real time the system dynamics over a varying and unknown finite time horizon. It is constituted by a state representation, which provides an efficient process in realizing the state transitions that occurred in the Markov domain. The convergence of POD to the stationary distribution of the Markov chain was proven in Ref. [23], hence, establishing POD as a robust model toward making autonomous intelligent systems that can learn to improve their performance over time in stochastic environments. While the driver drives the vehicle, the model progressively perceives the driver’s driving style by means of the transition probability matrix, \( P(\cdot, \cdot) \). In addition, the model captures specified engine performance criteria, e.g., fuel economy, pollutant emissions, and engine performance, by means of the elements of the transition cost matrix, \( R(\cdot, \cdot) \).

The learning process of the controller, illustrated in Fig. 3, transpires while the engine is running the vehicle and interacting with the driver. Taken in conjunction with assigning values of the controllable variables from the feasible action space, \( A \), this interaction portrays the progressive enhancement of the controller’s “knowledge” of the driver’s driving style with respect to the controllable variables. More precisely, at each of a sequence of decision epochs \( k = 0, 1, 2, \ldots, M \), the driver introduces a state \( s_k \in S \) to the controller, and on that basis the controller selects an action, \( a_k = \mu(s_k) \). This state arises as a result of the driver’s driving style corresponding to particular engine operating points. One epoch later, as a consequence of this action, the engine transits to a new state \( s_{k+1} = j \in S \) and receives a numerical cost, \( R(k, s_{k+1}, s_k, a_k) \in \mathbb{R} \).

At each epoch, the controller implements a mapping from the Cartesian product of the state space and action space to the set of real numbers, \( S \times A \rightarrow \mathbb{R} \), by means of the costs that it receives, i.e., the transition cost matrix, \( R(\cdot, \cdot) \). Similarly, another mapping from the Cartesian product of the state space and action space to the closed set \([0, 1]\) is executed, \( S \times A \rightarrow [0, 1] \), i.e., the transition probability matrix, \( P(\cdot, \cdot) \). The latter essentially perceives the incidence in which particular states or particular sequences of states arise. The implementation of these two mappings aims the controller to derive the control policy designated by the stochastic control algorithm. This policy is expressed by means of a mapping from states to probabilities of selecting the actions, resulting in the minimum expected accumulated cost.

The objective of the control algorithm is to evaluate in real time the action at each epoch that is optimal not only for the current state but also for the next two subsequent states over the following epochs. The requirement of real-time implementation imposes a computational burden in allowing the algorithm to look further ahead of time and, thus, evaluating an action over additional succeeding states. Suppose that the current state is \( s_k \) and the following state given an action \( a_k \in A(s_k) \), is \( s_{k+1} \). The immediate cost incurred by this transition is \( R(s_{k+1}, s_k, a_k) \). The minimum expected cost for the next two subsequent states is perceived in terms of the magnitude, \( V(s_{k+1}) \), and is equal to

\[
V(s_{k+1}) = \min_{a_k \in A(s_k)} \mathbb{E} \left\{ R(s_{k+1}, s_k, a_k) \right\}
\]

(7)

All uncertain quantities are described by probability distributions and the expected value of the overall cost is minimized. The control policy \( \pi \) realized by the algorithm is based on the minimax
control approach, whereby the worst possible values of the uncertain quantities within the given set are assumed to occur. This essentially assures that the control policy will result in at most a maximum overall cost. At state $s_k$, the control algorithm provides the policy $\pi = \{\mu_0, \mu_1, \ldots, \mu_{M-1}\}$, in terms of the values of the controllable variables as

$$\pi(s_k) = \arg \min \max \left[ R(s_{k+1}|s_k, \mu_i) + V(s_{k+1}) \right] \quad (8)$$

