Logical Foundations of Syllable Representations

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Objectives

- To show how model theory and logic [1] can be used to translate between different formal representations.
- To demonstrate that three popular representations of syllable structure are notationally equivalent, so long as there is a bound on syllable size.

Notational Equivalence

- Differences in notation do not necessarily entail differences in the information encoded in a particular formalism.
- Fonts are notationally equivalent, in the sense that *cat* and `cat` are the same word.
- In contrast, there are meaningful differences between bracket types in mathematical notation: $\{1, 2\}$ is an unordered set, while $(1, 2)$ is an ordered pair.

Research question: are different syllable representations notationally equivalent?

Model Theory

Model theory addresses the question of notational equivalence of representations [2, 3]. Some definitions:

- *Model* (M): a type of graph that encodes certain labels and relations over a domain of positions (elements).
- *Graph transduction* (Γ): a way of translating information from one model to another (as in [4]).
- If one can write a graph transduction from M_1 to M_2 using logic L, then M_2 is *L-interpretable* from M_1.
- If M_1 is L-interpretable from M_2 and vice versa, then the two are *L-bi-interpretable*.
- Informally, L-bi-interpretability means the two models are notationally equivalent with respect to logic L.

Key point: the weaker the logic needed for bi-interpretability, the less meaningful the differences are between the two models.

Logics and Their Expressivity

- Three logics in decreasing power are Monadic Second Order (MSO), First Order (FO), and Quantifier-Free (QF).
- MSO logic quantifies existentially and universally over *sets* of elements of the domain, in addition to quantifying over domain elements themselves.
- A MSO sentence without quantification over sets is FO.
- A FO sentence with no quantification at all is QF.
- For formal definitions of MSO and FO, see [5], [6], and [7].

Key point: QF is a very weak logic.

Three Types of Syllable Representations

The Dot Model

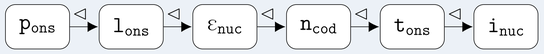
- Encodes successor (immediate precedence) relations ($\lhd$).
- Explicitly labels segments and syllable boundaries ($\bullet$).
- As in [8, 9] and others.

Figure 1: The dot word model for *plenty* [plɛnti].

The Flat Model

- Encodes successor relations ($\lhd$).
- Explicitly labels segments along with the syllable constituent each segment belongs to (onset, nucleus, or coda).
- As in [10]; see also [11].

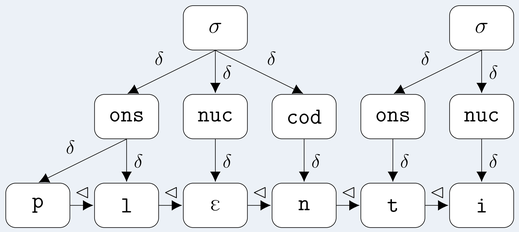
Figure 2: The flat word model for *plenty*.



The Tree Model

- Encodes successor relations ($\lhd$) and immediate dominance relations (δ).
- Explicitly labels segments, syllable constituents, and whole syllables.
- Represents tree-like structures where σ nodes dominate onset, nucleus, and coda nodes, which in turn dominate segment nodes.
- As in [12, 13] and others.

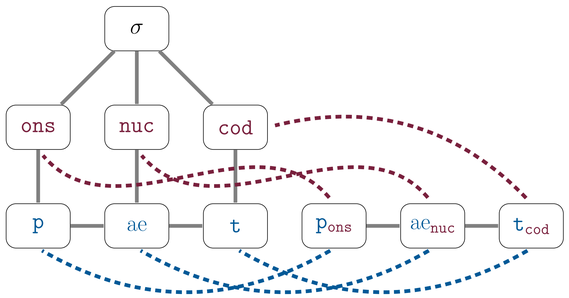
Figure 3: The tree word model for *plenty*.



The Tree-to-Flat Transduction

- Successor relations are preserved.
- Dominance relations in the input are used to deduce syllable constituents in the output.
- E.g., an input segment dominated by an onset node is labeled `ons` in the output.
- The output has no dominance relations.

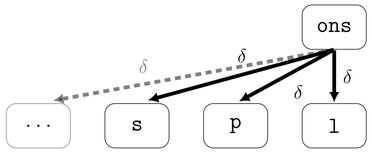
Figure 4: The Tree-to-Flat transduction for *pat*.



The Flat-to-Tree Transduction

- Successor relations are preserved.
- Dominance relations in the output must be deduced.
- E.g., a segment labeled `ons` in the input must have two copies: one with the phoneme label, and one labeled `ons` which dominates the other.
- There must be a bound on syllable size for it to be QF.
- Without a bound, the transduction would require either:
  - Infinitely many predicates to assign syllable constituents (impossible).
  - A universally quantified expression (FO).

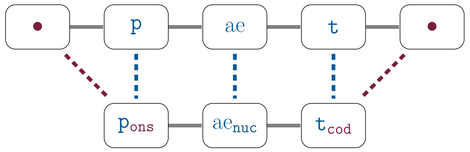
Figure 5: An onset of indeterminate length.



The Dot-to-Flat Transduction

- Successor relations are preserved, except where they must be modified to 'skip' over the dots.
- Onset and coda labels can be deduced because these positions are between a dot and a nucleus, or vice versa.
- Quantification (FO) is only needed if syllables may be of unbounded size.

Figure 6: The Dot-to-Flat transduction for *pat*.



The Flat-to-Dot Transduction

- Successor relations are preserved, except where dots intervene.
- As above, dot placement without universal quantification (FO) necessitates a bound on syllable size.

Consequences

Theorem: each transduction above is QF.

- QF-interpretability is preserved under composition.
- Thus, every pair of representations is QF-bi-interpretable.
- Thus, every constraint expressed in one representation can be expressed in another without introducing any quantifiers.

Conclusion

- Model theory addresses the question of whether some representations are more expressive than others.
- There are no constraints on syllable structure expressible in any one of the Dot, Flat, and Tree models that are not expressible in any other of the three, provided there is a bound on syllable size.

References

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