Chapter 2: Linear Functions

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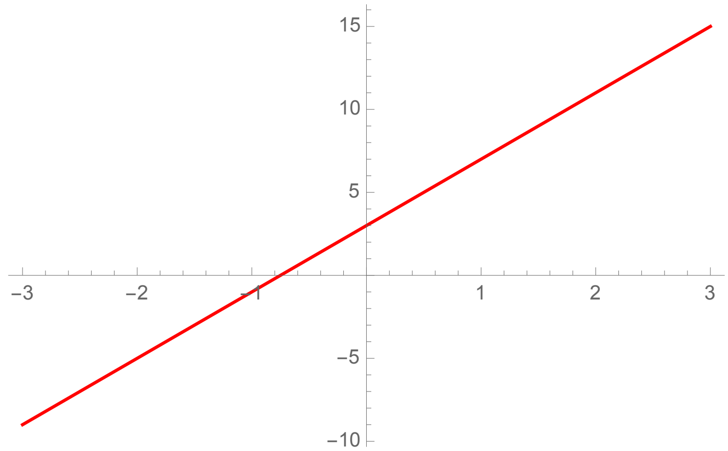
1 Chapter 2: Linear Functions

1.1 2.1 Linear Functions

Page 110, Question 1
A town's population has been growing linearly. In 2003, the population was 45,000, and the population has been growing by 1700 people each year. Write an equation, $P(t)$, for the population t years after 2003.
$P(t) = 17t + 45,000$

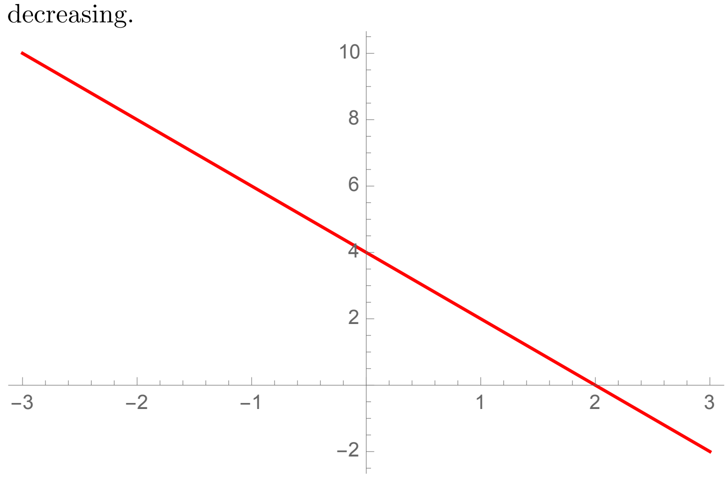
Page 110, Question 3
Sonya is currently 10 miles from home, and is walking further away at 2 miles per hour. Write an equation for her distance from home t hours from now.
$D(t) = 2t + 10$

Determine if the function is increasing or decreasing.
$f(x) = 4x + 3$
The function is increasing. 4 is the m value in $f(x) = mx + b$ The slope can be determined by the distance between two points on a graph using rise over run, or by dividing the 2 y-coordinates by the 2 x-coordinates of the points on the graph (i.e $m = \frac{y2-y1}{x2-x1}$). Since the m value in this function is a positive 4, meaning a positive rise value, the function can be determined as increasing.



Above is the graph of $f(x) = 4x + 3$. In this image there is a better understanding of what is meant by an "increasing function"(the y-value is increasing on the y-axis while the x-value is moving positively across the x-axis.)

Page 110, Question 11
$h(x) = -2x + 4$
This function is decreasing, just like in the last problem, 2 is the m value in $f(x) = mx+b$. The slope of a line can be determined by the distance in between two points on a graph using rise over run, or dividing the 2 y-coordinates by the 2 x-coordinates (i.e. $m = \frac{y2-y1}{x2-x1}$). Since the m value in this function is a negative 2, meaning a negative rise value, the function can be determined as decreasing.



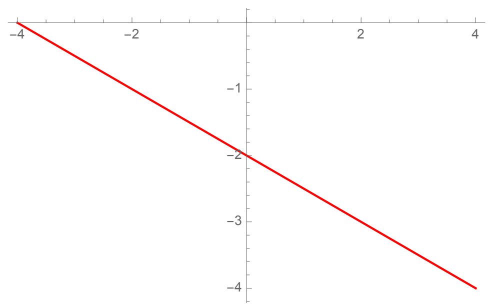
Above is the graph of $h(x) = -2x + 4$. Including this image allows a better understanding of a "decreasing function" We can see that the line has a negative slope (the y coordinate is decreasing while the x value is moving positively along the x-axis.)

