Chapter 1: Functions

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1 Chapter 1: Functions

1.1 Functions and Function Notation
Page 17, Question 21
Evaluate: $f(-2)$,$(f-1)$,$f(0)$,$f(1)$,$f(2)$ when $f(x)$=:$4-2x$
When $x=-2$
$f(-2)=4-2(-2)$= $f(-2)=4+4$= $f(-2)=8$ $f(-2)=8$
When $x=-1$
$f(-1)=4-2(-1)$= $f(-1)=4+2$= $f(-1)=6$
$f(-1)=6$
When $x=0$
$f(0)=4-2(0)$
$f(0)=4$
When $x=1$
$f(1)=4-2(1)$
$f(1)=2$
When $x=2$
$f(2)=4-2(2)$= $f(2)=4-4$= $f(2)=0$
$f(2)=0$

Page 17, Question 26
Evaluate: $f(-2), f(-1)f(0), f(1), f(2)$ when $f(x)=5x^4+x^2$
When $(x)=-2$
$f(-2)=5(-2)^4+(-2)^2$= $f(-2)=5(16)+4$= $f(-2)=84$
$f(-2)=84$
When $x=-1$
$f(-1)=5(-1)^4+(-1)^2$
$f(-1)=6$
When $x=0$
$f(0)=5(0)^4+(0)^2$
$f(0)=0$
When $x=1$
$f(1)=5(1)^4+(1)^2$
$f(1)=6$
When $x=2$

$f(2) = 5(2)^4 + (2)^2 = f(2) = 5(16) + 4$
$f(2) = 84$

Page 18, Question 36
Evaluate: $f(x) = x^2 + x + 3$ When $f(-2), f(4)$
Part A:$f(-2) + f(4)$
$f(-2) = (-2)^2 + (-2) + 3$
$f(-2) = 5$
$f(4) = (4)^2 + (4) + 3$
$f(4) = 23$
$f(-2) + f(4) = 28$
Part B: $f(-2) - f(4)$
$f(-2) = 5$ and $f(4) = 23$
$f(-2) - f(4) = -22$

Page 19, Question 44.
Write an equation of the circle centered at $(9, -8)$ with a radius of 11.
$(x - 9)^2 + (y - (-8)^2) = 11^2$

In this section, 1.1 Function and Function Notation, I found the questions I chose were easy to complete. In this section, questions such as Evaluate $f(x) = 2x^3 + 2$ when $x = -2, -1, 0$ were present. As an example, lets take Question 21 on page 17 in more depth. When evaluating $f(x)$, we assume that all x values are real numbers. When plugging in lets say -2 into the equation $f(x) = 4 - 2x$, all x values in that problem will now have a value of -2. So solving that would be $f(-2) = 4 - 2(-2) = 8$, and our $f(x)$ value would be 8. In a function we need a x value and a y value. We know that f(x) is equal to y, meaning our y value would be -2. Therefore, if we wanted to plot that on a graph, it would be located at (8,-2).

1.2 Domain and Range
Page 34, Question 11
Determine the domain of the function.
$f(x) = \frac{9}{x-6}$
$x - 6 = 0$
$x + 6 = 0 + 6$
$x = 6$
Domain= $(-\infty, 6) \cup (6, \infty)$

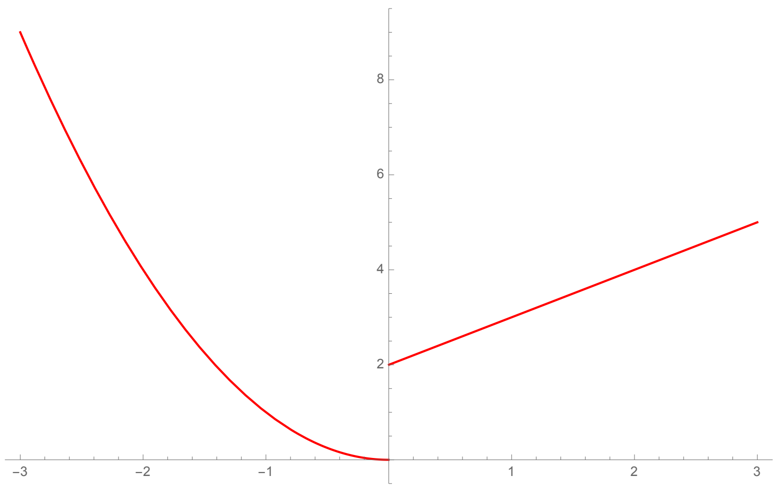
Page 34, Question 23
Given Each Function, Evaluate: $f(-1), f(0), f(2), f(4)$