3 Application: Gasoline Engine Calibration With Respect to Spark Ignition Timing

In gasoline engines, the fuel and air mixture is prepared before it is ignited by the spark discharge. The major objectives for the spark ignition are to initiate a stable combustion and to ignite the air-fuel mixture at the crank angle resulting in maximum efficiency, while fulfilling emissions standards and preventing the engine from knocking. Simultaneous achievement of the aforementioned objectives is sometimes inconsistent; for instance, at high engine loads, the ignition timing for maximum efficiency has to be abandoned in favor of prevention of engine destruction by way of engine knock. Two key parameters are controlled with the spark ignition: ignition energy and ignition timing. Control of ignition energy is important for assuring combustion initiation, but the focus here is on the spark timing that maximizes engine efficiency. Ignition timing influences nearly all engine outputs and is essential for efficiency, drivability, and emissions. The optimum spark ignition timing generating the maximum engine brake torque is defined as maximum brake torque (MBT) timing. Any ignition timing that deviates from MBT lowers the engine’s output torque, as illustrated in Fig. 4. The BSFC, defined as the fuel flow rate per unit power output evaluates how efficiently an engine is utilizing the fuel supplied to produce work

$$BSFC(g/kW \cdot h) = \frac{\dot{m}_f(g/h)}{P(kW)} \quad (9)$$

where $\dot{m}_f$ is the fuel mass flow rate per unit time, and $P$ is the engine’s power output. Continuous engine operation at MBT ensures optimum fuel economy with respect to the spark ignition timing.

In an ideal gasoline engine calibration with respect to spark ignition timing, the engine should operate at the MBT timing for each engine operating point. By achieving MBT timing for all steady-state and transient operating points, an overall improvement of the BSFC is expected. Aspects of preventing knocking are not considered in this example; however, they can be easily incorporated by defining the spark ignition space to include the maximum allowable values.

The software package ENDYNA by TESIS [24] suitable for real-time simulation of internal combustion engines is employed. The software utilizes thermodynamic models of the gas path and is well suited for testing and development of engine electronic control units. In this example, a four-cylinder gasoline engine is used from the ENDYNA model database. The electronic control unit of the engine model including the baseline calibration of spark ignition timing with respect to steady-state engine operating points is bypassed to incorporate the self-learning controller embedded with the control algorithm. The model with the baseline engine calibration incorporates a static map for spark ignition timing (without considering knocking) corresponding to steady-state operating points.

Before initiating the first simulation, the model with the self-learning controller has no knowledge regarding the particular transient engine operation and spark timing associated with it. The model is run repeatedly over the same driving style represented by a pedal position rate to represent a situation in which the driver desires a particular acceleration deemed characteristic of his/her driving style. The belief implicit here is that if the controller can successfully capture this profile, then it will also be able to capture engine realization designated by a driver in long term.

To evaluate the efficiency of our approach in transient engine operation, the pedal position rate is chosen to represent an aggressive acceleration, as illustrated in Fig. 5. The spark ignition timing derived by the self-learning controller is shown in Fig. 6 and compared with the baseline calibration. The ignition timing of the self-learning controller results in higher engine brake torque compared with the baseline calibration indicating that the engine is able to operate closer to MBT timing. It should be emphasized
here that the baseline calibration has been optimized only for fuel economy without considering emission regulations. So, the overall improvement of the BSFC, illustrated in Fig. 7, demonstrates the efficiency of the proposed approach in deriving optimal calibration in terms of spark timing with respect to fuel consumption. To evaluate the learning efficiency of the controller, the vehicles were simulated for three additional acceleration profiles, shown in Fig. 8. In all cases, the controller specified successfully the control policy in terms of the spark ignition timing minimizing the BSFC compared with the baseline calibration, as illustrated in Figs. 9–11.

4 Concluding Remarks

We presented the theoretical framework and a control algorithm for making the engine of a vehicle into an autonomous intelligent system that can learn its optimal calibration in real time while the driver is driving the vehicle. The engine was treated as a stochastic system, engine operation was modeled as a controlled Markov decision process, and engine calibration was formulated as a sequential decision-making problem under uncertainty.

The research presented here contributes to engine calibration schemes that can capture transient engine operation associated with common driving habits, e.g., stop-and-go driving, rapid acceleration, or braking. Each individual driving style is different and rarely meets test driving conditions, e.g., calibration over steady-state operating points or vehicle speed profiles for highway and city driving. Although the application presented limited evidence of the efficiency of the proposed approach, the results look promising.

The proposed approach may significantly reduce the discrepancy between posted gas mileage estimates and actual mileage of a vehicle [25,26], while an overall improvement of emissions may be also expected [27]. Future research should explore the impact of traffic patterns, and terrain, on the general applicability of having the engine learn its optimal calibration for an individual driving style. Drivability issues that may be raised in implementing this approach in a real vehicle should also be investigated.

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References