Page 110, Question 17
Find the slope of the line that passes through the 2 given points.
$(2,4)$ and $(4,10)$
The slope of the line can be found by diving the 2 y-values by the 2 x-values ($m = \frac{y2-y1}{x2-x1}$).
$m = \frac{10-4}{4-2} = \frac{6}{2} = \frac{3}{1}$
The slope of the line that passes through the two points $(2,4)$ and $(4,10)$ is 3.
Page 111, Question 27
A city's population in the year 1960 was 287,500. In 1989 the population was 275,900. Compute the slope of the population growth (or decline) and make a statement about the population rate of change in people per year.
$m = \frac{275{,}900-287{,}500}{1989-1960} = \frac{-11{,}600}{29} = -400$
Between 1960 and 1989, the city's population had decreased by 400 citizens a year.

In this section, 2.1, Linear Functions, I had a very easy time understanding what the questions were asking me to solve for. I remember working on linear functions while I was in high school, and I remembered what I had to do to solve for the equation or how to write an equation for a question. For example, lets take question 1 on page 111 into more depth. For this question it states that a towns population in 2003 was 45,000. It also states that since 2003, an annual increase of 1700 people were added to the population since 2003. Since the staring value of the population was 45,000 people we know that that can be our b value in the equation $f(x) = mx + b$ Secondly, we know that since 2003, the population has been growing by 1,700 people per year. That increase is equal to our slope, which would look like $1700t$. The t is to represent the years (time) since 2003. Lastly, $f(x)$ for this equation can be $p(t)$, the p to represent population, and t to represent time. Therefore, our equation for this question would be $p(t) = 1700t + 45000$

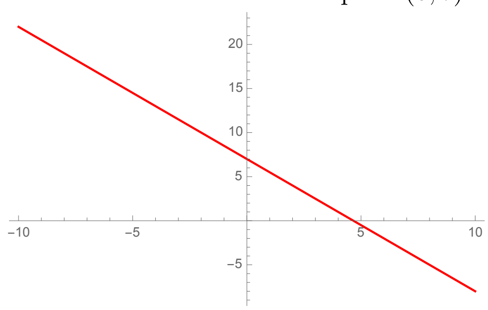
1.2 2.2 Graphs of Linear Functions

Sketch a Line with the Given Features:
Question 7, Page 125
A function with a x-intercept of $(-4,0)$ and a y-intercept of $(0,-2)$



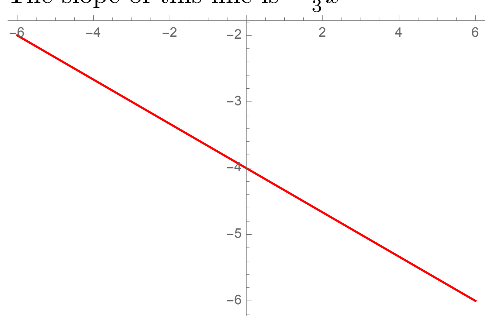
The function for this line is $f(x) = -\frac{1}{2}x - 2$

Question 9, Page 125
A line with a vertical intercept of $(0, 7)$ and a slope of $\frac{-3}{2}$



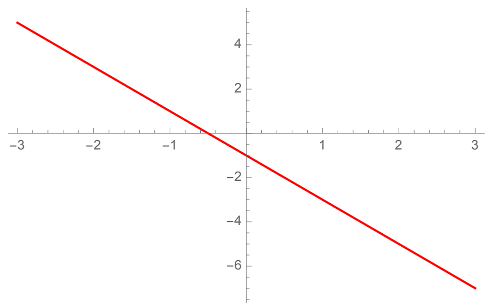
The function of this line is $f(x) = -\frac{3}{2}x + 7$

Question 11, Page 125
A Line passing through $(-6, -2)$ and $(6, -6)$
Finding the slope of the line:
$m = \frac{-6+2}{6+6} = -\frac{4}{12} = -\frac{1}{3}$
The slope of this line is $-\frac{1}{3}x$