$$f(x) = \begin{cases} \text{5x} & \text{if x¡0} \\ 3 & \text{if } 0 \le x \le 3 \\ x^2 & \text{if } x > 3 \end{cases}$$

$f(-1) = 5(-1) = -5$
$f(0) = 3$
$f(2) = 3$
$f(4) = (4)^2 = 16$

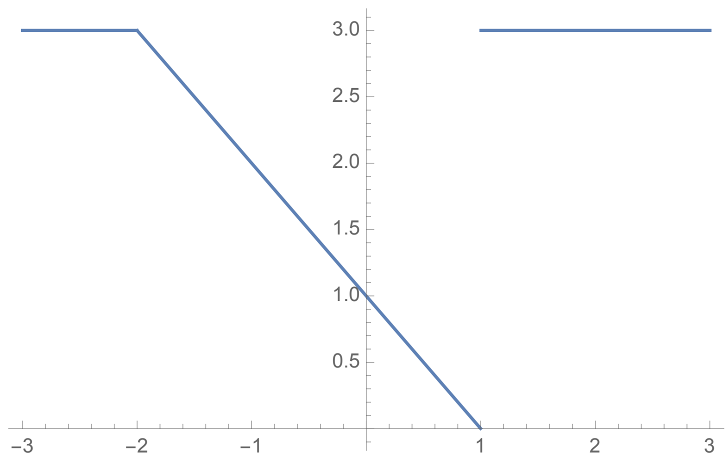
Page 35, Question 33
Sketch a Graph of each Piecewise Function

$$f(x) \begin{cases} x^2 & \text{if } x < 0 \\ x+2 & \text{if } x \leq 0 \end{cases}$$



Page 35, Question 35

$$f(x) = \begin{cases} 3 & \text{if } x < -2 \\ -x & \text{if } -2 < x \leq 1 \\ 3 & \text{if } x > 1 \end{cases}$$



In Section 1.2, I had a harder time understanding the concepts that were mentioned. I watched all the videos that were linked on the course website, and then went straight to the questions. After looking at a few, I was left feeling confused and unsure on how to properly answer them. For some of the questions, I emailed you for assistance. I now understand that to to find the domain of the function if it is a fraction the denominator cannot be equal to zero. For an example, Question 11 on page 34 had this come up. It states, "determine the domain of the function" $f(x) = \frac{9}{x-6}$. The denominator cannot equal zero, meaning that x cannot be equal to 6. It can get infinitely close, but can never reach it. Therefore, for the domain of that function it would be $(-\infty, 6) \cup (6, \infty)$. The graphing section of 1.2 was the hardest part of the section for me to complete. Originally, I was quite stuck on how to graph the function into Mathematica, however I watched videos on how to plot a graph and how to insert them into LaTeX. I did so with ease and fully understand what to do from this point forward.

1.3 Rates of Change and Behavior of Graphs
Page 48, Question 5
Find the average rate of change of each function on the interval specified.
x^2 on $[1, 5]$

1	1
5	25

Table 1: Input and Outputs

In the table above are the inputs, 1 and 5, and their corresponding outputs, 1 and 25. To determine the average rate of change in a function, finding the slope between the points $(1, 1)$ and $(5, 25)$ is necessary. To calculate the slope, divide the y values $(y2 - y1)$ by the x values, $(1, 25)$.
$m = \frac{1-25}{1-5} = \frac{24}{4} = 6$

The Average Rate of Change for the function for the function x^2 between the points $(1,1)$ and $(5,25)$ is 6.
Page 48, Question 7
Find the average rate of change of each function on the interval specified.
$3x^3$ on $[3,-3]$

