What is school choice?

Many people think of school choice as government provision of financing for private schools, but market designers who study school choice tackle a much more fundamental problem: how can we assign students to public schools so that no two students are better off if they switch schools? This sounds straightforward, but the most simple mechanisms, wherein students submit a rank-order list of the schools they’d like to attend, are fraught with problems of thinness, congestion, and safety.

What causes these problems?

**Thinness:** too few market participants.
- prevalent in rural areas
- mechanisms restricting transfers by geographic location may cause thinning

**Congestion:** students left unassigned
- too many algorithm iterations
- short rank-order lists
- too much back and forth interaction

**Safety:** incentive to misrepresent preferences
- match dependent on rank-order list

A Fictional Market

The rank-order lists below represent the preferences of students in a market generated randomly in Python (numpy seed(10)). There are nine students and four schools. \( s_i \) denotes student \( i \), \( c_j \) denotes school \( j \).

**Student Preferences**

\[
\begin{align*}
S_1 & : c_2 > c_3 > c_1 > c_4 > c_5 > c_6, \\
S_2 & : c_1 > c_2 > c_3 > c_4 > c_5 > c_6, \\
S_3 & : c_1 > c_2 > c_3 > c_4 > c_5 > c_6, \\
S_4 & : c_1 > c_2 > c_3 > c_4 > c_5 > c_6, \\
S_5 & : c_1 > c_2 > c_3 > c_4 > c_5 > c_6, \\
S_6 & : c_1 > c_2 > c_3 > c_4 > c_5 > c_6, \\
S_7 & : c_1 > c_2 > c_3 > c_4 > c_5 > c_6, \\
S_8 & : c_1 > c_2 > c_3 > c_4 > c_5 > c_6, \\
S_9 & : c_1 > c_2 > c_3 > c_4 > c_5 > c_6.
\end{align*}
\]

This rank-order list represents the priorities of schools generated in the same way. I examine the allocations generated by the Top Trading Cycles algorithm and the Deferred Acceptance algorithm as interpreted in the school choice literature.

**School Priorities**

\[
\begin{align*}
C_1 & : s_9 > s_8 > s_7 > s_6 > s_5 > s_4 > s_3 > s_2 > s_1, \\
C_2 & : s_8 > s_7 > s_6 > s_5 > s_4 > s_3 > s_2 > s_1, \\
C_3 & : s_7 > s_6 > s_5 > s_4 > s_3 > s_2 > s_1, \\
C_4 & : s_6 > s_5 > s_4 > s_3 > s_2 > s_1, \\
C_5 & : s_5 > s_4 > s_3 > s_2 > s_1, \\
C_6 & : s_4 > s_3 > s_2 > s_1, \\
C_7 & : s_3 > s_2 > s_1, \\
C_8 & : s_2 > s_1, \\
C_9 & : s_1.
\end{align*}
\]

New York method: each iteration requires a decision by both students and schools. After every iterate the decision set must be updated for both parties. This mechanism suffers from congestion and justifiable envy. Justifiable envy arises when a student attends a school ranked lower on her list than someone else’s.

In order to combat congestion in New York City, where roughly 8% of students when unassigned every year, the deferred acceptance algorithm was implemented.

**Deferred Acceptance Algorithm**

The deferred acceptance algorithm is safe and stable. Below we see the results of the DA algorithm in our fictional market.

The ultimate matching appears as follows:

**Top Trading Cycles Matching**

The mechanism design approach to school choice abstracts from normative issues of social welfare to develop methods for matching students and schools so that students have incentive to reveal their true preferences. Recent work has called into question the efficacy of these methods for accomplishing the initial normative task, including empirical studies that show “cream-skimming” and theoretical work that examines the impact of stable mechanisms on school quality. Much work remains to be done to better understand the implications of incomplete information and network effects in these markets.

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