The function of this line is $f(x) = -\frac{1}{3}x - 4$

Sketch a Graph of Each Equation
Question 13, Page 125
$f(x) = -2x - 1$



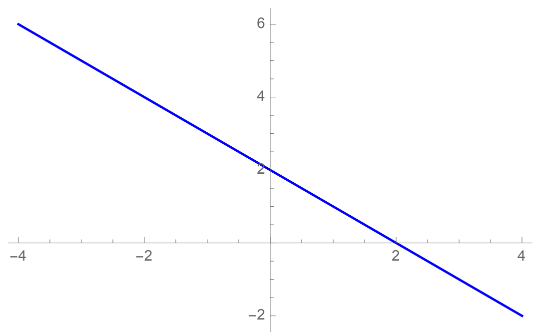
Question 23, Page 126
If $g(x)$ is the transformation of $f(x) = x$ after a vertical compression by $\frac{3}{4}$, a shift left by 2, and a shift down by 4
$g(x) = \frac{3}{4}(x + 2) - 4$

23A) Write an equation for $g(x)$
$g(x) = \frac{3}{4}x - \frac{5}{2}$
As we learned from chapter 1, Section 5, Transformation of Functions, in order to write an equation for $g(x)$ we have to understand how Transformation of functions work, and how to write it. First, as we know, there is a vertical compression of $\frac{3}{4}$ and that the there is a shift to the left by 2, and a shift down by 4. The function for transformations is $f(x) = af(b(x + c)) + d$. The vertical compression is equal to the a value in this equation. Any horizontal shifts, in this case moving left by 2 units, is equal to the c value in the equation. Lastly, Vertical shifts, which is 4 units down in this equation, is equal to the d value. Now that all the values are known, plugging them into the equation is rather easy. $f(x) = \frac{3}{4}(x + 2) - 4$. This however can be simplified further. After simplifying, the equation for $g(x)$ would be $g(x) = \frac{3}{4}x - \frac{5}{2}$.

23B) What is the slope of this line?
The slope of the line $\frac{3}{4}x - \frac{5}{2}$ is $\frac{3}{4}$.

23C) Find the vertical intercept of this line.
The vertical intercept would be $-\frac{5}{2}$ We know this since it is equal to the b value in the equation $f(x) = mx + b$.

Question 29, Page 126
Find the horizontal and vertical intercepts of each equation.
$f(x) = -x + 2$
Vertical Intercept: $(0, 2)$
Horizontal Intercept: $(2, 0)$
If this equation was plugged into Mathematica, seen below, the Vertical and Horizontal Intercepts are seen to be located at $(0, 2)$ and $(2, 0)$.



Question 35, Page 127
Given below are descriptions of two lines. Find the slopes of Line 1 and Line 2. Is each pair of lines parallel, perpendicular or neither?

Line 1: Passes through $(0, 6)$ and $(3, -24)$.
Line 2: Passes through $(-1, 19)$ and $(8, -71)$.
To begin, using the equation, $m = \frac{y2-y1}{x2-x1}$ would help find the slope for both lines.
Line 1: $m = \frac{-24-6}{3-0} = -\frac{30}{3} = -10$.
The slope of the line is -10.
Line 2: $m = \frac{-71-19}{8+1} = -\frac{90}{9} = -10$.
The slope of Line 2 is -10.
Both Line 1 and Line 2 have the slope -10 meaning that the lines are parallel.

Question 41, Page 127
Write an equation for a line parallel to f $(x) = -5x - 3$ and passing through the point $(2, -12)$.
Because the two lines will be parallel, we know that they will have to have the same slope, meaning that Line 2 will have a slope of -5. Secondly, if we plug the known x value and y value into the equation $f(x) = mx + b$ we would get $-12 = (-5)(2) + b$. All that is left to do is to solve for b.
$-12 = (-5)(2) + b = -12 = -10 + b = -2 = b$.
The equation for a line to be parallel to the line $f(x) = -5x - 3$ and passing through the point $(2, -12)$ would be $f(x) = 5x - 2$.