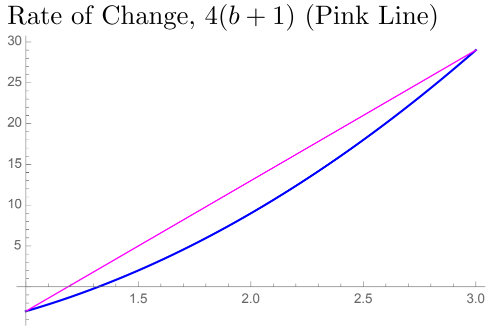
3	80
-3	82

Table 2: Inputs and Outputs

Above are the inputs 3, and -3, and their corresponding outputs, 80, and 82. To determine the average rate of change in this function, finding the slope between $3,80$ and $-3,-82$ is necessary.
$m = \frac{y2-y1}{x2-x1} = m = \frac{80-(-82)}{3-(-3)} = \frac{162}{6} = 27$
The Average Rate of Change for the function $3x^3 - 1$ between $(3,80)$ and $(-3,-82)$ is 27.

Page 48, Question 11
Find the average rate of change of each function on the interval specified. Your answers will be expressions involving a parameter (b or h).
$f(x) = 4x^2 - 7$ on $[1,b]$
$f(1) = 4(1)^2 - 7 = 4 - 7 = -3$
$f(b) = 4(b)^2 - 7 = 4b^2 - 7$
To find the Average Rate of Change between these two points, $(1,-3)$ and $(b, 4b^2-7)$ finding the slop is necessary.
$\frac{(4b^2-7),-(-3)}{b-1} = \frac{(4b^2-7+3)}{b-1} = \frac{4b^2-4}{b-1} = \frac{4(b^2-1)}{b-1} = \frac{4(b+1)(b-1)}{(b-1)} = 4(b+1)$
The Average Rate of Change for $f(x) = 4x^2 - 7$ on $[1,b]$ is $4(b+1)$

Below is an image of the line $f(x) = 4x^2 - 7$ (Blue Lines) and the Average Rate of Change, $4(b+1)$ (Pink Line)



Page 48, Question 13
Find the average rate of change of each function on the interval specified. Your answers will be expressions involving a parameter (b or h).
$h(x) = 3x + 4$ on $[2, 2+h]$

2	10
2+h	$3h + 10$

Table 3: Inputs and Outputs

Now that the outputs for $[2, 2+h]$ are known, finding the Average Rate of Change of the line requires knowing the slope of the line in between points $(2, 10)$ and $(2+h), (3h+10)$
m=$\frac{(3h+10)-10}{(2+h)-2}$= $\frac{3h}{h}$= 3
The Average Rate of Change for $h(x) = 3x + 4$ between the points $(2, 10)$ and $(2+h), (3h+10)$ is 3.

For section 1.3, I understood the process of finding the Rate of Change in between to points rather easily. Finding the Rate of Change is similar to finding the slop of a line. For example, Question 5 on page 48 asks to find the rate of change when $f(x) = x^2$ on $[1, 5]$. To answer this question, finding the output value of 1 and 5 when it is plugged into f is necessary. After that, the x and y cordinates are known. For example, $(1, 1)$ and $(5, 25)$. From there, subtracting y2 by y1 and then dividing by x2 minus x1 will give you the slope of the line between the two points, alongside the rate of change. $m = \frac{25-1}{5-1} = \frac{24}{2} = 6$. Therefore, the average rate of change between $(1, 1)$ and $(5, 25)$ when $f(x) = x^2$ would be 6. The part that i got stuck on was when the questions were asked to be solved using a parameter. I wasn't sure on how to solve using a parameter or what it was asking me to do. To help myself understand during our zoom class I asked to go over question 11 to better understand what was being asked in the question. I now understand how to solve for a parameter and find it easier than finding the average rate of change with two known numbers.
1.4 Composition of Functions
Page 60, Question 3a, and 3b
Given Each Pair of Functions, Calculate $f(g(0))$ and $g(f(0))$ When $f(x) = \sqrt{x = 4}$ and $g(x) = 12 - x^3$
3a. $f(g(0))$
$g(0)$=$12 - (0)^3 = 12$
$f(12) = \sqrt{12 + 4} = \sqrt{16} = 4$
$f(g(0)) = 4$
3b. $g(f(0))$
$f(0) = \sqrt{(0) + 4} = \sqrt{4} = 2$
$g(2) = 12 - (2)^3 = 12 - 8 = 4$
$g(f(0)) = 4$