Similar to section 2.1, I found section 2.2, Graphs of Linear Functions, to be rather easy and a review of what I learned while I was in high school. I found myself flying through the questions, and was done in the matter of 15 minutes (my work on paper before coding it into LaTeX). All in all, reviewing and relearning certain topics helps the brain embed the process and the understanding better, allowing us to remember how to solve problems like this for years to come. For a better understanding of this section, reviewing how to solve Question 13 on page 125 would be helpful. The question states, "Sketch a graph of the line $f(x) = -2x - 1$." To solve, begin with the y-intercept. As seen in the equation, the y intercept, or known as the b value in the equation $f(x) = mx+b$, is -1. The y intercept would be placed at $(0, -1)$ after this, plot the rest of the points on the graph, and draw the line. We know that the slope of the line is $-2x$ meaning that it is moving down 2 units for each unit moving to the right. Connecting the y intercept, and the new point, $(1, -4)$ would give you the graph of the line as seen in the graph I used for that equation on page 4 of my project.

1.3 2.3, Modeling With Linear Functions

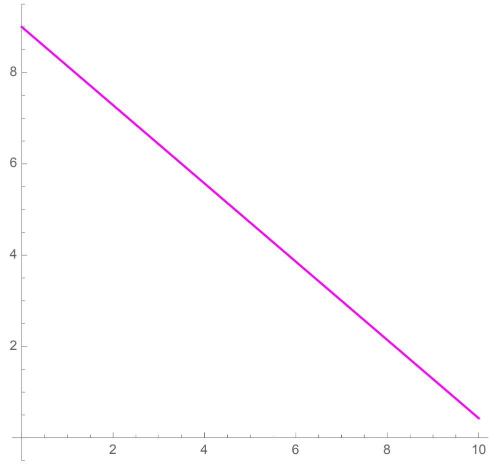
Question 1, Page 137
In 2004, a school population was 1001. By 2008 the population had grown to 1697. Assume the population is changing linearly.
1a. How much did the population grow between the year 2004 and 2008?
$1697 - 1001 = 696$
The population of the school population grew by 696 students between 2004 and 2008.
1b. How long did it take the population to grow from 1001 students to 1697 students?
$2008 - 2004$=4
It took 4 years for the population of the school to grow by 696 students.
1c. What is the average population growth per year?
This can be solved by using $m = \frac{y2-y1}{x2-x1}$
$f(x) = \frac{1697-1001}{2008-2004}$=$\frac{696}{4}$ =174
The Average Rate of Growth per year is 174 Students.
1d. What was the population in the year 2000?
Assuming that the Average rate of Growth was the same in 2000 as it is in between 2004-2008, the population would be 305 Students. This can be solved knowing that the population of students in 2004 was 1001, and the average rate of growth in population is 174, you could multiply 174 by 4, which is 696, and then subtract that number by 1001, you would get the amount of students in the year 2000.
1e. Find an equation for the population, P, of the school t years after 2000.
$p(t) = 174t + 305$
1f. Using your equation, predict the population of the school in 2011.
Substitute 11 into all t values in the function to find your answer. $p(11) = 174(11) + 305$= $p(11) = 1914 + 305$= $p(11) = 2219$. The amount of students in 2011 was 2,219.

Question 3, Page 137 A phone company has a monthly cellular plan where a customer pays a flat monthly fee and then a certain amount of money per minute used on the phone. If a customer uses 410 minutes, the monthly cost will be 71.50 dollars. If the customer uses 720 minutes, the monthly cost will be 118 dollars.
3a. Find a linear equation for the monthly cost of the cell plan as a function of

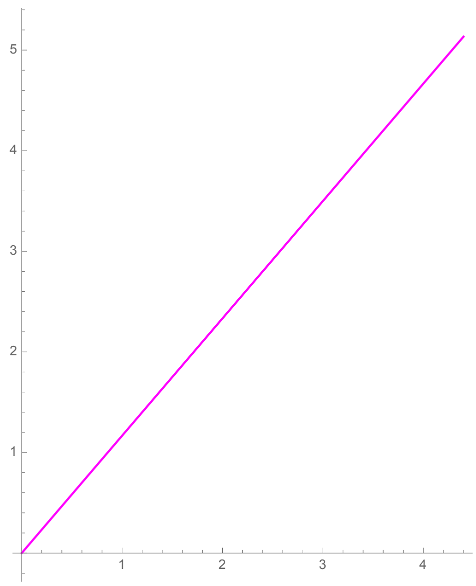
x, the number of monthly minutes used.
The first step I took was finding the slope, and after that finding, the y intercept.
$(410, 71.50)$, $(720, 118)$
$m = \frac{118-71.50}{720-410} = \frac{46.50}{310} = .15x$
For this problem, the slope will be $.15x$ (x being the amount of minutes used).
Secondly, finding the y-intercept, or the flat monthly fee, Plugging in any of the values, I used $(720, 118)$, will give the the y intercept.
$(118) = .15(720) + b = 118 = 108 + b = 10 = b$
The flat monthly fee is 10 dollars.
With this information we can form the equation for this question. $f(x) = .15x + b$
3b. Interpret the slope and vertical intercept of the equation.
if x is equal to the amount of minutes, the customer is spending 15 cents per minute, with a monthly flat fee of 10 dollars.
3c. Use your equation to find the total monthly cost if 687 minutes are used.
Substitute 687 minutes into the x value of the equation, $f(x) = .15x + 10$.
$f(x) = .15(687) + 10 = 103.05 + 10 = 113.05$
If 687 minutes were used the total bill would be 113.05 dollars.

Question 13, Page 137 Find the area of a triangle bounded by the y axis, the line $f(x) = 9 - \frac{6}{7}x$, and the line perpendicular to $f(x)$ that passes through the origin.

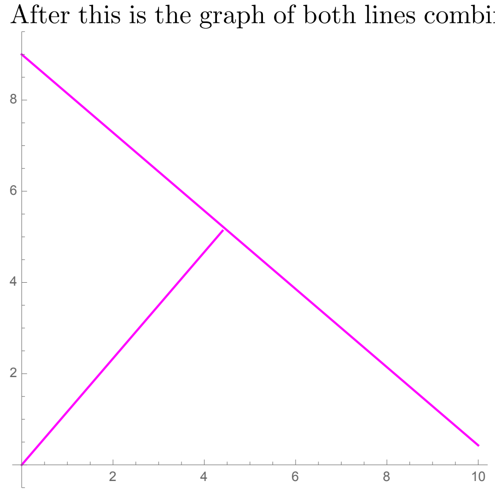
Here is were I will insert my graphs for this problem.
Below is the graph for the line $f(x) = 9 - \frac{6}{7}x$.



The graph below is for the line $g(x) = \frac{7}{6}x$.



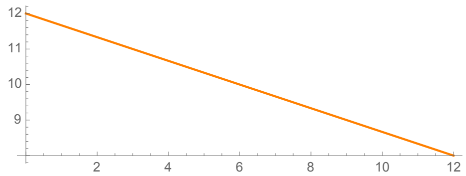
After this is the graph of both lines combined, showing were the two lines meet.



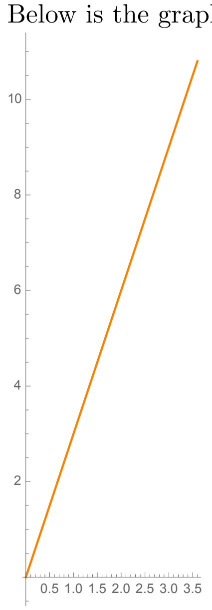
Question 14, Page 139
Find the area of a triangle bounded by the x axis, the line $f(x) = 12 - \frac{1}{3}x$, and the line line perpendicular to $f(x)$ that passes through the origin.
Just like the last question, I used Mathematica to solve, however I will explain what I did in words before showing my graphs.It originally asks us to find the area of the triangle bounded by the y axis. We know that the equation to find area is $a = \frac{1}{2}bh$. We already know the height of this triangle is 12. Now we just have to find the base. In order to find the base, we need to find the point were $f(x) = 12 - \frac{1}{3}x$ and the line perpendicular to $f(x)$ that passes through the origin meet. We know that anything perpendicular to something has to be the opposite-reciprocal to the line we already know. Therefore, the equation of that

line would be $g(x) = 3$ Since it passes through the origin, there is no y-intercept for this line. After this, finding the point on the graph is rather easy, they meet at points $(3.6, 10.8)$. Now that we know the base is 3.6, we can use the formula $a = \frac{1}{3}(3.6)(12)$= 43.2. The area of this triangle is 43.2 units^2 (Since the unit is unknown for this problem, using unit would work).

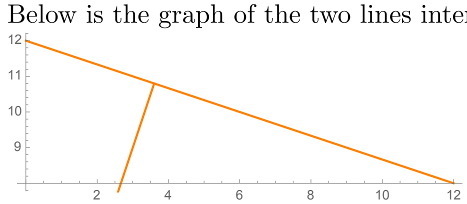
After this, I will show the graphs for this equation.
Below is the graph of the line $f(x) = 12 - \frac{1}{2}x$.