Page 60, Question 23a, and 23b
Given each Pair of Functions, Calculate $f(g(x))$ and $g(f(x))$ when $f(x) = x^2 + 1$ and $g(x) = \sqrt{x+2}$
23a. $f(g(x))$=$f(\sqrt{x+2})$=$\sqrt{x+2}^2 + 1$=$x + 3$

$f(g(x))$=$x+3$
23b. $g(f(x))$=$g(x^2+1)$=$\sqrt{(x^2+1)+2}$=$\sqrt{x^2+3}$
$g(f(x))$=$\sqrt{x^2+3}$

Page 60, Question 25a, and 25b
Given each Pair of Functions, Calculate $f(g(x))$ and $g(f(x))$ when $f(x) = |x|$ and $g(x) = 5x+1$
25a. $f(g(x)) = f(5x+1) = |5x+1|$
$f)g(x)) = |5x+1|$
25b. $g(f(x)) = g(|x|) = |5x|+1$
$g(f(x)) = |5x|+1$

Page 61, Question 27
If $f(x) = x^4+6, g(x) = x-6$, and $h(x) = \sqrt{x}$, find $f(g(h)))$
$g(h) = g(\sqrt{x})$=$\sqrt{x-6}$
$f(\sqrt{x-6}) = \sqrt{x-6}^4+6$

For Section 1.4, I found it mostly self explanatory. Before doing any of the work, I watched the 5 videos that you posted on the website to gain some basic knowledge on the topic, and then got to work. In this section, questions such as "Calculate $f(g(x))$ When $f(x) = \frac{1}{x}$ and $g(x) = x-5$. To solve that problem, knowing that $(f \circ g)$ means f composed with g, or, "f after g" is important. To begin, we know that $g(x) = x-5$. Then, multiplying the f component by the g component would give you your answer. That would look like $f(g(x)) = f(x-5) = \frac{1}{x-5}$. If the problem looked like $g(f(x))$, simply multiplying the g component by the f component would give you the answer. Even though I found this section easy to work through, I checked the solution manual to make sure that my answers were correct.

1.5 Transformation of Functions
Page 85, Question 11
Write a formula for $f(x) = \sqrt{x}$ shifted up 1 unit and 2 units left.
$\sqrt{(x+2)}+1$

Page 87, Question 33
Starting with graph of $f(x) = 6^x$, write equation of the graph that results from:
a. Reflecting $f(x)$ about the x-axis and y-axis
b. Reflecting $f(x)$ about the y-axis. Shifting left left 2 units and down 3 units.
33a. -6^-x
33b. $(6^{x+2})-3$
Page 88, Question 39
Determine if the following functions are odd, even or neither.
a. $f(x) = 3x^4$ This function is even. if x is replaced with a $-x$, the answer would remain the same. For example:
$f(-x) = 3(-x)^4$= $81x$ $f(x) = 3(x)^4$=$81x$

b. $g(x) = \sqrt{x}$ This function is neither an even function or an odd function. A value of a square root cannot be negative, assuming that the number(s) under the radica1 are real numbers.

c. $h(x) = \frac{1}{x} + 3x$ This function is odd. if the x value was negative, the answer would no longer be positive, but negative. For example, $h(-x) = \frac{1}{-x} + 3(-x) = -\frac{1}{x} - 3x$
Page 89, Question 69
Determine the interval(s) on which the function is increasing and decreasing.
$f(x) = \sqrt{-x} + 4$
This function is decreasing by negative for when $x \leq 4$

For Section 1.5, Transformation of Functions, I found it slightly more difficult than the rest of the sections so far. I understand how to translate a function on a graph and how to write the question when it asks a question such as, "Graph $2x + 4$ with a translation of 4 to the right, and up 5. However question 39 on page 88 I got stuck on and had to look up the solutions in the solution manual. I now understand how to determine if a function is odd, even, or neither. Originally when a question asked me if a function was odd, even, or neither I didn't understand what it wanted me to do, or even what it meant. However I now understand that for instance, $f(x) - x^3$ is an odd function. That is because if x was only squared, the function would look like, $f(x) = (-x)(-x)$. The two negatives would cancel each other out. However, with x being cubed, the function would look like, $f(x) = (-x)(-x)(-x)$. The first two negatives would be canceled out, meaning that the function would then look like, $(x^2)(-x)$ Multiplying the positive x^2 and the negative x would result in a negative x^3, meaning that the function $f(x) = -3^3$ would be an odd function.