Below is the graph of the line $f(x) = 3x$



Below is the graph of the two lines intersecting at points $(3.6, 10.8)$.



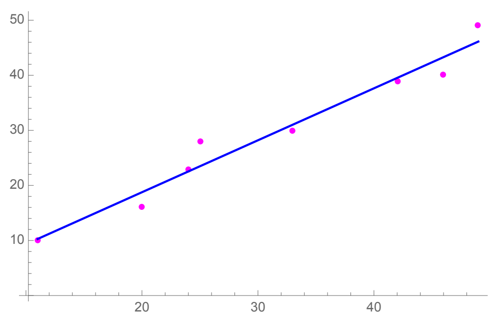
In this section, 2.3, Modeling with Linear Functions, I understood how to do the first two required questions easily. However, when it came to the third required question, 11, I understood what it was asking me to do, but I wasn't sure how to graph it into Mathematica. However, you covered how to solve this problem at our class meeting on Wednesday the 23rd. That helped me tremendously, and I flew through the questions quickly once I understood how to solve them. Because of the depth I went into while solving the problems, I will not be covering how to work through them in this section, simply because I do not want to repeat myself completely on something I have already explained.

1.4 2.4 Fitting Linear Models to Data

Question 1, Page 147
The following is data for the first and second quiz scores for 8 students in a class. Plot the points, then sketch a line that fits the data.

11	20
20	16
24	23
25	28
33	30
42	49
46	40
49	49

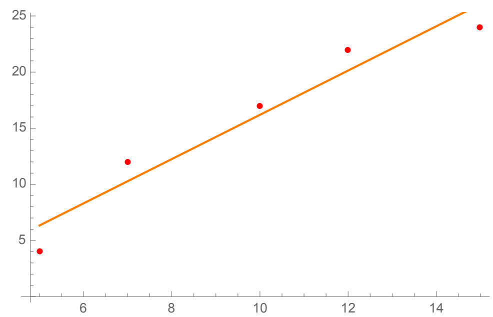
Table 1: Question 1



Question 3, Page 147
Based on each set of data given, calculate the regression line using your calculator or other technology tool, and determine the correlation coefficient.

5	4
7	12
10	17
12	22
15	24

Table 2: Question 3



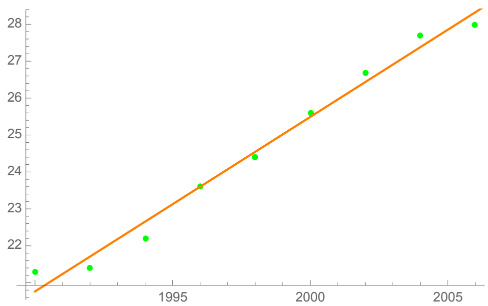
Regression Line= $3.519x - 1.97$
Correlation Coefficient= $R = .966$

Question 7, Page 148
A regression was run to determine if there is a relationship between hours of TV watched per day (x) and number of situps a person can do (y). The results of the regression are given below. Use this to predict the number of situps a person who watches 11 hours of TV can do.
$y = ax + b$
$a = -1.341$
$b = 32.234$
$r2 = 0.803$
$r = -0.896$
Plug in the known values into $y = ax + b$.
$y = (-1.341)(11) + (32.234)$= $y = -14.751 + 32.234$= 17.483
A person that watches 11 hours of tv a day can do approximately 17.5 push-ups.

Question 13, Page 157
The US census tracks the percentage of persons 25 years or older who are college graduates. That data for several years is given below. Determine if the trend appears linear. If so and the trend continues, in what year will the percentage exceed 35 percent?

1990	21.3
1992	21.4
1994	22.2
1996	23.6
1998	24.4
2000	25.6
2002	26.7
2004	27.7
2006	28
2008	29.4

Table 3: Question 13

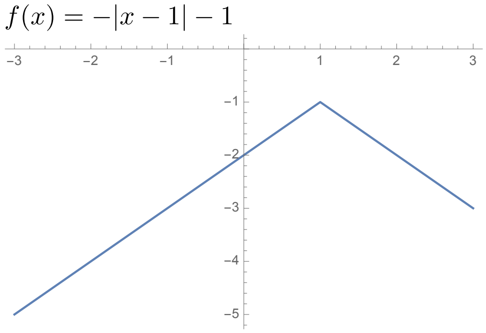


As we can see, the trend is linear. The percentage of graduates would be at 35 percent at the very end of 2019.

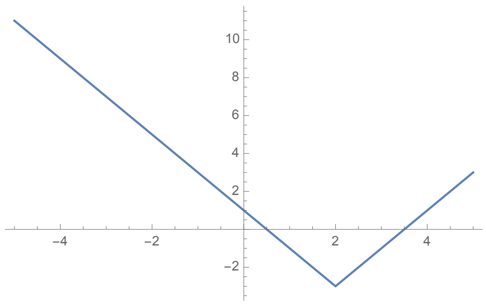
For section 2.4, Fitting Linear Models to Data, I had issues trying to graph the regression line in Mathematica to upload my graph into the document. However, you went over it in class and afterwards I was able to graph the problems into Mathematica without hassle. Before this section, I never heard of a regression line and was confused on what was meant by it. I watched some of the videos linked for that section and found them helpful.

1.4.1 2.5 Absolute Value Functions

Question 5, Page 156
Sketch a graph of each function
$f(x) = -|x-1| - 1$



Question 9 Page 156
$f(x) = |2x-4| - 3$



Question 11, Page 156
Solve each of the equations
$11 = |5x - 2|$
$5x - 2 = 11 = 5x = 13 = x = \frac{13}{5} = 2.6 = x$

Question 17, Page 157
Find the horizontal and vertical intercepts of each function.
$f(x) = 2|x + 1| - 10$
$2|x + 1| - 10 = 0 = |x + 1| = 5$
$x + 1 = 5\ x = 4\ x + 1 = -5\ x = -6$
The horizontal Intercepts are $(4, 0)$ and $(-6, 0)$

Question 19, Page 157
$f(x) = -3|x - 2| - 1$
$-3|x - 2| - 1 = 0 = |x - 2| = -\frac{1}{3}$
Since the absolute value of the quantity of the function is negative, there are no horizontal intercepts.

Question 21, Page 157
Solve each inequality
$|x + 5| < 6$
$x + 5 = 6\ x = 1$
$x + 5 = -6\ x = -11$
$-11 < x < 1$

Question 25 Page 157
$f(x) = |3x + 9| < 4$
$3x + 9 = 4 = 3x = -5 = x = -\frac{5}{3}$
$3x + 9 = -4 = 3x = -13 = x = -\frac{13}{3}$
$x = -\frac{13}{3}$ or $x = -\frac{5}{3}$
$-\frac{13}{2} < x - \frac{5}{3}$

In this section, 2.5, Absolute Value Functions, I remembered how to solve each problem for the most part. I got stuck on question 5 on page 156 because I was unsure on how to graph the equation into mathematica, however you explained it to me in class on September 28th. After that, I understood how to solve all of the questions that I had to work out in chapter 2.5. To take a deeper look at the section, lets take question 5 into more depth. As previously stated, I was stuck on that equation to begin with before you explained it to the class. Question 5 is,"Sketch a graph of each function, $f(x) = -|x-1| - 1$" Similar to transformations in chapter one, the b value, in this case it is -1, will determine what the y coordinate is (1 below zero). Secondly, inside of the absolute signs, $x-1$ indicates that the vertex will be places horizontally on (1,0). People tend to get confused as to why is would be placed to the right of the origin, however it is because you can think of $x-1$ as how far away from 0 you are. For example, $x-1=0$= $x=1$. Combining the y coordinate and the x coordinate to get $(1,-1)$ Since the absolute value is being multiplied by a negative, it will be opening downwards, as shown in the graph for that question on page 11.

This chapter I think was the same difficulty level as chapter 1. The section I found the easiest to solve was definitely 2.1. I remembered it from high school, and solved the problems with ease. However, I did struggle a lot in 2.4. I was unsure on how to graph the problems into Mathematica, and even after you explained how to do it, I was still having issues. Even though you sent us a template, I personally like trying to solve it on my own. Section 2.4 Covered regression lines which I didn't learn about in high school whatsoever, so that section as a whole was new information for me. Unlike chapter 1, this time around I was not intimidated by the amount of questions you assigned us. Personally, finishing this project did not take me very long, even if I didn't work on it for a day or two (I work consistently I promise). Overall, this chapter was mostly a review for me and if the next chapter is similar, I should be all set.