1.6 Inverse Functions
Question 13, Page 100
Find $f^-1(x)$ for each function.
$f(x) = x + 3$ (Original function)
$x = y + 3$ (Interchanging x and y variables in function)
$x - 3 + y$
$f^-1(x) = x - 3$ (Inverse function)

Question 17, Page 100
Find $f^-1(x)$ for each function.
$f(x) = (x + 7)^2$ (Original function
$x = 11y + 7$ (Interchanging the x and y variables in function)
$x + 7 = 11y$
$\frac{x-7}{11} = y$
$f^{-1}(x) = \frac{x-7}{11}$ (Inverse Function)

Question 19, Page 100:

Find domain of which $f(x)$ is one-to-one and non-decreasing, then find the inverse f restricted to that domain.
$f(x) = (x+7)^2$ (At $x > 7$, the function is increasing, meaning that the function will be one-to-one and non-decreasing at $x > -7$)
Domain: $[-7, \infty)$
Determine inverse function:
$f(x)$=$(x+7)^2$ (Original Function)
$x = (y+7)^2$ (Interchanging x and y values in function)
$\sqrt{x} = \sqrt{(y+7)^2}$
$\sqrt{x} = y + 7$
$\sqrt{x} - 7 = y$
$f^{-1}(x) = \sqrt{x} - 7$ (Inverse Function)

Question 21, Page 100:

Find domain of which $f(x)$ is one-to-one and non-decreasing, then find the inverse f restricted to that domain.
$f(x) = x^2 - 5$ (At $x > 0$ the function will be one-to-one and decreasing) Domain: $[5, \infty)$
Determine Inverse function:
$f(x) = x^2 - 5$ (Original function)
$x = y^2 - 5)$ (Interchanging x and y values in function)
$\sqrt{x} = y - 5$
$\sqrt{x} + 5 = y$
$f^{-1}(x) = \sqrt{x} + 5$ (Inverse Function)

This section, 1.6 Inverse Functions, I found very straight forward. After watching the videos linked on your website I understood how to confirm if a function was a one-to-one or not, and how to make an inverse function for a one-to-one function. For example, questions such as, "find $f^{-1}(x)$ for $f(x) = 5x+2$ For this equation, first thing to do would be interchange the x and y values in the function so that the equation would now look like $x = 5y+2$. After this, just like any function, y needs to be isolated. $x - 2 = 5y$ then divide by 5. $\frac{x-2}{5} = y$ Now that y is isolated, and the function is symmetrical on a graphical scale with $f(x) = 5x + 2$, you can write the inverse function as $f^{-1}(x) = \frac{x-2}{5}$. The hardest part for me in this section was understanding how to find the domain for the function when it was non-decreasing or non-decreasing. Simply putting the function into a graphing calculator and seeing the function on the graph fixed that situation for me.

To sum up chapter 1, Functions, I was originally overwhelmed by the amount of questions that had to be done (a total of 24). I felt as though trying to space everything out would be difficult for me to make sure that the work would be done by the due date, September 20th. Overall, the math itself was not too hard once I got the hang out it. The part that had me nervous was trying to figure out how to properly code my work into LaTex, and then learn how to plot and make graphs in Mathematica to then upload into the LaTex document. The section I struggled with the most out of chapter one, was section 1.5, primarily due to not understanding functions that were odd, even, or neither. After practicing a few of those questions and making sure I did them correctly I felt more secure while doing the questions for section 1.5 The section I found the most easy was section 1.1 or section 1.4 I remember learning how to do Composition of functions, and how to do function notation while I was in high school, so doing those sections were a breeze for me to solve the questions for them. All in all, I felt as though chapter one was an easy chapter and a good starting point for the semester.